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P. Tymoshchuk,
L'viv Polytechnic National University,
CAD Department

ANALOGUE NEURAL CIRCUIT OF LARGEST MAGNITUDE SIGNAL SET IDENTIFICATION IN UNKNOWN RANGE

A continuous-time analogue neural circuit which is capable of identifying the *K* largest of unknown finite value *N* distinct inputs, where $1 \le K < N$, located in an unknown range is proposed. The circuit model is described by a state equation and by an output equation. A corresponding functional block diagram of the circuit is presented as *N* feed-forward hard-limiting neurons and two feedback neurons, which are used to determine the dynamic shift of inputs. The circuit combines such properties as high accuracy and speed, low hardware implementation complexity, and independency of initial conditions. Simulation example demonstrates that the circuit state variable trajectories are globally stable and globally convergent to the KWTA operation from each initial value.

Keywords: continuous-time model, analogue neural circuit, functional block diagram, feed-forward hard-limiting neuron, hardware implementation complexity, KWTA operation.

1. Introduction

K-winners-take all (KWTA) neural networks are known to choose *K* largest out of a set of *N* inputs, where $1 \le K < N$ is a positive integer [1]. In the special case when *K* is equal to unity, the KWTA network is the winner-takes-all (WTA) one, that determines the maximal out of *N* inputs [2], [3].

KWTA neural networks have various applications, particularly, in data and signal processing, in decision making, for pattern recognition, in competitive learning, and in sorting [4] - [6]. The KWTA networks are used in telecommunications [7] and vision systems [8], for solving problems of filtering [9], decoding [10], image processing [11], clustering [12], and classifying [13], [14]. The KWTA operation is used in machine learning, in mobile robot navigation, and in feature extraction [15], [16]. The KWTA mechanisms are used for modeling cognitive phenomena and spiking neural networks [17], [18].

Continuous-time KWTA neural networks implemented in analogue hardware can be faster, more compact and more power-efficient compared to digital implementations [19]. Many different analogue neural networks have been proposed to solve the KWTA problem [1], [5], [20] - [22]. In particular, a continuous-time model of the KWTA neural circuit which can select the *K* maximal out of *N* unknown inputs, where $1 \le K < N$, located in a definite range of change was proposed in [21]. The operation of the model depends on the initial values of the state variable. A modification of this circuit was derived and simulated [23]. In contrast to the predecessor, the modified circuit is independent of initial conditions and uses a simplified residual function. Computer simulations showed that the circuit convergence speed to the KWTA operation is close to that of one of the fastest Hopfield type analogue KWTA neural networks, whereas a computational and hardware implementation complexity of the circuit is lower than the complexity of this network. The hardware implementation complexity of the circuit is close to that of one of the simplest continuous-time KWTA networks, whereas the convergence time to the KWTA operation of the model is lower than that of this comparable model. A discrete-time version of the circuit model and a corresponding functional block-diagram of a digital neural circuit have been proposed in [24].

In this paper, a generalized continuous-time model and a corresponding functional block-diagram of analogue KWTA neural circuit are presented. In contrast to the predecessors proposed in [21], [23], the circuit is capable of selecting K largest out of N unknown inputs, where $1 \le K < N$, located in an unknown range of change. The circuit is described by a differential equation with discontinuous right-hand side and an output equation. Computer simulations show that the circuit state variable trajectories are globally stable and globally convergent to the KWTA operation from each initial value.

2. Continuous-time model of the circuit

Let us generalize a continuous-time model of an analogue KWTA neural circuit presented in [23] based on the case of identifying K maximal out of N unknown input signals, where $1 \le K < N$, located in an unknown range. We assume that there is an input vector $\boldsymbol{a} = \left(a_{n_1}, a_{n_2}, ..., a_{n_N}\right)^T \in \Re^n$, $1 < N < \infty$ with unknown finite value elements, the inputs are distinct and can be arranged in a descending order of magnitude satisfying the inequalities

$$\infty > a_{n_1} > a_{n_2} > ... > a_{n_N} > -\infty, \tag{1}$$

where $n_1, n_2, ..., n_N$ are the unknown numbers of the first largest input, the second largest input and so on up to the N-th largest input inclusive. Let us design a continuous-time model of an analogue neural circuit that should identify the K largest of these inputs, which are referred to as the winners. The designed model should process the input vector $\boldsymbol{a}$ to obtain, after a finite convergence time, such an output vector $\boldsymbol{b} = \left(b_{n_1}, b_{n_2}, ..., b_{n_N}\right)^T$ that the following KWTA property is satisfied:

$$b_{n_i} > 0, i = 1,2,...,K;\ b_{n_j} < 0, j = K+1, K+2, ..., N. \tag{2}$$

We assume that the outputs of the model are given by

$$\begin{aligned} b_{n_i} &= a_{n_i} - x > 0, i = 1,2,...,K; \\ b_{n_j} &= a_{n_j} - x < 0, j = K+1, K+2, ..., N, \end{aligned} \tag{3}$$

where x is a scalar dynamic shift of inputs [21].

Let us describe the model of a designed KWTA neural circuit by the following state equation:

$$dx/dt = \alpha\left(|x| + p\right)\left(\sum_{k=1}^{N} S_k(x) - K\right), \tag{4}$$

and an output equation

$$b_{n_k} = a_{n_k} - x, k = 1,2,...,N, \tag{5}$$

where

$$R(x) = \sum_{k=1}^{N} S_k(x) - K \tag{6}$$

is a residual function,

$$S_k(x) = \begin{cases} 1, & \text{if} \quad a_{n_k} - x > 0; \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

is a step function, $\sum_{k=1}^{N} S_k(x)$ is the number of positive outputs of the model, α is the gain which can be used to control a convergence speed of the model state variable trajectories to the KWTA operation, p is a constant parameter, $-\infty < x_0 < \infty$ is an initial condition. Note that the state equation (4) can also be transformed to the following special form:

$$dx/dt = \gamma\left(|x| + c\right) sgn\left(R(x)\right) \tag{8}$$

where

$$sgn\left(R(x)\right) = \begin{cases} 1, & \text{if} \quad R(x) > 0; \\ 0, & \text{if} \quad R(x) = 0; \\ -1, & \text{if} \quad R(x) < 0 \end{cases} \tag{9}$$

is a signum (hard-limiting) function, γ is a gain, and c is a constant parameter.

3. Analogue functional block-diagram of the circuit

The functional block-diagram of a generalized analogue KWTA neural circuit built based on the model described by the state equation (4) and output equation (5) is shown in Fig. 1. The diagram consists of inputs $a_1, a_2, ..., a_N$, summers $\sum$, an integrator I with the gain α, external sources of constant signals K, x_0, p, blocks $S_1, S_2, ..., S_N$ of step functions $S_k(x)$, $k=1,2,3,...,N$, outputs $b_1, b_2, ..., b_N$, block of multiplication ×, and a block of module function *Abs*. Note that the outputs of blocks $S_1, S_2, ..., S_N$ can also be used as the circuit outputs. However, in this case, the only K winners out of N inputs will be identified. No information is available on the ordering of inputs by magnitude which can be useful for some applications [25].

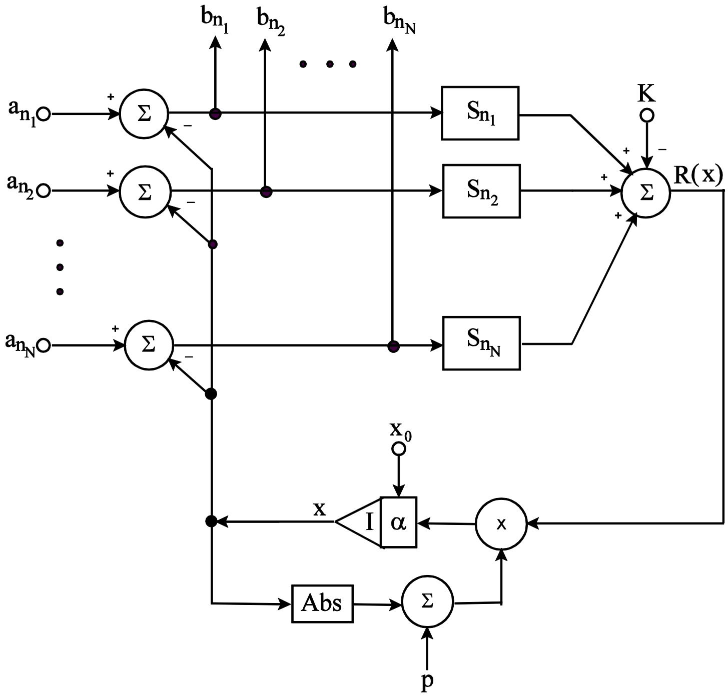

Fig. 1. *Architecture of the KWTA neural circuit described by a state equation (4) and by an output equation (5).*

As we can see, from an analogue hardware implementation complexity point of view, the circuit contains $N+2$ summers, N switches, one integrator, one multiplier, one absolute function block and three sources of constant signals (or two sources of constant signals if $x_0 = 0$). Note that the block Abs can be realized, for instance, by using a switch and an inverter. Therefore, the circuit presented can be implemented in modern hardware using such traditional electronic circuit components as analogue summers, multiplier, inverter, switches, integrator, and sources of constant voltage or current. For comparison, the previous continuous-time model of analogue KWTA neural circuit (A circuit) presented in [23] needs $N+2$ summers, $N+2$ switches, one integrator and four sources of constant signals (or three sources of constant signals if $x_0 = 0$). An implementation of one of the simplest KWTA networks with a single state variable and the Heaviside step activation function, presented in [20], requires $N+1$ summers, N switches, one integrator and one source of constant signals. Thus, the hardware implementation complexity of the circuit described by a state equation (4) and by an output equation (5) is close to that of these comparable analogs.

A resolution of the circuit is theoretically infinite and does not depend on its parameter values. Since the circuit is capable of correctly processing any finite value distinct inputs, its resolution is the same as that in other comparable neural networks with the same property [1], [20], [21], [23].

Since the present circuit can operate correctly with any finite initial condition $-\infty < x_0 < \infty$, it requires neither a periodical resetting for repetitive processing of input sets, nor corresponding analogue supervisory circuit, nor spending additional processing time. This simplifies the hardware and decreases the convergence time to the KWTA operation.

4. Computer simulation results

Let us consider the example with corresponding computer simulations which illustrates the performance of the herein presented analogue KWTA neural circuit.

Example. We set 200 uniformly distributed random initial values $x_0 \in [-250, 250]$ of inputs a_{n_k}, $k=1,2,3,\ldots,N$ uniformly distributed within the interval *[-250, 250]* for $N=400$, $K=100$, $\alpha = 10^6$, and $p=1$. A 1.81 GHz desktop PC and the variable order Adams-Bashforth-Moulton solver of non-stiff differential equations ODE113 with relative and absolute error tolerances equal to 10^{-5} are employed. Fig. 2 presents, in normalized units, the state variable transient behaviors showing that the state variable trajectories are globally stable and globally convergent to the KWTA operation from each initial value.

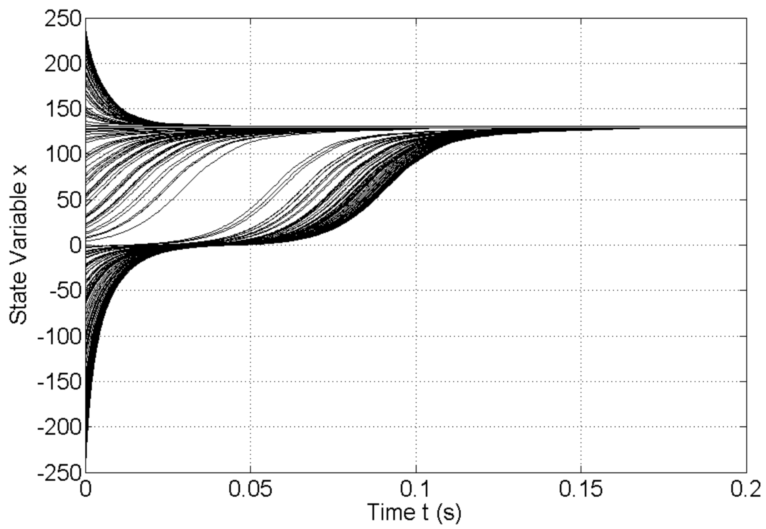

Fig. 2. *Convergence behaviors of state variable x of the KWTA circuit model (4), (5) with inputs* a_{n_k}, *$k=1,2,3,\ldots,N$, uniformly distributed in the interval [-250,250], where N=400, K=100,* $\alpha = 10^6$*, starting from 200 uniformly distributed random initial values* $x_0 \in [-250, 250]$.

5. Conclusions

This paper presents a continuous-time mathematical model and a corresponding functional block-diagram of an analogue *K*-winners-take-all neural circuit. In contrast to the predecessor, the proposed KWTA neural circuit is capable of selecting *K* maximal among unknown finite value *N* distinct inputs located in an unknown range, where $1 \le K < N$. The hardware implementation complexity of the proposed KWTA circuit is close to that in these comparable analogs. Computer simulations show that the circuit is globally stable and globally convergent to the KWTA operation from each initial value.

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