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## MATHEMATICAL MODEL OF ELEMENTS OF AUTOMATED SYSTEM OF LOOSE MATERIALS DOSING

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**Abstract.** Automatic dosing system for loose materials is widely used in construction, food and pharmaceutical industries to prepare various mixtures. The main criterion for optimization of such systems is the accuracy of dosing of each component of the mixture, which depends on the speed component dosing and other process factors. Technological requirements for product quality in production and the high cost of individual components of the mixture strictly regulate the developers of the automated system loose materials size dosing errors. Ensuring compliance with the maximum possible mix recipe requires dosing of each component with high accuracy, which is a daunting task for engineers.

Precision dispensing in automated systems is a function of the large number of systematic and random factors: size, shape and relative position of individual particles of the material; coupling coefficient of each other and of the structural elements of the dispenser; relative humidity of material dosage and indoor air; the height of the fall of the material in the receiving container; the value of the dispenser tilt to a level of horizon; electromechanical vibration and noise, etc.

Developed simulation model of frequency-controlled electric screw feeder that is different from the known one facts that modeling of screw feeder is considering mass “falling pillar” which is constantly changing as the accumulation of material in the hopper.

### Introduction. Problem statement

According to the classification of basic processes of chemical technology dosing refers to mechanical processes, which are basically determined by the laws of solid state physics. Dosing of loose materials is widely used in food industry and related industries. In many cases dosing is one of the basic operations so far as it specifies the quality of final product and regulates the consumption of raw materials.

Of the variety of electrotechnical systems of loose materials dosing it is necessary to highlight automated systems constructed on the basis of modern asynchronous electrical drive with microcontroller operation of weight-dosing device, which use the principle of multicomponent dosing. Considerable contribution to development and modernization of dosing systems was made by such outstanding scientists as B. A. Fedosenko, H. A. Rohinskyi, A. V. Katalymov, V. A. Liubartovych, A. M. Grygoriev, S. P. Orlov, A. V. Pugachov, Yu. D. Vidineiev etc.

The problem of improving the efficiency of controlling of automated electrotechnical system of loose materials dosing acquires specific importance because its solution will significantly improve the system productivity and decrease erroneous dosing of especially important small-sized and expensive components. That's why development and investigation of electrotechnical system of automated weight-dosing of loose materials with frequency-regulated electrical drive which ensures improved dosing accuracy with maximal productivity are urgent problems and will ensure satisfaction of heightened requirements to technological process of loose materials dosing in complicated manufacturing conditions.

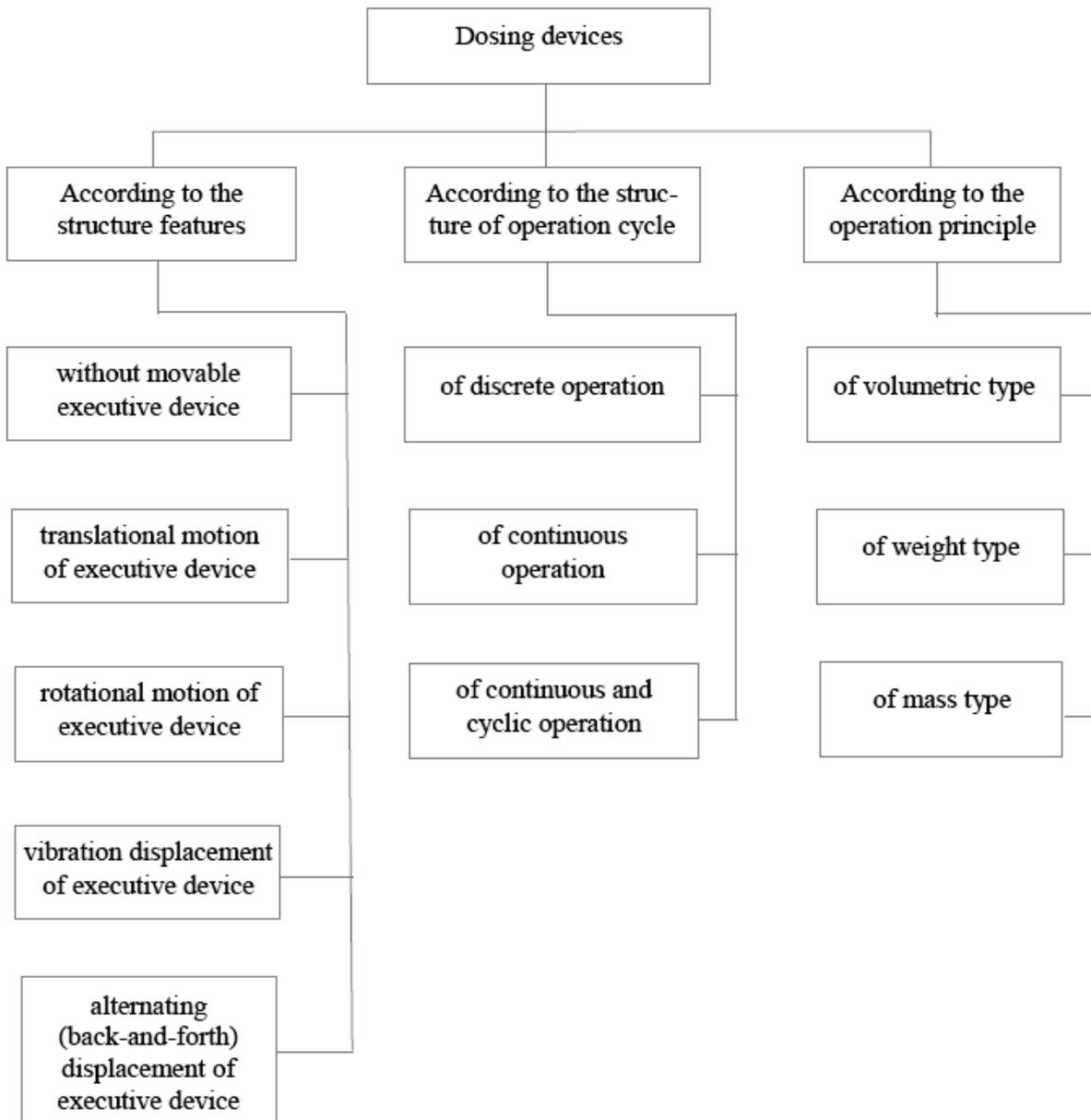
### Analysis of modern information sources on the subject of the article

Today it is developed a great number of dosing devices. This is due to the fact that loose materials have wide range of physical and mechanical and technological characteristics, besides specific requirements are often preferred to dosing equipment depending on the process features.

The classification of feeders may be conducted according to the following three criteria (Fig. 1):

- the structure of operation cycle;
- structure features;
- operation principle.

According to the structure of operation cycle it is distinguished feeders of continuous operation and feeders of discrete operation.



*Fig. 1. Classification of dosing devices*

Weight feeders of continuous operation are used for automatic reproduction of prescribed values of the loose material mass in a given time (productivity), which are being conveyed by transporters in technological (production) lines of various branches of industry, agriculture and commerce.

The principle of feeder operation is based on transforming of deformation of elastic elements of weight-measuring resistive-strain sensors, which appears as a result of the influence of gravity force of the weight, into the analogue-digital signal being changed in proportion to the weight mass, and also on transforming of linear speed of transporting string into proportional frequency signal. After this analogue

signal from weight-measuring speed sensors come to weight-measuring device. The value of feeder productivity, string speed and total product mass is being induced in weight-measuring device.

Discrete feeders convey material in the form of equal doses in a given time stretches. In this case the control of the quantity of material being dosed may be carried out by means of regulation of the number of doses per unit of time or the volume of the dose. This type of feeders is less accurate, however, it has such advantages as servicing simplicity and reliability in hard operation conditions.

Three main types of dosing is being distinguished: volumetric; weight; mass. Volumetric feeders are easy-to-use but have insufficient measuring accuracy during the operation with some production types [9]. Weight feeders are the optimal solution for dosing of loose materials with fractions of any dimensions and of fluids. The popularity of such devices is explained by the universality, optimal accuracy and high productivity. Resistive-strain weight-measuring devices equipped by feeders of such type are extremely easy-to-use; the processes of weighing and dosing are fully automated; the control of weighing machine is reduced to feeding device manipulating. The only fault of weight feeders is comparatively low speed of their operation [9]. Mass feeders may be equally successfully used for dosing rigid, loose and fluid substances. They find their application in various branches of industry. Mass feeders may be characterized by high reliability, measuring accuracy and comparatively high operation speed [9].

The classification of dosing devices according to the structure features is the largest one. The classification according to the kind of motion and to the type of executive devices of dosing devices is the most essential one. Physical and mechanical features of material have a strong hold over the classification of dosing equipment according to the structure features. First of all, these are dimensions of parts, bulked density, fluidity, adhesion etc.

Average size of parts of loose materials is classified in such a way:

- lump ( $d > 10$  mm);
- coarse-grained ( $d = 2-10$  mm);
- small-grained ( $d = 0,5-2,0$  mm);
- powder-like ( $d = 0,05-0,50$  mm);
- pulverized ( $d < 0,05$  mm).

### Raising of the task

The formation of the mathematical model of screw feeder is necessary for checking serviceability of synthesized control system of dosing complex in order to test the developed algorithms of decrease in errors of dosing taking into account the specialties of materials being dosed. These specialties consist of random changing of the mixture features during some period of time and the influence of “falling pillar” of material on the dosing error.

For forming the mathematical model of electrotechnical system of loose materials dosing of enhanced accuracy let us use block principle the point of which consists in the fact that the model is being formed by separate logically completed blocks.

The generalized functional diagram of electrotechnical system of loose materials dosing which consist of expendable bunker (EB) and screw feeder (SF), which transports the material to the hopper feeder (HF), is presented in Fig. 2. Microprocessor control of screw feeder is carried out by the frequency converter – induction motor (FC-IM) system with scalar control.

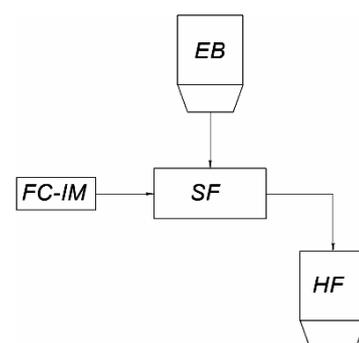


Fig. 2. Electrotechnical system of loose materials dosing

### Mathematical model of screw feeder with variable characteristics of loose material

In technology loose materials are concerned as materials which can be transported and cured in bulk. The “loose” term is used for the majority of materials which consist of parts of various dimensions: pulverized, powder-like, granular and lump ones. The volume of loose material consists of various in shape and dimensions volumes of rigid parts being touched and voids between them, which are thickened with air or water. The parts which form the structure (framework) of the loose material have different relations one

with another, which depends on the dimensions of the parts, their shapes, humidity etc. As a rule, most loose materials have complicated fractional structure and may be concerned as the assembly of parts with random undetermined arrangement.

In dosing systems provided that the main criterion is the accuracy of dosing screw feeders have indisputable advantages over other types of dosing materials transporting [17-19]. The necessary dosing accuracy can be reached due to speed control at the moment of screw shutdown. The development of the control system is carried out taking into account known information about the dose volume of the material, which is being transported to hopper feeder. However, the priority of dosing accuracy influences the productivity of screw feeder which is directly proportional to the speed of screw rotation. That's why it is necessary to obtain optimal correlation of dosing accuracy and productivity of the screw feeder when setting up the control system. At the design stage the calculation of coefficients which characterize dosing accuracy and productivity should be conducted with the help of mathematical model.

The productivity of screw feeder may be determined by the formula [1]:

$$Q = \frac{60 \cdot p \cdot D^2}{4} \cdot S \cdot n \cdot c \cdot g \cdot \gamma, \quad (1)$$

where  $D$  is the outer diameter of the auger screw, m (Fig. 3);  $S$  is the pitch of the auger screw, m;  $n$  is the rotation frequency of the auger, rpm;  $c$  is the coefficient which take into account the influence of the angle of auger axis inclination to a level of horizon on its productivity;  $g$  is the bulk weight of the material, kg/m<sup>3</sup>;  $\gamma$  is the coefficient of the auger body filling.

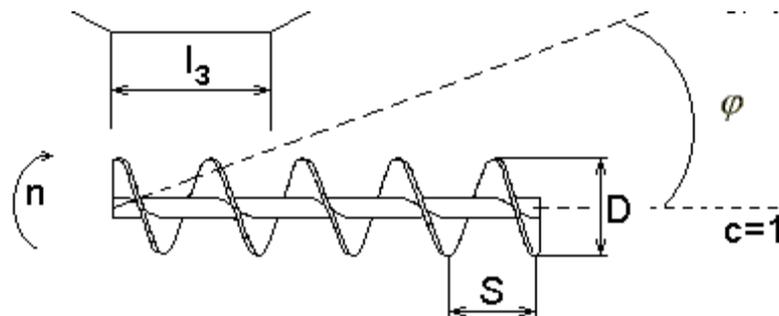


Fig. 3. Parameters of the auger

The productivity of the screw feeder is proportional to the screw diameter  $D$ , the pitch  $S$  of the screw, the rotation frequency of the auger  $n$  and also depends on the parameters of the material being transported, the coefficient of the auger body filling  $\gamma$  and the bulk weight  $g$  of the material. The productivity of the screw feeder is also influenced by the spatial location of the auger, the coefficient  $c$  which take into account the influence of the angle of auger axis inclination to a level of horizon etc.

The inclination of the screw feeder considerably reduces the productivity of auger operation. For example, when the inclination equals 45 % the productivity is reduced by 30 % [1]. At the stage of auger design it is possible to correct the productivity by means of selection of  $S / D$  correlation under defined inclination or in dependence of the material characteristics. The coefficient of the chute filling for different materials lies in the range of 0.45–0.2 and is directly proportional to the dimensions of the parts.

The change of the humidity of the ambient medium is also the factor of instability of loose materials dosing system because it causes the change of the coefficient of the chute filling and the bulk weight of the material being transported.

The bulk weight of the specific loose materials doesn't have constant value. The fluctuation of the volumetric weight and the filling coefficient depends on granulometric (grain-size) composition of the material: parts dimensions, presence of parts of different sizes in material, its humidity etc. The coefficient of the chute filling is influenced by the rotation speed of the auger and the length of intake part of the screw. Under the certain rotation speed of the auger shaft and intake part of the screw ( $l_3$ ) it is possible to

obtain maximal value of the coefficient of the chute filling (for different materials the maximal value will be different). The coefficient of the chute filling  $\gamma$  may be chosen in dependence of the type of material transportation.

The productivity of screw feeder depends on the rotation speed of the drive motor shaft, air humidity, granulometric (grain-size) features of the material and angle of auger inclination. Under the nonessential increase of humidity of the substance the productivity increases. The productivity under the inclined transportation of the material reduces because of the considerable decreasing of the coefficient of the chute filling. That's why in formula (1) for productivity calculation the correction coefficient  $c$  is inserted.

The volume of the feeder hopper may be determined by the formula:

$$V = \frac{m}{\rho}, \text{ m}^3. \quad (2)$$

In the case when feeder hoppers are designed in the shape of cube the height of the hopper may be defined by the formula:

$$h_0 = \sqrt[3]{V}, \text{ m}. \quad (3)$$

The height of the falling pillar when material is being transported from the auger to the hopper may be determined by the formula:

$$h_0 = \frac{g \cdot t^2}{2}, \text{ m}, \quad (4)$$

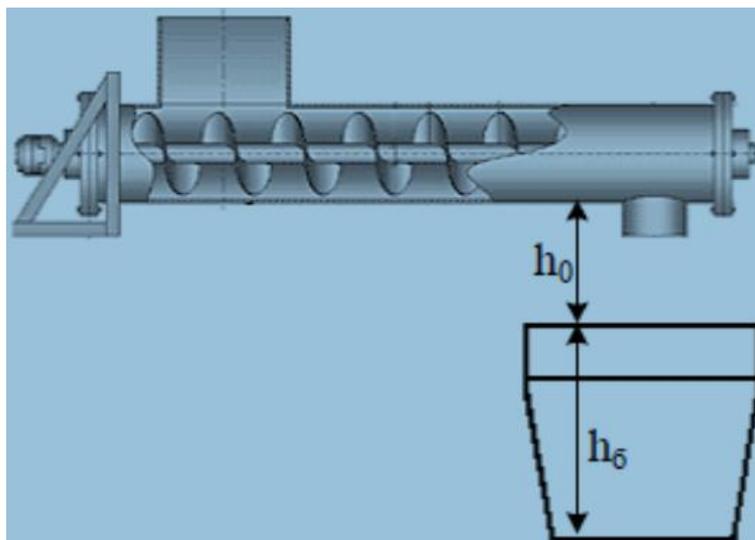
where  $t$  is time of material falling into the feeder hopper:

$$t = \sqrt{\frac{2 \cdot h_0}{g}}, \text{ s}. \quad (5)$$

As can be seen from the formula (5), the value of time depends on the height of material falling. This height may be divided into two sections (Fig. 4):

$$h = h_0 + h_0, \quad (6)$$

where  $h_0 = \text{const}$  is the distance between the auger and the top plane of the hopper;  $h_0$  is the distance between the material level in the hopper and its top plane.



**Fig. 4.** The height of dosed materia falling into the hopper

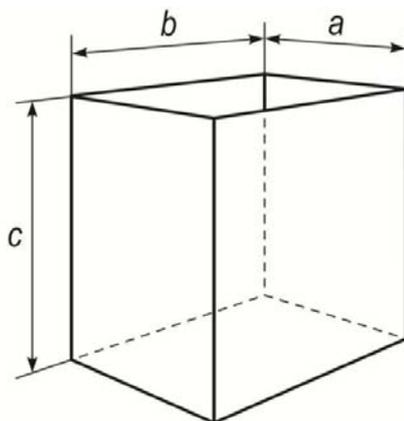
There are two possible methods to determine the dependence between the mass and the height. The first method is experimental one. Under various values of dosing weight the height  $h_0$  is to be measured and the graphic dependence is to be plotted on the basis of the obtained experimental points (data). This

method is highly accurate one but it is used very rarely because of different technological factors. The second method is analytical one. For hoppers with different shapes the dependence of the height  $h_{\phi}$  is different. Let us consider the dependences for different hopper shapes.

1. The hopper has the shape of parallelepiped (Fig. 5).

The volume of this hopper may be defined as:

$$V_{nap} = a \cdot b \cdot c. \quad (7)$$



**Fig. 5.** Box-like (rectangular) shape of the hopper

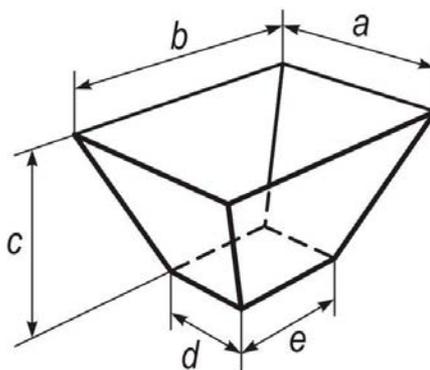
The volume of the material in the hopper:

$$V_{nap} = \frac{m}{\gamma}. \quad (8)$$

From the formula (7) the height may be determined as:

$$h_{\phi} = c = \frac{V_{nap}}{a \cdot b}. \quad (9)$$

2. The hopper has the shape of truncated four-cornered (tetragonal) pyramid which has a square in its base (Fig. 6).



**Fig. 6.** Trapezoidal shape of the hopper

The volume of this hopper may be determined as:

$$V_{nip} = \frac{1}{3} \cdot c \cdot (d \cdot e + \sqrt{d \cdot e \cdot a \cdot b} + a \cdot b). \quad (10)$$

From the formula (10) the height may be defined as:

$$h_{\phi} = c = \frac{3 \cdot V_{nip}}{(d \cdot e + \sqrt{d \cdot e \cdot a \cdot b} + a \cdot b)}. \quad (11)$$

The hoppers which have complicated shape (Fig. 7) should be divided into simpler component shapes, which were described earlier. In this case the dependence of  $h_{\sigma}$  should be formed from the dependences of the heights of separate sections.

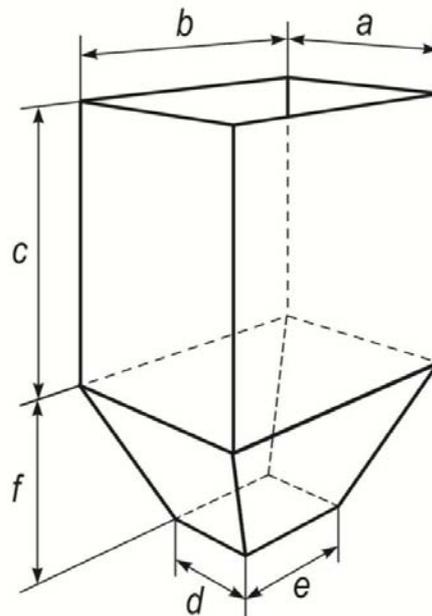


Fig 7. The hopper of complicated shape

The height of material falling into the hopper of complicated shape may be determined as:

$$h_{\sigma} = c + f . \tag{12}$$

Consequently:

$$h_{\sigma} = \frac{V_{nap}}{a \cdot b} + \frac{3 \cdot V_{nip}}{(d \cdot e + \sqrt{d \cdot e \cdot a \cdot b} + a \cdot b)} . \tag{13}$$

Taking into account all the previously presented calculations let us form the simulation model of mechanical system of electric drive (Fig. 8).

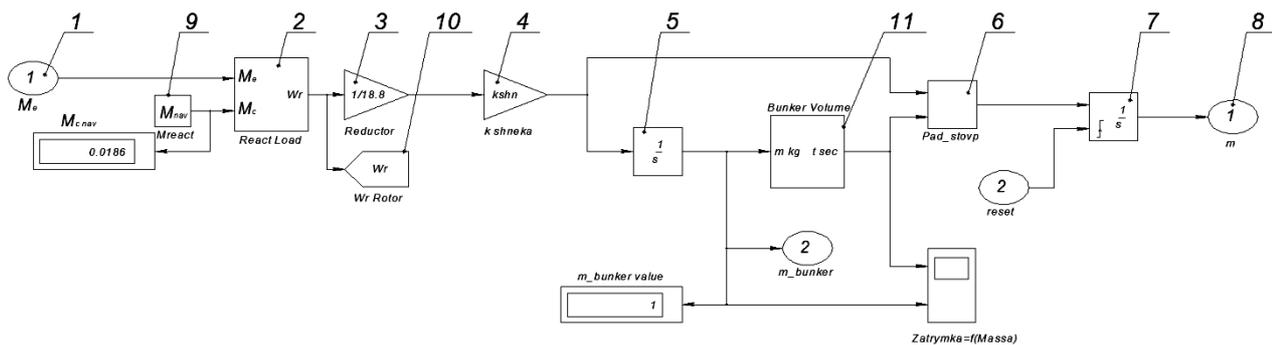


Fig. 8. Simulation model of mechanical system

The simulation model of mechanical system formed in MATLAB Simulink software environment consists of the following basic elements: 1 – the input signal in mechanical system which may be considered as electromagnetic moment of the motor; 2 – the block which takes into account the resistance moment in the mechanical system; 3 – the block which takes into account the reduction ratio of the reducing gear; 4 – the block which takes into account the transmission coefficient of auger feeder; 5 – the block of integrator which characterizes the accumulated mass of material in the hopper without temporary stop; 6 – the block which takes into account the delay when material is falling; 7 – the block which characterizes the accumulated mass of material in the hopper: the input RESET is necessary for zeroing the value of material mass in order to carry out

the following dosing cycle; 8 – the output of the mechanical subsystem in the form of mass accumulated in the feeder hopper in the current dosing process; 9 – the consolidated resistance moment on the shaft; 10 – the output of the mechanical subsystem in the form of motor speed; 11 – the block which takes into account the dependence of the time delay of material falling from the mass accumulated in the hopper.

### The model of the falling pillar of loose material

The falling pillar consists of some mass of material being dosed which increases the total value of dosing error. For reducing the influence of the mass of falling pillar on the dosing accuracy it is necessary to determine the mass of the falling pillar and to conduct the correction of the dose value. To find the mass of the falling pillar it is necessary to know the bulk density of the material being dosed, the diameter and the height of the falling pillar. The bulk density is the mass-to-volume ratio of the product in the bulked state.

If the geometrical dimensions of the hopper, the distance between the auger and the hopper, the material density are known it is possible to determine the height of the falling pillar using the formula (9) for the hopper in the shape of cube. The mass of the material in the hopper may be defined by the formula:

$$m = g \cdot V, \quad (14)$$

where  $g$  is the bulk density of the material;  $V$  is the volume of the filled hopper.

The volume of the filled hopper may be determined as:

$$V = a \cdot b \cdot h_m, \quad (15)$$

where  $h_m$  is the height of the hopper filled with the material.

The mass of the material dose is the technological parameter which is measured with a help of three strain indicators and is summed up into standard current normalized signal, which comes to the control system on the input of the programmed weight regulator.

Let us substitute (14) into (15) and by means of simple mathematical transformations we may define the height of the hopper filled with the material:

$$h_m = \frac{m}{g \cdot a \cdot b}. \quad (16)$$

For the hopper of the shape of truncated pyramid (Fig. 6) which has a square in its base the height of the hopper filled with the material may be determined as:

$$h_{m1} = \frac{3 \cdot m}{g \cdot c \cdot (d \cdot e + \sqrt{d \cdot e \cdot a \cdot b + a \cdot b})}. \quad (17)$$

For the hoppers of the complicated shape (Fig. 7) the height of the loose material in the hopper may be determined as:

$$h_{m2} = h_m + h_{m1}. \quad (18)$$

In this case the height of the falling pillar may be defined as:

$$h_{cm} = h_0 + (h_{\bar{o}} - h_m). \quad (19)$$

Let us consider that the material is uniformly dispersed on the area of the receiver hopper and the formation of the cone bank during the dosing process may be neglected.

The area of the falling pillar cross-section may be defined when the geometrical dimensions of the auger blade are known. Let us consider that the area of the cross-section is equal to the area of the working surface of the auger flight [18].

As it is presented in the Fig. 9, the flight has external and internal diameter the area of which may be determined by the formula:

$$S_{cm} = \frac{P}{4} \cdot (D_0^2 - d_0^2). \quad (20)$$

Let us define the bulk weight of the particular material being dosed and determine the mass of the falling pillar:

$$m_{cm} = S_{cm} \cdot h_{cm} \cdot g. \quad (21)$$

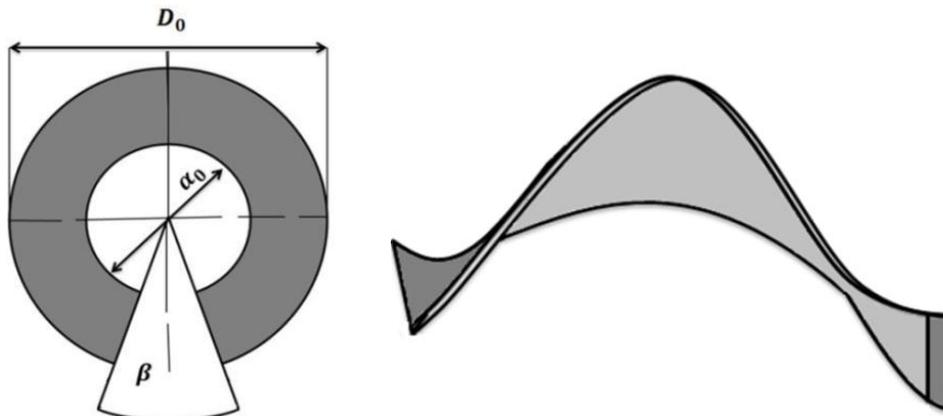


Fig. 9. General view of the blade and the auger flight

The obtained expression of the mass of the falling pillar allows real time monitoring of the change of the material mass which is in the air.

Dynamically changeable temporary delay of material falling into the feeder hopper during the material accumulation in the hopper may be calculated taking into account the following formula:

$$t_{m3} = \sqrt{\frac{2 \cdot h_{cm}}{g}} \quad (22)$$

The simulation model of the auger feeder taking into account the changeable temporary delay and the calculation of the falling pillar mass is presented in Fig. 10. This simulation model differs from the known ones because the modelling of the auger feeder is carried out taking into account the “falling pillar” mass and the temporary delay which continuously change as the materials of different features accumulate in the hopper.

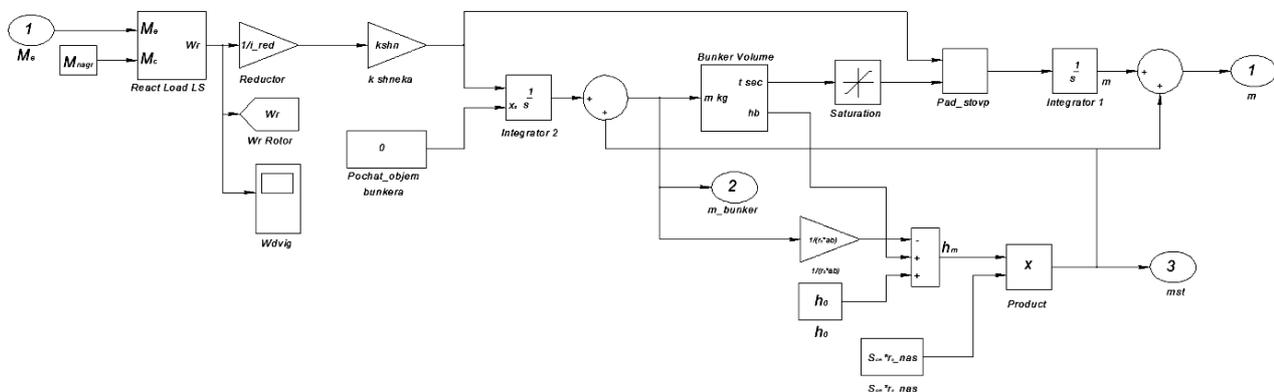


Fig. 10. Simulation model of the auger feeder taking into account the changeable temporary delay and the calculation of the falling pillar mass

### Conclusions

The mathematical and simulation model of the automated dosing system with frequency controlled electric drive of the auger taking into account variable features of the material and dynamically changeable height of the “falling pillar” into the hopper feeder.

The analysis of the dosing process with a help of the simulation model showed that horizontal position of the hopper feeder is the most optimal in terms of productivity.

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