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CONSTRUCTING OF REGIONAL MODEL OF IONOSPHERE PARAMETERS

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Aim. The widespread use of global navigation satellite systems (GNSS) has led to the development of new methods designed to determine and accumulate the index of ionosphere ionization (VTEC). Using these data it is possible to significantly improve the accuracy and reliability of determining the coordinates of the observation point. Therefore, the task of constructing a model of ionization index is relevant. **Method.** To construct a spatial model, we used the spherical Legendre functions of the first kind of real order with integer degree as a basic system of functions. We found the magnitude of order using the Sturm-Liouville theory since it depended on the size of the investigated region. Such system of functions form two orthogonal systems of functions in the region under study (the sphere segment), but does not have recurrence relations between functions, therefore, it is necessary to use function expansion in a hypergeometric series to find them. Also in order to find unknown coefficients of the model it is necessary to use the Tikhonov regularization parameter, since the matrix of normal equations will not be stable. For calculating the time model of the ionosphere the coefficients of different spatial models were expanded in series of power polynomials. **Results.** Based on the data of the ionization parameter values obtained of 19 permanent stations of the ZAKPOS network using the Trimble Pivot Platform software, the spatial-temporal model of this parameter was constructed using the Legendre spherical functions up to the 3rd order as well as with power polynomials up to 3rd order. The standard deviation between the measured and model values of the VTEC parameter does not exceed 1*TECU*. **The scientific novelty and practical significance.** We developed algorithm for construction of the space-time model of the ionosphere parameter. A ionosphere model of high resolution is obtained, which can be used to solve geodetic tasks in order to provide the necessary accuracy in determining the coordinates of the point, as well as to study and forecast the space weather.

Key words: ionosphere parameters, TEC, VTEC, spatial-temporal model, Legendre spherical functions, ionization index, system of functions.

Introduction

At present times the monitoring of the Earth's atmosphere has moved to a new methodological and technological level. This is due to the widespread use of GNSS as well as their modernization and development of new methods for determining the parameters of the Earth's ionosphere. During the work of active reference, the GNSS-station results are accumulated for the definition of the ionization index TEC (Total Electron Content). These data not only reflect the state of the ionosphere at the time of observation but also are an essential tool for improving the accuracy and reliability of determining the coordinates of the observation place [Yankiv-Vitkovska L. M., 2013].

For analyzing and using the data of the ionization index TEC we needed to create a regional model of this parameter of the ionosphere which reflected it with the necessary precision [Yankiv-Vitkovska L. M., 2014; Yankiv-Vitkovska L. M. et al., 2015]. Many regional models of the ionosphere have been constructed [Abdelazeem M. et al., 2017; Gao Y. et al., 2002; Schaer S., 1999]. Since all the data of the TEC ionization index are reduced to the sphere, for constructing such model we used the Legendre spherical functions and imposed boundary conditions on them in accordance with the form of part of the sphere in which they are located [Dzhuman B. B., 2013].

Aim

The aim of this work is to construct a high-precision regional spatio-temporal model of the mean integral value of electron concentration in the ionosphere to improve the accuracy of geodetic measurements by introducing an appropriate correction in them. To calculate such a model we chose the Legendre spherical functions of the real order with the integer degree as the basic system of functions [Hobson E. W., 1931], which form the non-orthogonal system of functions on the sphere segment.

Initial data

As a result of processing the geodetic measurements data, namely pseudo-distances from permanent stations to satellites, we can find the parameter of the ionosphere TEC. It enables not only to improve the accuracy of geodesic measurements by introducing an appropriate correction in them, but also to study the ionosphere itself [Yankiv-Vitkovska L. M., 2013; Yankiv-Vitkovska L. M. et al., 2015]. In order to study and operate the values of the parameters of the ionosphere we propose to construct a model based on the known vertical values of the ionosphere parameter – VTEC. Such values are obtained after processing geodetic measurements from the network of permanent stations ZAKPOS using the Trimble Pivot Platform software at 19 stations [Yankiv-Vitkovska L. M. et al., 2016] for four epochs: 0, 6, 12 at 18 hour intervals from January 16, 2017. Table 1 shows the name and coordinates of these stations in the IGb08 system.

The scheme of stations placement is shown in Fig. 1.

Table 1

Name and coordinates of permanent stations

Name	*X, m*	*Y, m*	*Z, m*
SULP	3765296.774	1677559.389	4851297.523
JRSL	3789362.976	1582622.631	4864201.687
MIZ1	3881155.006	1687860.780	4756281.930
RAH1	3895973.878	1751354.220	4721320.921
WLDW	3643581.000	1588599.400	4971661.200
HMEL	3706268.952	1885665.826	4820450.263
CRNI	3824750.141	1860009.032	4737620.488
MUKA	3909873.985	1637331.318	4750029.398
SKON	3673288.267	1884882.203	4845675.670
SARN	3569794.434	1788894.752	4957203.650
CHTK	3773817.572	1823744.106	4791846.472
FRAN	3815148.231	1755911.792	4784539.637
VRHV	3871108.919	1787573.460	4728428.526
STRY	3812850.798	1687385.965	4810814.650
SHPT	3645297.429	1863387.523	4874829.701
DUNA	3748012.720	1898753.736	4783151.029
RIVN	3635935.526	1794403.437	4907317.994
HUS1	3912065.379	1686037.865	4731343.980
TER2	3737869.749	1791704.220	4831815.981

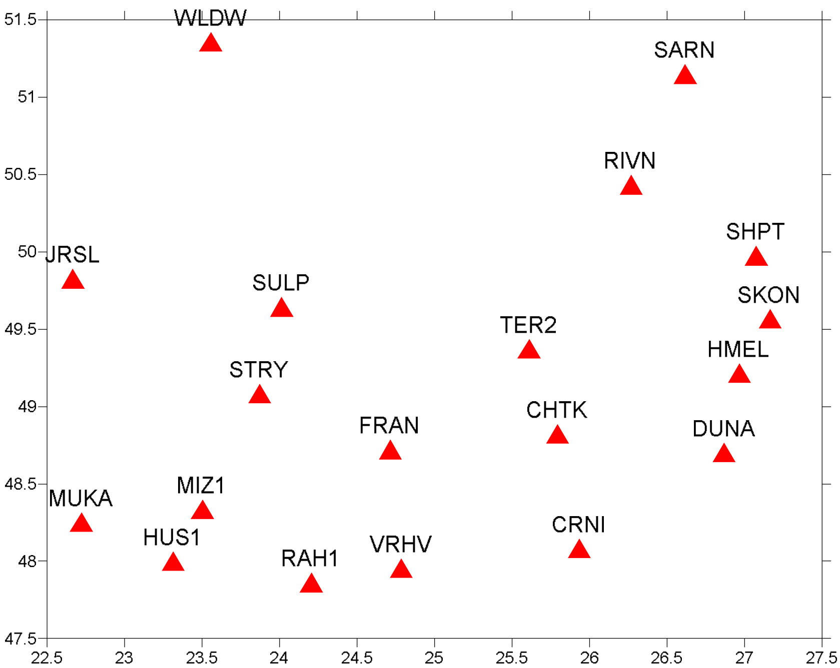

Fig. 1. The scheme of stations placement

Method

Legendre spherical functions with real eigenvalues

We received a daily solution of VTEC values at the above-mentioned stations. To calculate the model of the VTEC we decided to use the spherical cap harmonic analisys method (SCHA) [Haines G. V., 1985; Dzhuman B. B., 2014]. This method involves finding the eigenvalues *n* and *m* of Legendre spherical functions for the segment of sphere ($q \le q_0, r = 1$) using the Sturm-Liouville problem [Ohashi M., 2015]. As well known, in the spherical coordinate system the Laplace operator has the following form

$$\Delta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin q}\frac{\partial}{\partial q}\left(\sin q \frac{\partial}{\partial q}\right) + \frac{1}{r^2 \sin^2 q}\frac{\partial^2}{\partial l^2}, \qquad (1)$$

where r, q, λ is the spherical coordinate. According to the Sturm-Liouville theory any differential equation of the second order can be reduced to the form [Haines G.V., 1988]

$$-\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + q(x)y = vw(x)y, \qquad (2)$$

where *y* is a function of the variable *x*. In (2) functions $p(x) > 0$, $q(x)$ i $w(x) > 0$ are considered to be known. In the simplest cases all coefficients are continuous on a finite closed interval [*a*,*b*], and *p* has continuous derivatives. Function *y* is called the solution of the equation (2), and *y* is differentiated into the intercept [*a*, *b*] and satisfies equation (2) at each point [*a*, *b*]. An unknown function *y* usually satisfies some boundary conditions at points *a* and *b*. Function $w(x)$ is called the weight function or the density function.

Thus, finding the eigenvalues v for which there is a nontrivial solution of the equation (2), that satisfies the boundary conditions, is part of the Sturm-Liouville problem. The Sturm-Liouville problem is called regular if $p(x)$, $w(x) > 0$ and also $p(x)$, $p'(x)$, $q(x)$ and $w(x)$ are continuous functions on a finite interval [*a*,*b*] and have separated boundary conditions [Kelvin L., 1896]

$$a_1 y(a) + a_2 y'(a) = 0 \qquad (a_1^2 + a_2^2 > 0), \quad (3)$$

$$b_1 y(b) + b_2 y'(b) = 0 \qquad (b_1^2 + b_2^2 > 0). \quad (4)$$

It is easy to see that the differential equation of the Legendre spherical functions according to form (2) which are:

$$\frac{d}{dm}\left[(1 - m^2)\frac{dP_{mn}(m)}{dm}\right] +$$

$$+\left[n(n+1)-\frac{m^2}{1-m^2}\right]P_{mn}(m)=0, \qquad (5)$$

where $m=\cos q$.

The boundary conditions of the equation (1) along the longitude for the whole sphere are of the form

$$V_{nm}(r,q,l)=V_{nm}(r,q,l+2p); \qquad (6)$$

$$\frac{\partial V_{nm}(r,q,l)}{\partial l}=\frac{\partial V_{nm}(r,q,l+2p)}{\partial l}. \qquad (7)$$

Conditions (6) and (7) are the limit value of *m* to the real and integer.

However, for $q_0 \neq \pi$ the function *V* in q_0 as well as its derivative in q must be arbitrary [Schmidt M., 2007]:

$$V(r,q_0,l)=f_1(r,l); \qquad (8)$$

$$\frac{\partial V(r,q_0,l)}{\partial q}=f_2(r,l). \qquad (9)$$

Condition (8) is satisfied only if the eigenvalue $n_k(m)$ (*k* is index) assumes a value that the derivative from *V* in the zenith is equal to zero at the boundary of the segment

$$\frac{dP_{n_k(m)m}(\cos q_0)}{dq}=0. \qquad (10)$$

Condition (9) is performed when the eigenvalue n_k assumes such value that the function *V* is equal to zero at the boundary of the segment

$$P_{n_k(m)m}(\cos q_0)=0. \qquad (11)$$

Such Legendre spherical functions of real degree and integer order form two orthogonal systems of functions on a sphere segment.

Modeling of the ionosphere parameter

To use the above-described modeling method we first we recalculated the coordinates of the stations into a spherical coordinate system and transformed them into a sphere segment. After such a transformation it is found that the optimal size of the spherical segment is $q_0=2.5°$. For finding eigenvalues $n_k(m)$ it is necessary to find zeros of functions [Marchenko A., 2015]

$$\tilde{F}(n_k,m,m_0)=0; \qquad (12)$$

$$n_k m_0 \tilde{F}(n_k,m,m_0)-$$
$$-(n_k-m)\tilde{F}(n_k-1,m,m_0)=0; \qquad (13)$$

where $m_0=\cos 2.5°$, and $\tilde{F}$ is a symbol of the hypergeometric series, namely

$$\tilde{F}(n,m,m)=F\left(m-n,m+n+1,m+1,\frac{1-m}{2}\right), \qquad (14)$$

where $m=\cos q$.

Results

In order to compare the above-described algorithm with other methods first we constructed the spatial model of the ionosphere using the power polynomials to the 1st order in the epoch 18th hour on January 16, 2017. In this case the equation of the model has the following form

$$VTEC_m=aB+bL+c, \qquad (15)$$

where *a, b, c* are unknown coefficients. Finding the coefficients *a, b, c* using the least squares method we constructed respective map of model values of the ionosphere index VTEC. This map is shown in Fig. 2. Also the differences between the input VTEC values and the model values were found. These differences are shown in Fig. 3. The average square error of these differences reaches 1.201 *TECu*. The average square error of model reaches 1.324 *TECu*. The obtained accuracy is absolutely not sufficient to solve the problems posed in this paper. In addition, from the functional analysis it is well known that the power polynomials are at least closed, but are not a basic function system. Therefore, it is obviously necessary to use a different algorithm that would allow us to obtain a model with an average square error less than 1 TECu. We use for this a technique built on the using of spherical Legendre functions with real eigenvalues.

Since we have measurements at 19 stations it is advisable to compute a model up to 3 degree/order. Therefore we found the zeros of functions (12) and (13) up to the third order (see Table 2).

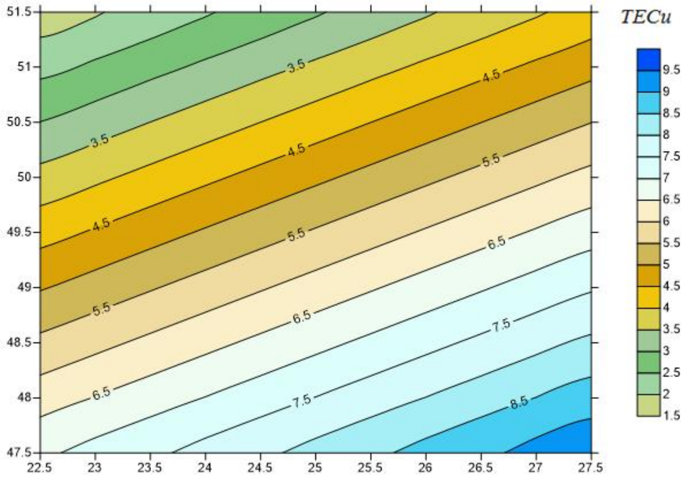

Fig. 2. Map of model values of the VTEC ionosphere index, constructed using power functions

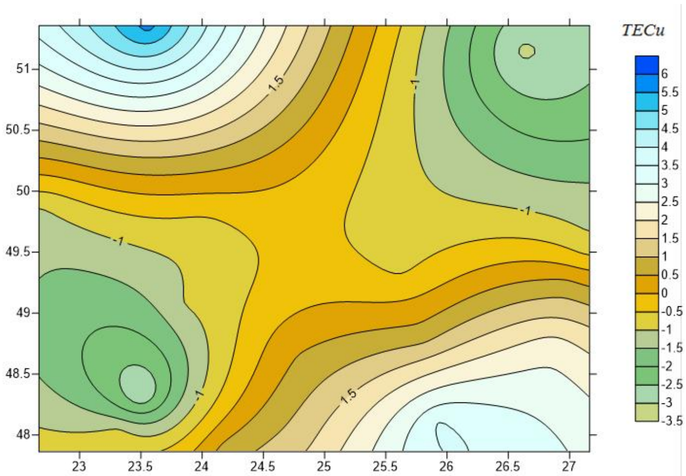

Fig. 3. Differences between input VTEC values and model values

Table 2

Eigenvalues $n_k(m)$ for the segment of the sphere $q_0 = 2.5°$

k/m	0	1	2	3
0	0.0000			
1	54.6138	41.7054		
2	87.3177	87.3177	69.5111	
3	126.0105	121.6901	117.2051	95.8028

The model equation will look like [Liu J., 2014]

$$VTEC(q, l) = \sum_{k=0}^{3} \sum_{m=0}^{k} P_{n_k(m)m}(\cos q) \cdot$$

$$\cdot (a_{km} \cos m l + b_{km} \sin m l), \qquad (16)$$

where a_{km} and b_{km} are unknown coefficients, and $P_{n_k m}$ can be found from the hypergeometric series

$$P_{n_k m}(m) = (1 - m^2)^{\frac{m}{2}} \cdot F\left(m - \right.$$

$$\left. - n_k, n_k + m + 1, 1 + m, \frac{1-m}{2} \right) \qquad (17)$$

Since the number of measurements is greater than the number of unknown coefficients, we solve the equation (16) using the least squares method introducing the Tikhonov regularization parameter [Tykhonov A.N., 1979]

$$V^T V + a X^T X \to \min, \qquad (18)$$

where V is the correction vector, vector X is the vector of unknown coefficients, and a is Tikhonov regularization parameter.

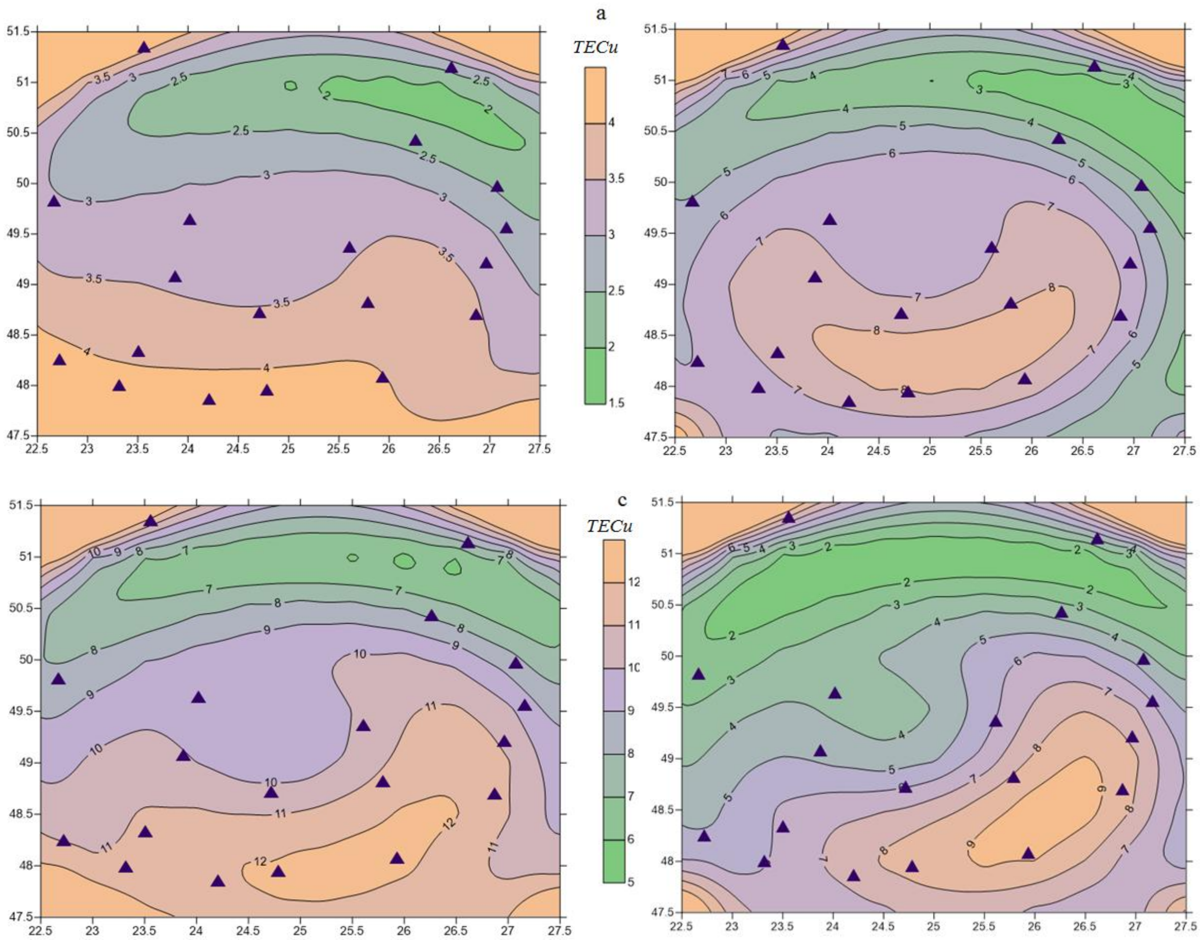

Fig. 4. Model of values of the VTEC parameter
at a – 0 o'clock, b – 6 o'clock, c – 12 o'clock, d – 18 o'clock

Using the above algorithm we constructed four models of the values of parameter VTEC for four epochs respectively. These models are shown in Fig. 4.

The found coefficients $\overline{a}_{km}$ and $\overline{b}_{km}$ were expanded in series by power polynomials up to the 3-rd order for completely restoring the time model of the state of the ionosphere:

$$\overline{a}_{km}(t) = \sum_{i=0}^{3} a_{kmi} t^i ;$$
$$\overline{b}_{km}(t) = \sum_{i=0}^{3} b_{kmi} t^i . \qquad (19)$$

It is easy to see from formula (18) that by finding the coefficients of the model a_{kmi} and b_{kmi} it is possible to completely recreate the spatial-temporal model of the ionosphere parameter VTEC during the studied time period.

We also calculated the difference between the measured and model values of VTEC. The maps of these differences are shown in Fig. 5.

The standard deviation of the measured and model values of the VTEC parameter, as well as their differences, was calculated and shown in Table 3.

Table 3

Standard deviation of the measured and model values of the VTEC parameter and their differences

Epoch, hours	Standard deviation of the measured values, *TECU*	Standard deviation of the model values, *TECU*	Standard deviation of the differences, *TECU*
0	0.785	0.624	0.385
6	1.688	1.388	0.778
12	1.838	1.466	0.840
18	2.376	1.996	0.988

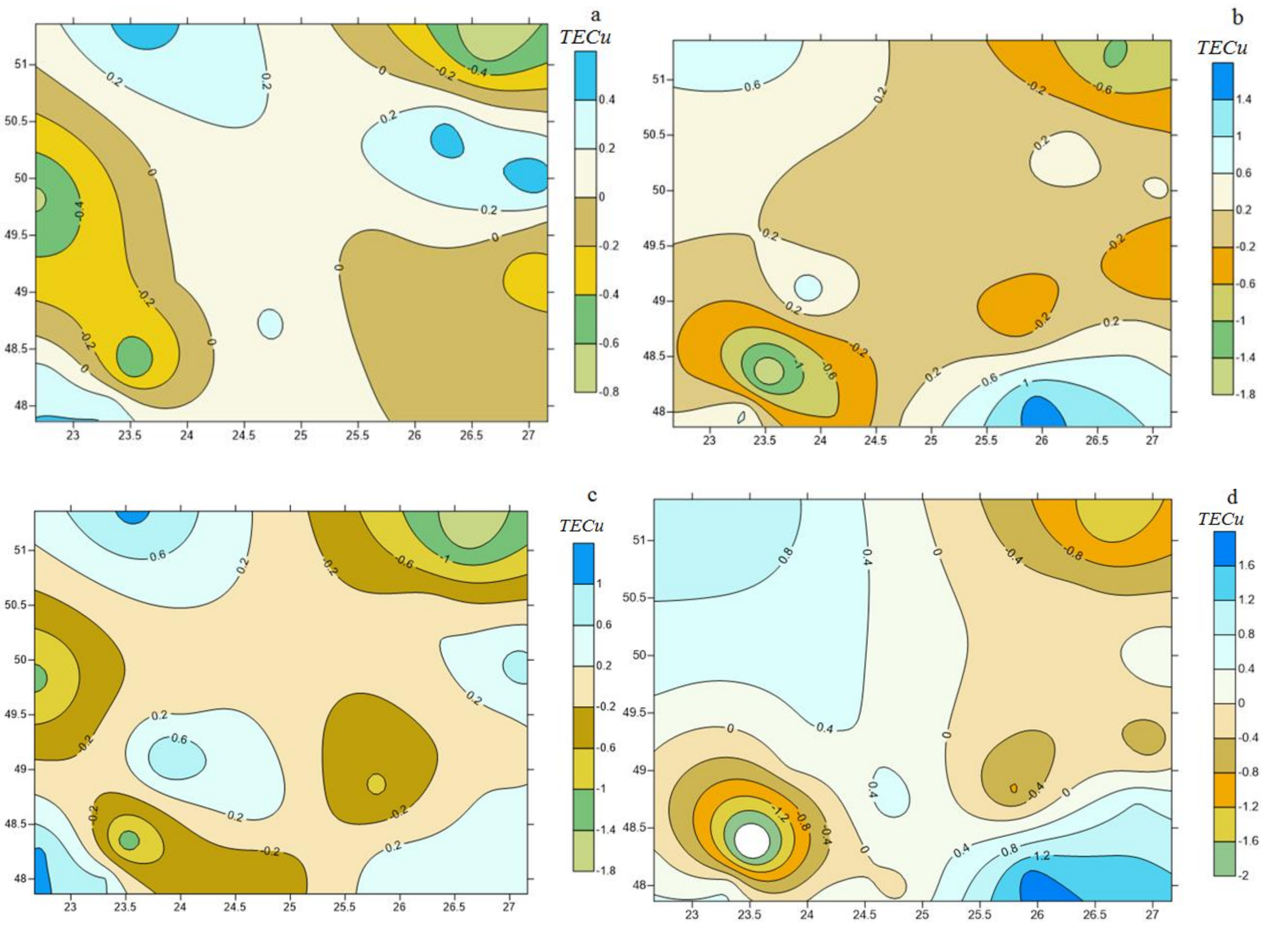

Fig. 5. The differences between the measured and model values of VTEC at a – 0 o'clock; b – 6 o'clock; c – 12 o'clock; d – 18 o'clock

The scientific novelty and practical significance

In this paper we propose to construct a space-time model of the VTEC parameter using a combination of Legendre spherical functions of real order and power polynomials. The above-mentioned methodology can be recommended for the development of technology for determining the spatial-temporal distribution of VTEC, the unstable state of the ionosphere and the construction of a dynamic map of changes of its parameters in real time over the network of active reference stations. This made it possible to obtain a high resolution model of ionosphere which can be used for research and forecasting space weather as well as for geodetic tasks to provide the necessary accuracy in determining the coordinates of the point, including single-frequency receivers, the use of which in Ukraine today reaches more than 50%. Accordingly in the future we plan to develop an algorithm for finding the correction in the coordinates of the defined point using such models of the ionosphere.

Conclusions

After analyzing the results of the research we can make the following conclusions:

– different systems of functions is suggested for the spatial-temporal modeling of the regional field of the VTEC parametr;

– we developed an algorithm for constructing the model of the VTEC parametr. For spatial modeling we used Legendre's spherical functions of real order, but of integer degree; in turn, for time modeling we used power polynomials;

– a spatial-temporal model of the VTEC parameter of ionosphere was constructed, and a standard deviation between the measured and model values was computing, which did not exceed 1 *TECU*;

– in compared to the input data the model constructed on the Legendre spherical functions keeps accuracy of at least 80 %, while the model based on linear interpolation keeps accuracy 55 %.

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ПОБУДОВА ПРОСТОРОВО-ЧАСОВОЇ МОДЕЛІ ПАРАМЕТРА ІОНОСФЕРИ VTEC

Мета. Широке застосування глобальних навігаційних супутникових систем спричинило розвиток нових методів, призначених для визначення і накопичення показника іонізації іоносфери. Оскільки за допомогою цих даних можливо суттєво підвищити точність та надійність визначення координат пункту спостережень, актуальною є створення моделі показника іонізації. **Методика.** Для побудови просторової моделі як базову систему функцій використано сферичні функції Лежандра першого роду дійсного порядку, але цілого ступеня. Величину порядку знаходили з використанням теорії Штурма–Ліувілля, оскільки вона залежить від розмірів регіону, що досліджується. Така система функцій формує дві ортогональні системи функцій на досліджуваному регіоні (сегменті сфери), проте не має рекурентних зв’язків між функціями, тому для їх знаходження необхідно використати розклад у гіпергеометричний ряд. Також для знаходження невідомих коефіцієнтів моделі необхідно використати параметр регуляризації Тіхонова, оскільки матриця нормальних рівнянь не буде стійкою. Для обчислення часової моделі іоносфери коефіцієнти різних просторових моделей розкладено в ряд за степеневими поліномами. **Результати.** На основі даних значень параметру іонізації, отриманих на 19 перманентних станціях мережі ZAKPOS за допомогою програмного забезпечення Trimble Pivot Platform, побудована просторово-часова модель цього параметра з використанням сферичних функцій Лежандра до 3-го порядку, а також з використанням степеневих поліномів до 3-го порядку. Стандартне відхилення між виміряним та модельним значеннями параметра іоносфери VTEC не перевищує 1 ***TECU***. **Наукова новизна і практична значущість.** Розроблено алгоритм для побудови просторово-часової моделі іоносфери. Отримано модель іоносфери високої розрізнювальної здатності, яку можна використовувати для розв’язання геодезичних задач щодо забезпечення необхідної точності у визначенні координат пункту, а також для дослідження і прогнозування космічної погоди.

Ключові слова: параметри іоносфери, TEC, VTEC, просторово-часова модель, сферичні функції Лежандра, показник іонізації, система функцій.

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