COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 3, No. 2, 2013

ANALYTICAL RELIABILITY MODEL OF A REDUNDANT REPAIRABLE SYSTEM WITH LIMITED NUMBER OF RESTORATIONS

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Abstract: Modern complex mission-critical systems are built as fault-tolerant systems, i.e. having the ability to function while some of their separate elements have a fault. The complexity of such fault-tolerant systems makes their reliability models quite big and complex. Therefore, such requirements for building models, as the high level of credibility and the appropriate level of formalization, are imposed on these models, which allows to implement the automation to their creating and analysis of reliability and, therefore, using modern computer tools. The combination of analytical methods for the research on reliability and computing capability of modern computers is a promising direction for further development of methods of reliability theory.

Key words: reliability, fault-tolerant systems, modelling, repairable systems

1. Introduction

The task of technical mission-critical systems is a continuous supply of information messages, energy resources etc. to users. Such systems are developed in such a way that the duration of their downtime caused by failures of the constituent elements of the system would be as small as possible. For this purpose the reservation (in most cases duplication) and renovation (repairing) of the components is used. At the design stage the developer should carry out the multivariant analysis of reliability indices of the system and select the best variant based on the comparative analysis of the reliability characteristics of design alternatives. The difficulty of this task lies in the fact that the mathematical reliability models of redundant and restorable systems are very complex, because these systems are characterized by a large number of states. There are known the analytical expressions for calculating of reliability indices of restorable systems with an unlimited number of restorations [1, 2, 3], but the assessment of reliability indices of systems with a limited number of restorations is of significant practical interest that is dictated by the terms of maintenance and economic indicators. Multi-analysis becomes practically impossible without the use of computer tools that allows you to get results of the analysis only for the specific numeric values of the parameters of system components. The prospective direction of development the multivariate reliability analysis methods of technical systems is the combination of general analytical methods with the computing capabilities of modern software [4].

In this paper, the analytical model of redundant repairable system with a limited number of restorations has been developed, whose constituent elements, each with single redundance (loaded reservation), are connected in series according to their reliability. While forming the model, the repeteability of structure topological properties has been used, that allows for simplifying the analysis of the time dependence of the probability of staying in specific states (normal operation, downtime (repairing), catastrophic failure) and calculating the reliability indicators. unit

2. Mathematical model of system reliability

The structure of the analyzed system is shown in Fig.1 in the form of series connection of N units, each of which is a pair of modules with loaded reserve. A repair team maintains every unit with the direct recovery priority. Modules of the system can be restored only once. With repeated failure of the module it stays invalid (inoperative) and cannot be repaired. If the component fails while the repair team is restoring another module, it waits for recovery in the queue.

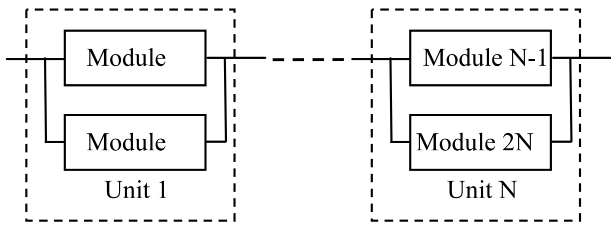

Fig. 1. Structure of the system.

The system passes to downtime condition when both modules of any link, that may be recovered, fail, or at least one of them can be recovered. If both modules were previously recovered, during their both repeated failure the system passes to a state of catastrophic failure.

Taking into account the system way of functioning, we can conclude that any module of the system can be in one of the following states:

- *F*- workability (normal functioning);
- *F*1- workability (normal functioning) after the first recovery;
- *R* – recovery (repair) state;
- *OR* - waiting its turn for repair in the absence of a free repair body;
- *ORK* - final failure state after carrying out the allowable number of recoveries;

The following component transitions from one state to another are possible: *F* → *R* → *F1* → *ORK*, or *F* → *OR* → *R* → *F1* → *ORK*. Note that the transitions F → R, *F* → *OR, F1* → *ORK* mean the failure of the unit, when transitions *R* → *F, R* → *F1* denote its recovery.

In its turn, any unit can dwell in one of the following conditions:

- *F*- unit workability state, when both its components are functioning without failure;
- *FR* - unit workability state, when one of its components is functioning without failure, while the other is in R state;
- *F1* - unit workability state, when one of its components is functioning without failure, and other is functioning after the first recovery;
- *F1R* - unit workability state, when one its components is functioning after the first recovery, and the other is in R state;
- *F11* - unit workability state, when both components are functioning after the first recovery;
- *FK* - unit workability state, when one of its components is functioning without failure, while the other is in ORK state;
- *F1K* - unit workability state, when one its components is functioning after the first recovery, and the other is in *ORK* state;
- *R1* - unit recovery (repair) state, in which one of its components is in *R* state, and the second – in *OR* state;
- *R1K* - unit recovery(repair) state, in which one of its components is in R state, and another one is in *ORK* state;
- *ORK* - final invalid unit state, in which both components are in *ORK* state;
- *PF* - state of work (or downtime) termination of workable unit caused by the failure or recovery of another unit of the system.

Such unit transitions from one state to another are possible:

F → *FR* → *F1* → *FK* → *R1K* → *F1K* → *ORK;*

and also:

F → *FR* → *R1* → *F1R* → *F11* → *F1K* → *ORK.*

Note that the unit can pass from any working state to functioning termination (downtime) state *PF*.

The matrix of an individual unit state is shown in Table 1.

Considering its content, we can see that states 0, 1, 2, 3, 4, 7, 8, 9, 10, 11, 14, 15 represent the unit workability, states 5, 6, 12, 13 are the states of recovery (repair) causing system downtime, and the 16th state is a unit final failure state which causes the final failure of the system as a whole.

Information about possible unit transitions from one state to another is described by means of a transition matrix, whose format is 2x17. The A_{ij} element of this matrix is the number of state to which the unit passes from the *j*-th state as a result of state changes of the *i*-th unit component (and = 1,2; *j* = 0,1, ..., 16).

The unit transition matrix is presented in Table 2:

Table 1

Matrix of individual unit state

State № / Unit №	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	*F*	*R*	*F*	*F1*	*F*	*R*	*OR*	*ORK*	*F1*	*R*	*F*	*F1*	*ORK*	*R*	*ORK*	*F1*	*ORK*
2	*F*	*F*	*R*	*F*	*F1*	*OR*	*R*	*F*	*R*	*F1*	*ORK*	*F1*	*R*	*ORK*	*F1*	*ORK*	*ORk*
State of modul	*F*	*FR*	*FR*	*F1*	*F1*	*R1*	*R1*	*FK*	*F1R*	*F1R*	*FK*	*F11*	*R1K*	*R1K*	*F1K*	*F1K*	*ORk*

Table 2

Unit transition matrix

State	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Module 1	1	3	6	7	9	8	-	-	-	11	13	14	-	15	-	16	-
Module 2	2	5	4	8	10	-	9	12	11	-	-	15	14	-	16	-	-

The parameters of an individual unit are λ, μ denoting the intensity of failures and recoveries of its components accordingly. In this case, the time dependences of unit probability of staying in certain condition is described by the following system of the Chapman– Kolmogorov equations (1):

Initial unit state at t = 0:

$P_0(0)=1$;

$P_1(0)=P_2(0)=P_3(0)=P_4(0)=P_5(0)=P_6(0)=P_7(0)=P_8(0)=P_9(0)=P_{10}(0)=P_{11}(0)=P_{12}(0)=P_{13}(0)=P_{14}(0)=P_{15}(0)=P_{16}(0)=0$.

$$\begin{aligned}
\frac{dP_0(t)}{dt} &= -(\lambda_1+\lambda_2)\cdot P_0(t) \\
\frac{dP_1(t)}{dt} &= \lambda_1\cdot P_0(t)-(\mu_1+\lambda_2)\cdot P_1(t) \\
\frac{dP_2(t)}{dt} &= \lambda_2\cdot P_0(t)-(\mu_2+\lambda_2)\cdot P_2(t) \\
\frac{dP_3(t)}{dt} &= \mu_1\cdot P_1(t)-(\lambda_1+\lambda_2)\cdot P_3(t) \\
\frac{dP_4(t)}{dt} &= \mu_2\cdot P_2(t)-(\lambda_1+\lambda_2)\cdot P_4(t) \\
\frac{dP_5(t)}{dt} &= \lambda_1\cdot P_1(t)-\mu_1\cdot P_5(t) \\
\frac{dP_6(t)}{dt} &= \lambda_1\cdot P_2(t)-\mu_2\cdot P_6(t) \\
\frac{dP_7(t)}{dt} &= \lambda_1\cdot P_3(t)-\lambda_2\cdot P_7(t) \\
\frac{dP_8(t)}{dt} &= \lambda_2\cdot P_3(t)-\mu_1\cdot P_5(t)-\mu_2\cdot P_8(t) \\
\frac{dP_9(t)}{dt} &= \lambda_1\cdot P_4(t)+\mu_2\cdot P_6(t)-\mu_1\cdot P_9(t) \\
\frac{dP_{10}(t)}{dt} &= \lambda_2\cdot P_4(t)-\lambda_1\cdot P_{10}(t) \\
\frac{dP_{11}(t)}{dt} &= \mu_2\cdot P_8(t)+\mu_1\cdot P_9(t)-(\lambda_1+\lambda_2)\cdot P_{10}(t) \\
\frac{dP_{12}(t)}{dt} &= \lambda_2\cdot P_7(t)-\mu_2\cdot P_{12}(t) \\
\frac{dP_{13}(t)}{dt} &= \lambda_1\cdot P_{10}(t)-\mu_1\cdot P_{13}(t) \\
\frac{dP_{14}(t)}{dt} &= \lambda_1\cdot P_{11}(t)+\mu_2\cdot P_{12}(t)-\lambda_1\cdot P_{14}(t) \\
\frac{dP_{15}(t)}{dt} &= \lambda_2\cdot P_{11}(t)+\mu_1\cdot P_{13}(t)-\lambda_1\cdot P_{15}(t) \\
\frac{dP_{16}(t)}{dt} &= \lambda_2\cdot P_{14}(t)+\lambda_1\cdot P_{15}(t)
\end{aligned} \tag{1}$$

It is advisable to solve the Chapman– Kolmogorov equations with the help of the Laplace transform, which allows accessing to the system of algebraic equations, recorded in relation to operator images of time probability dependences, by solving which we find the time probability dependences by using the inverse Laplace transform. For this purpose we can use the software MATLAB or MATHCAD in the field of symbolic mathematics.

Assume that both unit components are characterized by the same level of failure and repairs. As a result we obtain:

$$P_0(t)=e^{-2\cdot\lambda\cdot t}$$

$$P_1(t)=P_2(t)=-\frac{\lambda\cdot e^{-(\lambda+\mu)\cdot t}-\lambda\cdot e^{-2\cdot\lambda\cdot t}}{\mu-\lambda}$$

$$P_3(t)=P_4(t)=\frac{\mu\cdot\lambda\cdot e^{-(\lambda+\mu)\cdot t}-\mu\cdot\lambda\cdot e^{-2\cdot\lambda\cdot t}-\mu\cdot\lambda^2\cdot t\cdot e^{-2\cdot\lambda\cdot t}+\mu^2\cdot\lambda\cdot t\cdot e^{-2\cdot\lambda\cdot t}}{(\mu-\lambda)^2}$$

$$P_5(t)=P_6(t)=\frac{\lambda^2\cdot e^{-\mu\cdot t}+\lambda^2\cdot e^{-2\cdot\lambda\cdot t}-2\cdot\lambda^2\cdot e^{-(\lambda+\mu)\cdot t}+\mu\cdot\lambda\cdot e^{-(\lambda+\mu)\cdot t}-\mu\cdot\lambda\cdot e^{-\mu\cdot t}}{\mu^2-3\cdot\mu\cdot\lambda+2\cdot\lambda^2}$$

$$P_7(t)=P_{10}(t)=-\frac{\mu^2\cdot e^{-\lambda\cdot t}+\mu^2\cdot e^{-2\cdot\lambda\cdot t}+\lambda^2\cdot e^{-\lambda\cdot t}-\lambda^2\cdot e^{-(\lambda+\mu)\cdot t}-2\cdot\mu\cdot\lambda\cdot e^{-\lambda\cdot t}}{(\mu-\lambda)^2}+$$
$$+\frac{2\cdot\mu\cdot\lambda\cdot e^{-2\cdot\lambda\cdot t}+\mu^2\cdot e^{-\lambda\cdot t}+\mu\cdot\lambda^2\cdot t\cdot e^{-2\cdot\lambda\cdot t}+\mu^2\cdot\lambda\cdot t\cdot e^{-2\cdot\lambda\cdot t}}{(\mu-\lambda)^2}$$

$$P_8(t)=P_9(t)=-\left[\frac{\mu^3\cdot e^{-(\lambda+\mu)\cdot t}-\mu^3\cdot e^{-\mu\cdot t}-2\cdot\mu^2\cdot\lambda\cdot e^{-(\lambda+\mu)\cdot t}-\mu\cdot\lambda^2\cdot e^{-\mu\cdot t}+2\cdot\mu^2\cdot\lambda\cdot e^{-\mu\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)}+\right.$$

$$+\frac{\mu\cdot\lambda^2\cdot e^{-2\cdot\lambda\cdot t}-2\cdot\mu^2\cdot\lambda^2\cdot t\cdot e^{-\mu\cdot t}-\mu^2\cdot\lambda^2\cdot t\cdot e^{-2\cdot\lambda\cdot t}+\mu\cdot\lambda^3\cdot t\cdot e^{-\mu\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)}+$$

$$\left.+\frac{\mu^3\cdot\lambda\cdot t\cdot e^{-\mu\cdot t}+\mu\cdot\lambda^3\cdot t\cdot e^{-2\cdot\lambda\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)}\right]$$

$$P_{11}(t)=\frac{2\cdot\mu^5\cdot\lambda\cdot e^{-(\lambda+\mu)\cdot t}-2\cdot\mu^5\cdot\lambda\cdot e^{-\mu\cdot t}-16\cdot\mu^2\cdot\lambda^4\cdot e^{-(\lambda+\mu)\cdot t}+24\cdot\mu^3\cdot\lambda^3\cdot e^{-(\lambda+\mu)\cdot t}}{(\mu^2-3\cdot\mu\cdot\lambda+2\cdot\lambda^2)^3}+$$

$$+\frac{-12\cdot\mu^4\cdot\lambda^2\cdot e^{-(\lambda+\mu)\cdot t}+2\cdot\mu^2\cdot\lambda^4\cdot e^{-\mu\cdot t}-6\cdot\mu^3\cdot\lambda^3\cdot e^{-\mu\cdot t}+6\cdot\mu^4\cdot\lambda^2\cdot e^{-\mu\cdot t}}{(\mu^2-3\cdot\mu\cdot\lambda+2\cdot\lambda^2)^3}+$$

$$+\frac{14\cdot\mu^2\cdot\lambda^4\cdot e^{-2\cdot\lambda\cdot t}-18\cdot\mu^3\cdot\lambda^3\cdot e^{-2\cdot\lambda\cdot t}+6\cdot\mu^4\cdot\lambda^2\cdot e^{-2\cdot\lambda\cdot t}+12\cdot\mu^2\cdot\lambda^5\cdot t\cdot e^{-2\cdot\lambda\cdot t}}{(\mu^2-3\cdot\mu\cdot\lambda+2\cdot\lambda^2)^3}+$$

$$+\frac{-26\cdot\mu^3\cdot\lambda^4\cdot t\cdot e^{-2\cdot\lambda\cdot t}+18\cdot\mu^4\cdot\lambda^3\cdot t\cdot e^{-2\cdot\lambda\cdot t}-4\cdot\mu^5\cdot\lambda^2\cdot t\cdot e^{-2\cdot\lambda\cdot t}+4\cdot\mu^2\cdot\lambda^6\cdot t^2\cdot e^{-2\cdot\lambda\cdot t}}{(\mu^2-3\cdot\mu\cdot\lambda+2\cdot\lambda^2)^3}+$$

$$+\frac{-12\cdot\mu^3\cdot\lambda^5\cdot t^2\cdot e^{-2\cdot\lambda\cdot t}+13\cdot\mu^4\cdot\lambda^4\cdot t^2\cdot e^{-2\cdot\lambda\cdot t}-6\cdot\mu^5\cdot\lambda^3\cdot t^2\cdot e^{-2\cdot\lambda\cdot t}+\mu^6\cdot\lambda^2\cdot t^2\cdot e^{-2\cdot\lambda\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)^3}+$$

$$+\frac{2\cdot\mu^5\cdot e^{-(\lambda+\mu)\cdot t}-2\cdot\mu^5\cdot e^{-\mu\cdot t}-12\cdot\mu^4\cdot\lambda\cdot e^{-(\lambda+\mu)\cdot t}+12\cdot\mu^4\cdot\lambda\cdot e^{-\mu\cdot t}-16\cdot\mu^2\cdot\lambda^3\cdot e^{-(\lambda+\mu)\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)^3}+$$

$$+\frac{24\cdot\mu^3\cdot\lambda^2\cdot e^{-(\lambda+\mu)\cdot t}8\cdot\mu^2\cdot\lambda^3\cdot e^{-\mu\cdot t}-18\cdot\mu^2\cdot\lambda^3\cdot e^{-2\cdot\lambda\cdot t}-6\cdot\mu^3\cdot\lambda^2\cdot e^{-2\cdot\lambda\cdot t}-4\cdot\mu^2\cdot\lambda^4\cdot t\cdot e^{-\mu\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)^3}+$$

$$+\frac{2\cdot\mu^4\cdot\lambda^2\cdot t\cdot e^{-2\cdot\lambda\cdot t}+2\cdot\mu^5\cdot\lambda\cdot t\cdot e^{-\mu\cdot t}-8\cdot\mu^4\cdot\lambda^2\cdot t\cdot e^{-\mu\cdot t}+4\cdot\mu^2\cdot\lambda^4\cdot t\cdot e^{-2\cdot\lambda\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)^3}+$$

$$+\frac{-6\cdot\mu^3\cdot\lambda^3\cdot t\cdot e^{-2\cdot\lambda\cdot t}+2\cdot\mu^4\cdot\lambda^2\cdot t\cdot e^{-2\cdot\lambda\cdot t}+2\cdot\mu^5\cdot\lambda\cdot t\cdot e^{-\mu\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)^3}$$

$$P_{12}(t)=P_{13}(t)=-\frac{4\cdot\lambda^4\cdot e^{-\lambda\cdot t}-4\cdot\lambda^4\cdot e^{-(\lambda+\mu)\cdot t}+4\cdot\lambda^3\cdot\mu\cdot e^{-(\lambda+\mu)\cdot t}-\mu\cdot\lambda^3\cdot e^{-\mu\cdot t}-8\cdot\mu\cdot\lambda^3\cdot e^{-\lambda\cdot t}}{(\mu^2-3\cdot\mu\cdot\lambda+2\cdot\lambda^2)^2}+$$

$$+\frac{-\mu^3\cdot\lambda\cdot e^{-\lambda\cdot t}+5\cdot\mu\cdot\lambda^3\cdot e^{-2\cdot\lambda\cdot t}+\mu^3\cdot\lambda\cdot e^{-2\cdot\lambda\cdot t}-\lambda^2\cdot\mu^2\cdot e^{-(\lambda+\mu)\cdot t}+\mu^2\cdot\lambda^2\cdot e^{-\mu\cdot t}}{(\mu^2-3\cdot\mu\cdot\lambda+2\cdot\lambda^2)^2}+$$

$$+\frac{5\cdot\mu^2\cdot\lambda^2\cdot e^{-\lambda\cdot t}-5\cdot\mu^2\cdot\lambda^2\cdot e^{-2\cdot\lambda\cdot t}-3\cdot\mu^2\cdot\lambda^3\cdot e^{-2\cdot\lambda\cdot t}+\mu^3\cdot\lambda^2\cdot e^{-2\cdot\lambda\cdot t}+2\cdot\mu\cdot\lambda^4\cdot e^{-2\cdot\lambda\cdot t}}{(\mu^2-3\cdot\mu\cdot\lambda+2\cdot\lambda^2)^2}$$

$$P_{14}(t)=P_{15}(t)=\frac{2\cdot\mu^6\cdot e^{-2\cdot\lambda\cdot t}-2\cdot\mu^6\cdot e^{-\lambda\cdot t}-16\cdot\mu\cdot\lambda^5\cdot e^{-(\lambda+\mu)\cdot t}+16\cdot\mu\cdot\lambda^5\cdot e^{-\lambda\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^3}+$$

$$+\frac{16\cdot\mu^5\cdot\lambda\cdot e^{-2\cdot\lambda\cdot t}-16\cdot\mu^5\cdot\lambda\cdot e^{-2\cdot\lambda\cdot t}+24\cdot\mu^2\cdot\lambda^4\cdot e^{-(\lambda+\mu)\cdot t}-12\cdot\mu^3\cdot\lambda^3\cdot e^{-(\lambda+\mu)\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^3}+$$

$$+\frac{2\cdot\mu^4\cdot\lambda^2\cdot e^{-(\lambda+\mu)\cdot t}-2\cdot\mu^2\cdot\lambda^4\cdot e^{-\mu\cdot t}+4\cdot\mu^3\cdot\lambda^3\cdot e^{-\mu\cdot t}-2\cdot\mu^4\cdot\lambda^2\cdot e^{-\mu\cdot t}-56\cdot\mu^2\cdot\lambda^4\cdot e^{-\lambda\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^3}+$$

$$+\frac{76\cdot\mu^3\cdot\lambda^3\cdot e^{-\lambda\cdot t}-50\cdot\mu^4\cdot\lambda^2\cdot e^{-\lambda\cdot t}34\cdot\mu^2\cdot\lambda^4\cdot e^{-2\cdot\lambda\cdot t}-68\cdot\mu^3\cdot\lambda^3\cdot e^{-2\cdot\lambda\cdot t}+50\cdot\mu^4\cdot\lambda^2\cdot e^{-2\cdot\lambda\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^3}+$$

$$+\frac{20\cdot\mu^2\cdot\lambda^5\cdot t\cdot e^{-2\cdot\lambda\cdot t}-50\cdot\mu^3\cdot\lambda^4\cdot t\cdot e^{-2\cdot\lambda\cdot t}44\cdot\mu^4\cdot\lambda^3\cdot t\cdot e^{-2\cdot\lambda\cdot t}-16\cdot\mu^5\cdot\lambda^2\cdot t\cdot e^{-2\cdot\lambda\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^3}+$$

$$+\frac{4\cdot\mu^2\cdot\lambda^6\cdot t^2\cdot e^{-2\cdot\lambda\cdot t}-12\cdot\mu^3\cdot\lambda^5\cdot t^2\cdot e^{-2\cdot\lambda\cdot t}13\cdot\mu^4\cdot\lambda^4\cdot t^2\cdot e^{-2\cdot\lambda\cdot t}-6\cdot\mu^5\cdot\lambda^3\cdot t^2\cdot e^{-2\cdot\lambda\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^3}+$$

$$+\frac{\mu^6\cdot\lambda^2\cdot t^2\cdot e^{-2\cdot\lambda\cdot t}+2\cdot\mu^6\cdot\lambda\cdot t\cdot e^{-2\cdot\lambda\cdot t}+2\cdot\mu^4\cdot e^{-2\cdot\lambda\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^3}+$$

$$+\frac{-2\cdot\mu^4\cdot e^{-\lambda\cdot t}+4\cdot\lambda^4\cdot e^{-\lambda\cdot t}-4\cdot\lambda^4\cdot e^{-(\lambda+\mu)\cdot t}+4\cdot\mu\cdot\lambda^3\cdot e^{-(\lambda+\mu)\cdot t}-4\cdot\mu\cdot\lambda^3\cdot e^{-\lambda\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^2}+$$

$$+\frac{8\cdot\mu^3\cdot\lambda\cdot e^{-\lambda\cdot t}-\mu^2\cdot\lambda^2\cdot e^{-(\lambda+\mu)\cdot t}+\mu^2\cdot\lambda^2\cdot e^{-\mu\cdot t}-7\cdot\mu^2\cdot\lambda^2\cdot e^{-\lambda\cdot t}+7\cdot\mu^2\cdot\lambda^2\cdot e^{-2\cdot\lambda\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^2}+$$

$$+\frac{8\cdot\mu^2\cdot\lambda^3\cdot t\cdot e^{-\lambda\cdot t}-5\cdot\mu^3\cdot\lambda^2\cdot t\cdot e^{-\lambda\cdot t}+2\cdot\mu^2\cdot\lambda^3\cdot t\cdot e^{-2\cdot\lambda\cdot t}-3\cdot\mu^3\cdot\lambda^2\cdot t\cdot e^{-2\cdot\lambda\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^2}+$$

$$+\frac{-4\cdot\mu\cdot\lambda^4\cdot t\cdot e^{-\lambda\cdot t}+\mu^4\cdot\lambda\cdot t\cdot e^{-\lambda\cdot t}+\mu^4\cdot\lambda\cdot t\cdot e^{-2\cdot\lambda\cdot t}+2\cdot\mu^4\lambda\cdot e^{-(\lambda+\mu)\cdot t}}{(\mu^2-3\cdot\lambda\cdot\mu+2\cdot\lambda^2)^2}+$$

$$+\frac{-16\cdot\mu\cdot\lambda^4\cdot e^{-(\lambda+\mu)\cdot t}-2\cdot\mu^4\cdot\lambda\cdot e^{-\mu\cdot t}+16\cdot\mu\cdot\lambda^4\cdot e^{-\lambda\cdot t}-2\cdot\mu^4\cdot\lambda\cdot e^{-\lambda\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)^3}+$$

$$+\frac{2\cdot\mu^4\cdot\lambda\cdot e^{-2\cdot\lambda\cdot t}24\cdot\mu^2\cdot\lambda^3\cdot e^{-(\lambda+\mu)\cdot t}-12\cdot\mu^3\cdot\lambda^2\cdot e^{-(\lambda+\mu)\cdot t}-12\cdot\mu^2\cdot\lambda^3\cdot e^{-\mu\cdot t}+12\cdot\mu^3\cdot\lambda^2\cdot e^{-\mu\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)^3}+$$

$$+\frac{-24\cdot\mu^2\cdot\lambda^3\cdot e^{-\lambda\cdot t}12\cdot\mu^3\cdot\lambda^2\cdot e^{-\lambda\cdot t}+12\cdot\mu^2\cdot\lambda^3\cdot e^{-2\cdot\lambda\cdot t}+4\cdot\mu^2\cdot\lambda^4\cdot t\cdot e^{-\mu\cdot t}-6\cdot\mu^3\cdot\lambda^3\cdot t\cdot e^{-\mu\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)^3}+$$

$$+\frac{2\cdot\mu^4\cdot\lambda^2\cdot t\cdot e^{-\mu\cdot t}4\cdot\mu^2\cdot\lambda^4\cdot t\cdot e^{-2\cdot\lambda\cdot t}-6\cdot\mu^3\cdot\lambda^3\cdot t\cdot e^{-2\cdot\lambda\cdot t}+2\cdot\mu^4\cdot\lambda^2\cdot t\cdot e^{-2\cdot\lambda\cdot t}}{(\mu-\lambda)^2\cdot(\mu-2\cdot\lambda)^3}$$

From these analytical solutions probabilities can be determined:

unit workability state:

$$P_F(t)=P_0(t)+P_1(t)+P_2(t)+P_3(t)+P_4(t)+P_7(t)+P_8(t)+ +P_9(t)+P_{10}(t)+P_{11}(t)+P_{14}(t)+P_{15}(t);$$

recovery (repair) state:

$$P_R(t)=P_5(t)+P_6(t)+P_{12}(t)+P_{13}(t);$$

final invalid state:

$$P_K(t)=P_{16}(t)=1-\sum_{i=1}^{15}P_i(t);$$

Considering the system as a whole, we note that its state at any point of time is determined by the state of all units at the same time. The system as a whole may be in one of the following states:

- *F* - normal functioning if all parts are in working condition;
- *R*-recovery (repair) if any unit is in *R1* or *R1K* state, and others are in PF state;
- *V*-catastrophic failure, if any unit moves to ORK state. The probabilities of the system staying in specific conditions are determined using the expressions for the probability of individual units being in their respective states.
- The probability of system staying in the state of normal functioning

$$P_F(t)=P_{F1}(t)\cdot P_{F2}(t)\cdot\ldots\cdot P_{FN}(t),$$

- The probability of system staying in the state of recovery (repair)

$$P_R(t)=P_{R1}(t)\cdot P_{F2}(t)\cdot\ldots\cdot P_{FN}(t)+P_{F1}(t)\cdot P_{R2}(t)\cdot\ldots\cdot P_{FN}(t) +P_{F}1(t)\cdot P_{F2}(t)\cdot P_{R3}(t)\cdot\ldots\cdot P_{FN}(t)+\ldots+P_{F1}(t)\cdot P_{F2}(t)\cdot P_{R3}(t)\cdot\ldots\cdot P_{RN}(t),$$

where $P_{Fi}(t)$ is the probability of the *i*-th unit staying in working condition;

- $P_{Ri}(t)$ - the probability of the i-th unit dwelling in a state of recovery (repair)

$$P_V(t)=P_{K1}(t)+P_{K2}(t)+\ldots+P_{KN}(t)=1-[P_F(t)+P_R(t)].$$

- $P_{Ki}(t)$ is the probability of system staying in a state of catastrophic failure.

3. Example of the application of the developed mathematical model

The subject of research is a fault-tolerant technical system, which consists of one module in working configuration and one module in a hot standby (both have equal failure rates: λ1 = λ2). When the working configuration module fails, it is replaced from reserve. Modules that are out of order, queue up for the repair. After repairing, the modules are transferred to a functioning state or to reserve. Repairs are carried out with intensity μ. Total number of repairs of each module are limited to one.

For the formation of reliability indices, (namely, availability function – probabilities of staying in working condition, downtime probabilities – repair probabilities) let us use the analytical solutions of the system of the Chapman-Kolmogorov differential equations, obtained above.

The research was carried out with the following inputs:

- failure intensity of basic and backup module (hot standby) - 1 * 10-3 1 /hour
- repair intensity - 0.001, 0.002, 0.05, 0.01, 0.1 1/hour.

When analyzing the system reliability, it has been found out that ignoring the downtime probability for systems, the repair of which is significant, leads to a significant overstating of availability function (Fig. 2). Moreover, the probability of downtime depends on the ratio of the reliability of components of fault-tolerant system (intensity of failures - λ) and repair parameters (intensity of recovery - μ). If the system failures are rare and quickly repaired, and repair time is less than 1% of the average time between failures, the downtime practically does not influence on the availability function.

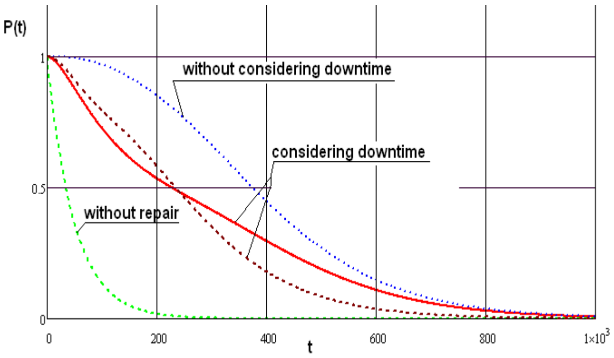

Fig.2. Dependence of the availability function on the time with downtime taking into account.

The research shows that the probability of downtime depends on the intensity ratio of failures and recoveries (Fig. 3). With shortening the duration of repair the probability of downtime decreases. Moreover, for systems in which a repair is done 100 times faster than the system fails, the probability of downtime can be ignored.

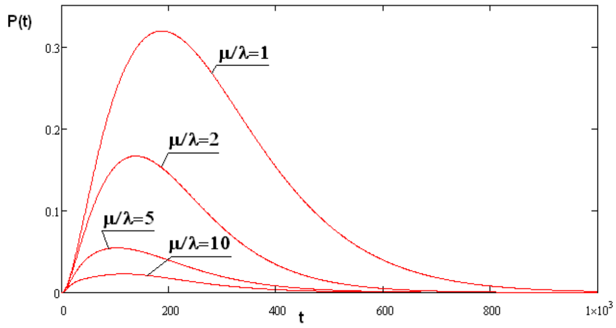

Fig.3. The dependence of the probability of downtime intensity ratio of failures and recoveries.

Therefore, taking into account the probability of downtime is necessary in cases where the system often fails and repair time is equal to or is more than 10% of the average time between failures.

Conclusion

The proposed method of forming the mathematical model of system reliability as a whole based on the reliability model of separate unit simplifies the analysis, because it does not require the construction of state matrix and transition matrix of the whole system, and is easily applied in the case of systems with an arbitrary number of units.

Using the developed analytical reliability model is convenient for solving the optimization task of selection the parameters of separate units by the criterion of minimum probability of system downtime.

References

[1] V.A. Belyayev, V.V. Bogatyryev, Bolotin, at al., *Reliability of complex systems. Handbook,* ed. I.A. Ushakov, Moscow: Radio i svyaz, 1985. (Russian)

[2] A.M Polovko and S.V. Gurov, *Fundamentals of reliability theory,* BHV- Peterburg, 2006. (Russian)

[3] A Birolini, *Reliability engineering. Theory and practice.* Berlin, Germany: Springer, 2004.

[4] H Pham, *Handbook of reliability engineering.* London: British Library Cataloguing in Publication Data, 2003.

[5] Y. Bobalo, B. Mandziy, B. Volochiy, and L. Osirkovskyy, “Reliability analysis of technical systems without redundancy with limited number of repairs”, in *Proc. XI International scientific conference "Modern Problems of Radielectronics, Telecommunications and Computer Engineering TCSET’2012”,* p. 83-84, Lviv-Slavsko, Ukraine, 2012.

АНАЛІТИЧНА МОДЕЛЬ НАДІЙНОСТІ РЕЗЕРВОВАНОЇ ВІДНОВЛЮВАНОЇ СИСТЕМИ З ОБМЕЖЕНОЮ КІЛЬКІСТЮ ВІДНОВЛЕНЬ

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Розроблено аналітичну модель резервованої відновлюваної системи з обмеженою кількістю відновлень, структура якої подана в розумінні надійності послідовним з'єднанням компонентів, кожен з яких зарезервований з кратністю 1 (навантажене резервування). При формуванні моделі використано повторюваність топологічних властивостей структури, що дає змогу спростити аналіз часових залежностей ймовірностей перебування системи у конкретних станах (нормального функціонування, простою (ремонту), катастрофічної відмови) та розрахунок відповідних показників її надійності. Використовувати розроблену аналітичну модель надійності зручно під час розв'язання оптимізаційної задачі вибору параметрів окремих ланок за критерієм мінімуму ймовірності простою системи.

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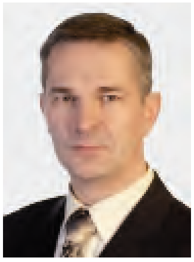

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