Methodology for Improving Mathematical Model of Ultrasonic Flowmeter
to Study Its Error at Distorted Flow Structure

Fedir Matiko, Vitalii Roman*, Roman Baitsar
Lviv Polytechnic National University, S. Bandery Str., 12, Lviv, 79013, Ukraine

Abstract
The paper presents a methodology for improving mathematical model of ultrasonic flowmeter (USM) results through the application of CFD simulation along with experimental reference data on measured consumption. On the basis of the proposed methodology, USM errors research method in terms of distortion of the flow structure has been developed. Using the proposed methodology mathematical model of two-path chordal USM has been improved and its study in terms of distortion of the flow structure after seven types of fittings has been conducted. According to the research results, concrete recommendations on the location of the USM towards the considered fittings have been proposed. The proposed recommendations will improve the accuracy of flow measurement by two-path USM by eliminating additional error due to the presence of distortions in flow structure. Testing of the proposed technique proves its correctness and application for any type of USM and various types of fittings.

Keywords: ultrasonic flowmeter; CFD simulation; flow structure; mathematical model; fitting; additional error.

1. Issue background
Ultrasonic methods and instrumentation for fluid flow rate and volume measurement are applied widely in various fields of industry including gas industry where natural gas metering is carried out during its transportation and supply to the consumers. The widespread application is caused by existence of new reliable ultrasonic flowmeters (USM) of gas providing high accuracy of measurement. For instance, multi-path USMs are developed for custody transfer of natural gas with the relative error of no more than ±0.15 % (USZ 08 of Honeywell RMG Company), ±0.2 % (FLOWSIC600 of SICK Company) or ±0.3 % (Daniel Senior Sonic of Emerson Company).

Given this, and the presence of the largest gas transport network and branched gas distribution networks in Ukraine, the introduction of USM in the gas industry in Ukraine is an important task whose solution requires joint efforts of both practicing engineers and scientists. In particular, PJSC “Ukrtransgaz” in 2014 analyzed the modern European experience in the field of flow measurement of gas and decided on the priority of USM (meters) use during the reconstruction and construction of new gas distribution stations [1]. In turn, “Naftogaz Ukraine” and the State Committee for Technical Regulation and Consumer Policy of Ukraine are implementing regulations on flow measurement, including standards implemented “Ultrasonic gas meters for commercial and technical accounting. General specifications (harmonized with ISO 17089-1: 2010)” [2].

It is known that the metrological characteristics of flowmeters depend on the operating conditions (in particular, on the extent to which the operating conditions are different from the calibration conditions). Based on the numerous experimental studies the flow disturbances in the pipe upstream of the USM were defined to lead to significant errors in flow rate measurement.

* Corresponding author. Email address: roman_vitaliy@ukr.net
According to ISO 17089-1: 2010 [3], which is the main regulatory instrument governing the application of USM for commercial purposes, to achieve steady flow rate profile during its previous asymmetry, straight sections of measuring pipeline without special flow conditioning devices, with the length of 5–50 D, while in case of twists – this value can reach 200D may be necessary. Installation of straight sections is expensive and may not always be implemented in terms of technological limitations.

That is why it is important to study the effect of a flow disturbance on the error of USM and to develop the up-to-date means for simulation in order to work out recommendations on installation of USMs to eliminate the additional errors of flow rate measurement caused by flow disturbances.

2. Analysis of publications

A powerful instrument for studying the gas dynamical processes in pipes together with experimental studies is a computer simulation by means of Computer Aided Design (CAD) and Computational Fluid Dynamics (CFD) software. The models of fluid flows in pipes of complicated configurations can be built by means of the software with high accuracy. These models provide the possibility to study the pipe configurations and USM constructions for which the experimental studies were not carried out sufficiently.

The results of CAD/CFD simulation for studying the effect of a disturbed flow on the error of USM are presented and discussed in works [4–7]. Based on the analysis of these works, it was defined that there are a lot of CAD/CFD software types both with open access and with licenses. There are different constructions of pipe fittings and positions of acoustic paths of USMs. That is why there is a need to develop a generalized methodology for application of CAD/CFD software to improve the mathematical model of USM and to study the effect of a disturbed flow on the error of USM.

The authors carried out such studies and the results of testing are partially outlined in [8]. However, topical is the use of proposed in [8] methodology to study the impact of flow structure distortions formed by different types of fittings on the concrete USM structures error and development of recommendations for its reduction.

3. Purpose of the work

The authors set the goal to investigate the effect of flow structure distortion after various types of fittings on the two-path chordal USM error and to make recommendations as to their installation.

4. Presentation of the basic material

Mathematical model of multi-path chordal USM can be obtained based on two equations: first, the equation of volume flow rate \( q_1 = Sv \), where \( S \) – cross-sectional area of the measuring pipe and \( v \) – average flow rate on this section; secondly, the equation for determining the coordinate position and weighting factor of acoustic USM paths [10]. With the use of mathematical USM models coupled with CFD simulations, which enables us to define the flow rate for each acoustic USM path, there may be a number of errors due to the following reasons:

1. Inaccurate drawing of the multi-path USMs and pipes which is caused by the complicated construction of the USM in the cases when the information on the geometrical dimensions of USM is lacking or is defined inaccurately (dimensions of the electro-acoustic transducers (EAT), their pockets, protection layer and length of the acoustic channel).

2. Inaccurate description of the behavior of the turbulent flow by the CAD/CFD software [6].

To eliminate the errors of USM simulation, it is proposed to improve the mathematical model of the meter by introducing the dependence of the calibration factor on the Reynolds number \( k_{cal} = f(Re) \) into the model [9]. The values of \( k_{cal} \) are defined on the basis of the reference values of volume flow rate and flow parameters derived experimentally or analytically (processing results of CFD simulations) for specific values of Reynolds number according to the following formula \( k_{cal} = q_{ref} / q_s \), where \( q_{ref} \) – reference value volume flow rate of gas in standard conditions, and \( q_s \) – values volume flow rate obtained by mathematical models of USM on the basis of CFD simulations and reduced to standard conditions. The flow rate reduced to standard conditions is calculated as follows \( q_s = q_pT / pTK \), where \( p \) and \( T \) – value of fluid absolute pressure and fluid thermodynamic temperature in operating conditions, and \( K \) – is the compressibility coefficient.
It should be noted that the reference value for the flow rate of operating conditions \( q_{v, \text{ref}} \) can be defined as follows:

- is selected from a measurement range for the specific USM model and size on condition of known USM designs modeling;
- is assumed to be the value of the reference flowmeter applied during the experiment.

Also, it should be emphasized that the value of the reference flow, fluid pressure and fluid temperature are used to configure the CFD simulations (boundary conditions), and the definition depending \( k_{cal} = f(\text{Re}) \) should be performed for the absence of flow structure distortion.

Thus, the proposed methodology of improving the mathematical model of an USM consists in defining the dependence of \( k_{cal} \) on Re based on the results of CFD simulation and the available reference data on the measured flow rate and subsequent introduction of this dependence into the mathematical model. The generalized mathematical model of a multi-path USM with taking into account the proposed methodology and the improved method for defining the coordinates of the location and weighting coefficients acoustic paths chordal USM [9] is presented as follows

\[
q_i = k_{cal} \frac{\pi D^2}{4} \sum_{i=2}^{N} \frac{2\sqrt{R^2 - x(i)^2}}{\pi R} w(i) \nu_i(i); \\
k_{cal} = f(\text{Re}), \; \text{Re}_{\text{min}} \leq \text{Re} \leq \text{Re}_{\text{max}}; \\
w(i) = \frac{1}{(1-x(i))^2} \int_{x(i)}^{1} PL(x, x(i))(1-x)^{i-1} dx; \\
PL(x, x(i)) = \prod_{j=1}^{N} \left( \frac{x-x(j)}{x(i)-x(j)} \right), \; i = 1, 2, ..., N; \\
PJ = f(x, N, k); \; k = 0,2; \; x(i) = \text{roots}(PJ),
\]

where \( D \) – pipe internal diameter of the measuring section of pipeline or USM; \( R = D/2 \); \( x(i), w(i), \nu_i(i) \) – position coordinate, weighting factor and flow rate along the \( i \)-th chordal acoustic path; \( PL \) – Lagrange polynomial; \( PJ \) – Jacobi polynomial; \( k \) – coefficient of weighting function of Jacobi polynomial; \( N \) – number of acoustic paths USM; Re – Reynolds number.

Mathematical model (1), applied in conjunction with CAD/CFD simulations tools, allows us to investigate the effect of individual design characteristics of multi-path chordal USM on measured values during their design, and to investigate the flow structure distortion effect on the value of the flow measured using acting USM dependence \( k_{cal} = f(\text{Re}) \) for which was obtained based on the experimental results in terms of undisturbed flow.

Technique for studying the error of flow rate measurement was developed on the basis of the mathematical model of USM (1). This technique is presented in [8] and it consists of the following steps:

- Definition of the main and supplementary constructional parameters of the USM and the pipe (see Fig. 1, a);
- Definition of the fluid flow parameters (type of fluid, flow rate, p and T etc.);
- Identification of \( k_{cal} \) of the USM model (drawing of 3D model of the USM and the pipe by means of CAD software (see Fig. 1, b); setting of 3D model parameters in CFD software; development of the dependence of \( k_{cal} \) on Re);
- Study of the USM error in a disturbed flow.

The developed technique was verified on the basis of the results of experimental study of flow rate measurement error for acting USM GUVR-011A2.2/VS in a gas test unit of “Energooblik” Company (Kharkiv, Ukraine) [8]. It should be mentioned that both the experimental study and the simulation of USM were carried out without any flow conditioner according to the requirements specified by the flowmeter manufacturer in the operational documentation [10].

Three dimensional model of the USM (see Fig. 1, b) together with the pipe and the fittings was built by means of CAD software. The pipe straight lengths were set according to the diagrams in Figs. 2. The simulated flow rate...
values were compared to the experimental results of flow rate measurement by means of the USM installed downstream of two typical fittings (single 90° bend, Fig. 2, a, and two 90° bends in perpendicular planes ($l \leq 5D$), Fig. 2, b). This way the conclusion about the adequacy of the proposed technique was made.

The mathematical model of the double-path chordal USM GUVR-011A2.2/VS (2) was built by means of the proposed methodology. The mathematical model is as follows

\[
q_c = k_{aw} \left[ \frac{\pi D^2}{4} \cdot 2\sqrt{R^2 - (0.5807R)^2} \right] \frac{1}{2(v_{h1} + v_{h2})} \text{ Re} = \frac{4q_{aw} \rho_{aw}}{\pi \mu D} = \begin{cases} 38.79 \text{ Re}^{-0.7857} + 1; & \text{ Re} = 1 \cdot 10^4 + 5 \cdot 10^3; \\ -1148 \cdot 10^{-6} \text{ Re}^2 + 2675 \cdot 10^{-6} \text{ Re} + 1049; & \text{ Re} = 5 \cdot 10^3 + 15 \cdot 10^3; \\ -3567 \cdot 10^{-4} \text{ Re} + 1063; & \text{ Re} = 1.5 \cdot 10^4 + 1.5 \cdot 10^5, \end{cases}
\]

where $v_{h1}$, $v_{h2}$ – flow velocity along the 1-th and 2-nd chordal acoustic path USM GUVR-011A2.2/VS for the results of CFD simulations; $\rho$ – fluid density, kg/m$^3$; $\mu$ – fluid dynamic viscosity, Pa·s.
Methodology for Improving Mathematical Model

This model was applied for calculating the flow rate measured by the flowmeters presented in Fig. 2, a, and 2, b. Here the values \( q_{v,ref} \) were taken equal to the experimental values of flow rate measured by the reference gas meter in the test unit.

The calculated values of flow rate using the mathematical model (2) were compared to the experimental values of flow rate measured by means of USM GUVR-011A2.2/VS in the test unit. To calculate the relative deviation \( \delta_M \) the values of flow rate were reduced to standard conditions. The curves of the relative deviation \( \delta_M \) versus flow rate are presented in Fig. 3, a, and b.

As we can see from Fig. 3, a, and 3, b the simulation results are close to the experimental values in the range of flow rate from \( 0.05 q_{v,max} \) to \( q_{v,max} \). For USM installed downstream of 90° bend the maximum value of \( \delta_M \) is 0.86 % in the specified range of flow rate. And for USM installed downstream of two 90° bends in perpendicular planes (\( l \leq 5D \)) the maximum value of \( \delta_M \) is 1.04 %. By means of these results the adequacy of the proposed technique is proved and the possibility of application of this technique for studying the effect of flow disturbances on the error of flow rate measurement by USMs is confirmed.

The developed technique was applied for studying the additional errors of USMs caused by flow disturbances downstream of typical fittings with relation to the distance between the fitting and the USM (\( l_{fit} \)). The following types of USM were taken to consideration:

1) USM1 – double-path chordal USMs with the angle of acoustic paths \( \varphi = 45^\circ \ldots 67^\circ \); calculation of the position coordinates and the weighting factors of the acoustic paths was carried out according to the improved method [9];

2) USM2 – double-path chordal USMs GUVR-011A2.2/VS (G400, Dn100) with the angle of acoustic paths \( \varphi = 67^\circ \) according to [10];

3) USM3 – double-path chordal USM with the angle of acoustic paths \( \varphi = 67^\circ \); calculation of the weighting factors of the acoustic paths was carried out according to the improved method.

The following fluid parameters were taken for simulation: type of fluid – air; fluid pressure and temperature – \( p = 400 \) kPa, \( T = 293.15 \) K (20 °C); reference values of flow rate: \( q_v = q_{v,max} \times (0.025; 0.05; 0.1; 0.25; 0.5; 0.75; 1) \), \( q_{v,max} = 650 \) m\(^3\)/h.

The additional error of flow rate measurement caused by flow disturbance was calculated according to the following formula

\[
\delta_s = \frac{q_s - q_{v,ref}}{q_{v,ref}} \times 100 ,
\]

where \( q_s = q_{v,CFD}T_s / p_s T_{CFD}K \) – values volume flow rate obtained by mathematical models of USM GUVR-011A2.2/VS (2) on the basis of CFD simulations and reduced to standard conditions.
The minimum pipe straight lengths \( l_{\text{min}} \) between a fitting and USM (see Table 1) were defined on the basis of the simulation results with taking into account the following criteria:

- \( l_{\text{min}} \) is equal to the minimum length at which the value of \( \delta_A \) error remains within the limits of the basic relative error of flow rate measurement declared by the manufacturer USM of;

- \( l_{\text{min}} \) is equal to the length at which prolonging the length by 10D would not lead to change of \( \delta_A \) error by more than 0.3 %.

The greater value of \( l_{\text{min}} \) was accepted in each case when making the Table 1 based on the two criteria mentioned above.

<table>
<thead>
<tr>
<th>No</th>
<th>Type of fitting</th>
<th>( l_{\text{min}} ), ≥ l/D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>USM1</td>
</tr>
<tr>
<td>1</td>
<td>Single 90° bend</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>Two 90° bends in perpendicular planes (l≤5D)</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>Gagged tee with change of flow direction</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>Two 90° bends in the same plane: U-configuration (l≤10D)</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>Two 90° bends in the same plane: S-configuration (l≤10D)</td>
<td>60</td>
</tr>
<tr>
<td>6</td>
<td>Expander (80/100)D</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>Reducer (130/100)D</td>
<td>40</td>
</tr>
</tbody>
</table>

The results of \( \delta_A \) calculation for various flow rates are presented in Fig. 4.

As we can see from Fig. 4, the minimum pipe straight length \( l_{\text{min}} \) defined on the basis of the two criteria for USM installed downstream of two 90° bends in perpendicular planes (l≤5D) is 33D. The additional error of flow rate measurement \( \delta_A \) is within the limits ±1 % [10] for the double-path chordal USMs under consideration in the range of flow rate measurement from \( q_{v,\text{min}} \) to \( q_{v,\text{max}} \).
Based on the results presented in Table 1 we can say that the shortest pipe straight lengths are needed for USMs installed downstream of an expander. And the longest pipe straight lengths are needed for USMs installed downstream of two 90° bends in the same plane: S-configuration (≤10D).

5. Conclusions
The paper proposes a methodology for improving mathematical model of multi-path USM which is to determine on the basis of CFD simulations and reference data on existing measured flow rate $k_{ref} = \Phi(Re)$ dependency and the introduction of this dependence in the mathematical model of USM.

An improved methodology was proposed in order to build a mathematical model of an USM of any construction. The technique for studying the error of flow rate measurement was developed and verified on the basis of the results of experimental study of flow rate measurement error for acting USM GUVR-011A2.2/VS in a test unit.

The recommendations to the minimum pipe straight lengths for the double-path chordal USMs without a flow conditioner were defined.

References


Методологія удосконалення математичної моделі ультразвукового витратоміра для дослідження його похибки за умов спотвореної структури потоку

Федір Матіко, Віталій Роман, Роман Байцар

Національний університет “Львівська політехніка”, вул. С. Бандери, 12, Львів, 79013, Україна

Анотація

Запропоновано методологію удосконалення математичної моделі ультразвукового витратоміра шляхом застосування результатів CFD-моделювання поряд із експериментальними еталонними даними про вимірювану витрату. На базі запропонованої методології розроблено методику дослідження похибки ультразвукового витратоміра у видах спотворень структурні потоку. Використовуючи запропоновану
методологію, удосконалено математичну модель двоканального хордового ультразвукового витратоміра та виконали її дослідження в умовах спотворень структури потоку після семи типів місцевих опорів. За результатами досліджень запропоновано конкретні рекомендації щодо місця встановлення ультразвукових витратомірів відносно розглянутих місцевих опорів. Запропоновані рекомендації дають змогу підвищити точність вимірювання витрати двоканальними ультразвуковими витратомірами шляхом усунення додаткової похибки зумовленої наявністю спотворень структури потоку. Апробація запропонованої методики підтверджує її правильність і можливість застосування для будь-якого типу ультразвукових витратомірів та різного типу місцевих опорів.

Ключові слова: ультразвуковий витратомір; CFD-моделювання; структура потоку; математична модель; місцевий опір; додаткова похибка.