COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 3, No. 1, 2013

CODING TECHNIQUE FOR OUT-OF-BAND POWER REDUCTION OF MULTICARRIER CDMA AND OFDM SYSTEMS

Igor Gepko

Ukrainian State Centre of Radio Frequencies, Kyiv, Ukraine
gepko@ucrf.gov.ua

Abstract: In this paper, we introduce a method for out-of-band power reduction of multicarrier (MC) communication systems. Recent years MC systems possess a dominated role in wireless access owing to the ability to achieve high data rates and simultaneously high robustness to multipath and fading. Despite all advantages, MC transmission produces an essential *out-of-band* (OOB) interference. The OOB radiation leads to the wastage of scarce spectral resources and severe threats to adjacent wireless channels. We propose a novel technique for reducing OOB radiations in Orthogonal Frequency Division Multiplexing (OFDM) and MC Code Division Multiple Access (MC-CDMA) systems. To reduce the OOB emissions in the MC-CDMA system, we propose an analytical criterion for spectrum efficiency estimation as well as a low complexity algebraic algorithm for the proper waveform selection. The structure of selected waveform provides suppression for radiation outside the signal necessary bandwidth. Being implemented in the OFDM system, the proposed algorithm is used for the calculation of phases of *cancellation carriers* suppressing most powerful OOB sidelobes in a transmitted signal. In the final part of the article we consider an example of a simple precoding procedure for OFDM systems reducing the OOB power by 10 dB or more at the cost of the insignificant decrease of the information data rate.[1]

Key words: out-of-band emission, interference cancellation, spectrum efficiency, sidelobe suppression, MC-CDMA, OFDM.

1. Introduction

Recently, multicarrier (MC) signals became the focal point in the wireless system development. Digital broadcasting, wireless access and communication systems which include Wi-Fi, WiMAX, LTE/4G cellular – this is not a complete list of their applications. The key idea of MC transmission is to transform a broadband frequency-selective channel into the group of parallel non-selective narrow-band channels. Its advantage over single-carrier schemes is their ability to cope with severe channel conditions such as frequency-selective fading caused by multipath and narrowband interference.

Orthogonal frequency division multiplexing (OFDM) has attracted great interest in the last decade for its ability to transmit a high rate data stream splitting it to a number of orthogonally-spaced slower data streams. The division of the available spectrum into a number of orthogonal subcarriers makes the transmission system robust to multipath channel fading [1]. These features have led to the adoption of OFDM as a standard for digital audio broadcasting (DAB), broadband local and metropolitan wireless area networks [2], mobile broadband wireless access (MBWA) [3]. OFDM provides high computational efficiency by using FFT techniques in modulation/demodulation functions and perfect coexistence with current and future wireless systems.

Multicarrier code-division multiple-access (MC-CDMA) modulation scheme benefits from the advantages of both multicarrier and CDMA techniques: multiple access capability with the high flexibility and spectral efficiency, robustness in frequency selective channels with the low complexity of a receiver considering simple one-tap equalization and narrow-band interference rejection [1].

Despite all benefits, a potential drawback of both systems is high out-of-band (OOB) power due to the sidelobes of the subcarriers. The OOB spectrum decreases slowly according to a sinc function. For example, for 256 subcarriers the bandwidth on the level –40 dB is almost four times wider than the –3 dB bandwidth. Both systems produce substantial OOB interference which may disrupt communications in adjacent wireless channels. Conventional disabling a set of OFDM subcarriers on the left and right side of its spectrum sometimes is not sufficient to avoid interference.

Several approaches to mitigate this source of interference are known [4]. One way is using conventional filters to reduce the out-of-band spectrum. But a digital filter requires at least a few multiplications per sample, so filtering can increase the system complexity and introduce long delays.

The second way is windowing – – the convolution of a spectrum with the set of impulses at the carrier frequencies (for those samples which fall into the roll-off region). A commonly used type is a raised cosine window. Only a few percent of the samples are in the

[1] Part of this paper was presented at the 10th Int. Symposium on Electromagnetic Compatibility, York, UK, Sep. 26-30, 2011.

roll-off region, therefore windowing is the order of magnitude less complex than filtering.

The third approach is sidelobe suppression via spectral compensation, where the total number N of subcarriers to be transmitted is divided into two sets: the set of K information subcarriers and the set of S compensation subcarriers. Using properly chosen tones as compensation subcarriers leads to a more compact spectral mask. Spectral compensation does not require any shaping-related processing at the receiver: simple tone-wise equalization is possible since the orthogonality of the received basis functions is preserved [5]. Consequently, such a technique conforms to any standard.

Known methods of compensation include subcarrier weighting [6], the use of interference cancellation carriers [7], and multiple-choice sequences (MCS) [8]. First approach referred to as subcarrier weighting is based on weighting the individual subcarriers so that their sidelobes cancel each other. The other, using *cancellation carriers* (CCs) is a promising technique which inserts carriers with optimized amplitudes and phases on the reserved positions (usually at the left and right edges of a spectrum) in order to cancel the sidelobes of the transmitted signal. These carriers do not carry any data and are calculated to cancel out the OOB interference. Sidelobes are reduced significantly at the expense of a slightly increased bit error rate or a reduced throughput. Thirdly, with the method of multiple choice sequences, the transmitted symbol is mapped into multiple equivalent transmit sequences and the one with the lowest sidelobe power is then transmitted.

In this paper, we consider a simple algebraic criterion which provides us with a basis for selection sequences with the low spectrum power sidelobes within the MCS method as well as a simple algorithm for determining subcarrier phases within the CC_S method.

The rest of this paper is organized as follows. In Section II, we consider a concept of a multicarrier data transmission scheme and discuss the similarities and differences of MC-CDMA and OFDM air interfaces. Then we examine spectral properties of multicarrier signals depending on their phase structure. The spectrum efficiency criterion for MC waveform is introduced in Section III. In Section IV, we consider a simple precoding algorithm for the OFDM system. The existence of spectrum efficient sequences as well as their number is studied in Section V. The paper is concluded with a summary in Section VI.

2. Spectral properties of multicarrier signals

The logic diagram of multicarrier data transmission is depicted in the Fig. 1.

The main difference between the OFDM and MC-CDMA schemes is that in the first most of the coefficients c_n represent data symbols (excluding subcarriers on reserved positions) while in the second one they describe the spreading code. Therefore, in the OFDM system the choice of the waveform is limited.

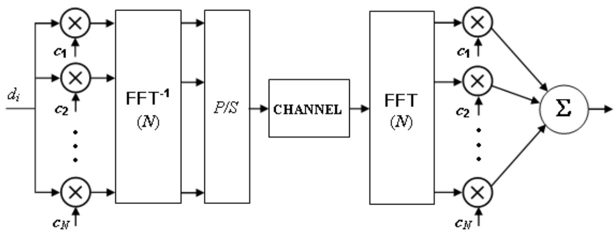

Fig. 1. The basic MC-CDMA (OFDM) scheme.

In general case coefficients c_i which describe amplitudes and phases of subcarriers are complex. Note that the QPSK (or QAM) signal can be represented by the sum (weighted sum) of binary signals which phases are in quadrature. Therefore its spectrum is a superposition of independent binary signals spectra. That is why the binary case is the most appropriate for studying.

Let the N-carrier signal be represented as

$$s_i(t) = d_i \cdot \sum_{n=1}^{N} c_n \cdot U(t)\cos(\omega_n t)\,, \qquad (1)$$

where $U(t)$=1, $0 \le t < T_{MC}$ is a rectangular pulse envelope and T_{MC} is the symbol duration. For MC-CDMA system d_i is a i^{th} data symbol, $c_n=\pm 1$ is N-chip binary spreading code sequence. For OFDM, c_n is a data vector (n=1,2,…N) and d_i is set to 1.

Cyclic frequency $\omega_n=\omega_0+\Omega_n$ of the n-th subcarrier is equal to the sum of the carrier ω_0 and the n-th cyclic subcarrier

$$\Omega_n = n \cdot \Omega_1\,,$$

where $\Omega_1 = 2p\,/\,T_{MC}$ is the basic cyclic frequency of the spectrum and $1/T_{MC}$ is a frequency separation between any two adjacent subcarriers. The spectrum of the signal (1) near the baseband can be represented as

$$S_{MC}(\Omega) = T_{MC} \cdot \sum_{n=1}^{N} c_n \frac{\sin[(\Omega-\Omega_n)T_{MC}/2]}{(\Omega-\Omega_n)T_{MC}/2}\,.$$

Maximums of its sidelobes moduli are reached where the sinus argument is equal to $\pm p\,/\,2$ – namely, in discrete points of the frequency axis $\Omega_q = q\Omega_1 + p$. Assuming $T_{MC} = 1$ for simplicity, one can consider

$$S_{MC}(\Omega_q) = \sum_{n=1}^{N} c_n \frac{\sin\pi(q-n+1/2)}{\pi(q-n+1/2)} = \frac{(-1)^q}{\pi}\sum_{n=1}^{N}\frac{c_n\cdot(-1)^n}{q-n+1/2} \qquad (2)$$

Let’s determine the ratio of spectrum sidelobes of single carrier and multicarrier pulses which have equal necessary bandwidths $\Delta F_{DS} = \Delta F_{MC} = N\,/\,T_{MC}$ (therefore $T_{DS} = T_{MC}\,/\,N$). If both pulses have equal powers then

$$A_{DS} = \sqrt{N \cdot A_{MC}^2 T_{MC}\,/\,T_{DS}} = N$$

In the points q^* of maximum spectrum sidelobe moduli (at cyclic frequencies $\Omega^* = p\left(2q^*+1\right)/T_{DS}$) we have

$$|S_{DS}(\Omega^*)| = \frac{|\sin(\Omega^* T_{DS}/2)|}{\Omega^* T_{DS}/2} = 2/\pi(2q^*+1)$$

Since $T_{MC}=NT_{DS}$, the corresponding points on the frequency axis are $q=N\cdot q^*+N/2$. As far as $q>>n$, the value of $(q-n+1/2)^{-1}$ gradually less depends of n, showing constant behavior. From (2) we obtain the asymptotic expression

$$\left|\frac{S_{MC}(\Omega^*)}{S_{DS}(\Omega^*)}\right| \le \frac{2q^*+1}{2Nq^*+N+1}\cdot\sum_{n=1}^{N}(-1)^n c_n \approx \frac{1}{N}\cdot\sum_{n=1}^{N}(-1)^n c_n \quad (3)$$

It follows that for MC system the worst waveform can be represented as a code with a regularly alternating sign, therefore $\sum_{n=1}^{N}(-1)^n\cdot c_n = N$. From (3) we make reasoning for the idea of average waveform in the sense of OOB emissions. Let us find the sum of discrete series $\sum_{n=1}^{N}(-1)^n c_n$ averaged over the complete set of binary code. The inversion of the odd elements of $\{c\}_N$ produces a sequence $\{c^*\}_N$ with $\sum_{n=1}^{N} c_n^* = \sum_{n=1}^{N}(-1)^n c_n$. Accordingly, the weight of $\{c^*\}_N$ is equal to $\sum_{n=1}^{N}(-1)^n c_n$.

For any codeword $\{c\}_N$ in a complete binary code set there always exists some "dual" $\{c^*\}_N$ such that the distributions of the sums $\sum_{n=1}^{N}(-1)^n c_n^*$ and $\sum_{n=1}^{N} c_n$ are identical. The weight of a binary sequence $\{c\}_N$ represented in multiplicative elements $\left(c_n \in \{-1;1\}\right)$ can always be defined in terms of its weight W in additive group $\left(c_n \in \{0;1\}\right)$: $\sum_{n=1}^{N} c_n = N-2W$. That is why

$$\left\langle\left|\sum_{n=1}^{N}(-1)^n c_n\right|\right\rangle = \left\langle\left|\sum_{n=1}^{N} c_n\right|\right\rangle = \frac{1}{2^N}\cdot\sum_{W=0}^{N}|N-2W|\cdot\mathbf{C}_W^N \quad (4)$$

where $\mathbf{C}_W^N = \begin{pmatrix}N\\W\end{pmatrix} = \frac{N!}{W!(N-W)!}$ is a number of combinations of N on W. The values of $\sum_{n=1}^{N}(-1)^n c_n$ calculated for different N according to (4) are represented in the Table 1.

Table I

Spectrum compactness of the "average" waveform

N	3	5	7	9	11	15	19	23	27
	4	6	8	10	12	16	20	24	28
$\langle\lvert\sum_{n=1}^{N} c_n\rvert\rangle$	1.50	1.87	2.19	2.46	2.71	3.14	3.52	3.87	4.18

N	32	48	64	80	96	112	128	144	160
$\langle\lvert\sum_{n=1}^{N} c_n\rvert\rangle$	4.48	5.50	6.36	7.11	7.80	8.43	9.00	9.56	10,1

Substituting (4) into (3), for the frequency offset $\Delta f >> \Delta F$ one can get

$$\frac{|S_{MC}(\Omega^*)|}{|S_{DS}(\Omega^*)|} \approx \frac{1}{N\cdot 2^N}\cdot\sum_{W=0}^{N}|N-2W|\cdot\mathbf{C}_W^N.$$

It follows that due to the mutual intercarrier suppression OOB emissions of a multicarrier system outside the necessary bandwidth may fade much faster than those of its single carrier counterpart. From the Table 1 it follows that the weight of average codeword is decreasing compared to N as far as N is increasing, and

$$\frac{1}{N}\cdot\sum_{n=1}^{N}(-1)^n c_n < 1$$

3. A new criterion for OOB spectrum suppression

Equations Let us consider the next part of (2) expanding it so as $\left(q-n+1/2\right)^{-1} = q^{-1}+o\left(\Delta_1\right)$, where $\Delta_1 = q^{-1}$ is the first term of the OOB-emission, which decreases proportionally to 1st degree of frequency offset (its power decreases in proportion to the 2nd degree of the frequency offset). The second element $o\left(\Delta_1\right)$ is always a small value compared to Δ_1. Then

$$\left(q-n+1/2\right)^{-1} = \frac{1}{q}+\frac{2n-1}{2q^2}+o(\Delta_2),$$

So $o\left(\Delta_1\right) = \Delta_2 + o\left(\Delta_2\right)$, where $\Delta_1 = \left(\frac{2n-1}{2}\right)^0\cdot q^{-1}$, $\Delta_2 = \left(\frac{2n-1}{2}\right)^1\cdot q^{-2}$ is a second element of the expansion, decreasing proportionally to square value of the frequency offset. In its turn, $o(\Delta_2) = \frac{1}{q-n+1/2} - \left(\frac{1}{q}+\frac{2n-1}{2q^2}\right) = \frac{4n^2-4n+1}{4q^3-4nq^2+2q^2} = \left(\frac{2n-1}{2}\right)^2\cdot q^{-3}+o(\Delta_3)$ (it follows that $\Delta_3 = \left(\frac{2n-1}{2}\right)^2\cdot q^{-3}$).

Thus continuing, we obtain $\Delta_4 = \left(\frac{2n-1}{2}\right)^3 \cdot q^{-4}$.

Assume that for some integer m there is

$$\Delta_m = \left(\frac{2n-1}{2}\right)^{m-1} \cdot q^{-m}, \; o(\Delta_m) = \Delta_{m+1} + o(\Delta_{m+1})$$

then $\Delta_{m+1} + o(\Delta_{m+1}) = (q-n+1/2)^{-1} - \sum_{i=1}^{m} \Delta_i$. The subtrahend here appears to be the sum of the geometric progression

$$\sum_{i=1}^{m} \Delta_i = \sum_{i=1}^{m} \left(\frac{2n-1}{2}\right)^{i-1} \cdot \frac{1}{q^i} = \frac{1}{q} \sum_{i=1}^{m} \left(\frac{2n-1}{2q}\right)^{i-1}$$

$$= \frac{1}{q} \cdot \frac{\left(\frac{2n-1}{2q}\right)^m - 1}{\frac{2n-1}{2q} - 1} = -\frac{\left(\frac{2n-1}{2q}\right)^m - 1}{q-n+1/2},$$

therefore,

$$o(\Delta_m) = \left(\frac{2n-1}{2}\right)^m \cdot q^{-(m+1)} + o(\Delta_{m+1})$$

Applying the mathematical induction as a method of proof we make a conclusion that $\Delta_{m+1} = \left(\frac{2n-1}{2}\right)^m \cdot q^{-(m+1)}$ is true for any integer m. For now, the module of spectrum (2) out of the baseband $(q > 1)$ can be written as

$$\left|S_{MC}(\Omega_q)\right| = \frac{1}{\pi} \left| \sum_{n=1}^{N} (-1)^n c_n \sum_{m=1}^{\infty} \left(\frac{2n-1}{2}\right)^{m-1} q^{-m} \right|. \quad (5)$$

It's clear from (5) that the structure and the intensity of the OOB-emissions in general are determined by the structure of the code (or waveform) c_n. When P equations

$$\sum_{n=1}^{N} (-1)^n c_n (2n-1)^{m-1} = 0, \text{ where } m \le P$$

come true, then P out-of-band components of the multicarrier signal decreasing proportionally to $1/q, 1/q\, 1/q, 1/q^2, \ldots 1/q^P$ (proportionally to $1/q^2, 1/q^4, \ldots 1/q^{2P}$ in terms of power) will be compensated due to the special phase relationships between subcarriers. For P=1, 2 and 3 respectively, we have $\sum_{n=1}^{N} (-1)^n c_n = 0$, $\sum_{n=1}^{N} n^1 \cdot (-1)^n c_n = 0$ and

$$\sum_{n=1}^{N} n^2 \cdot (-1)^n c_n = 0$$

where the first equation coincides with [9]. It's easy to prove

$$\sum_{n=1}^{N} n^m \cdot (-1)^n c_n = 0, \quad m=0, 1, 2, \ldots P-1. \quad (6)$$

Let us refer to (6) as to the compactness criterion of the MC signal spectrum. Then c_n is a "spectrum-efficient" code (SE-code) of the order P. The normalized right side of power spectra of a single carrier and multicarrier signals are shown in Fig. 2 (we consider the MC-CDMA system with N=8 subcarriers and three different spreading codes).

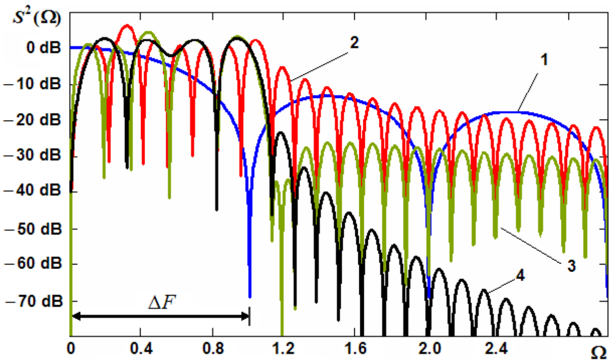

Fig. 2. DS-CDMA and MC-CDMA (N=8) spectra.

Here $\Delta\Omega = 2p \cdot \Delta F$ is the width of a frequency band, which is necessary for the undistorted transmission of the signal. Curve "1" depicts the spectrum of single carrier rectangular pulse, "2" – the spectrum of the multicarrier signal with the "bad" code sequence (P=0), "3" – the spectrum of the signal based on the "average" code with "random" properties, "4" – spectrum of the "best" waveform where only about 0,05% of signal power is OOB-radiated:

$$\int_{\Delta F}^{+\infty} S_{MC}^2(f)df \Big/ \int_{0}^{+\infty} S_{MC}^2(f)df \approx 0.0005.$$

Fig. 3 shows the power spectra of multicarrier (N=16) signals with different waveforms. Here curve "1" depicts the spectrum of a signal having "random" properties (P=0), the other corresponds to signals based on SE-codes (P=2, P=3 and P=4). The last, $\{1,-1,1,1,1,1,-1,1,-1,1,-1,-1,1,-1,1,1\}$ provides the lowest level of OOB emission (99.996% of its total radiated power falls within the necessary band $\Delta F = N / T_{MC}$).

For N=16 the average OOB power is about 2.5%, maximal – almost 45% from the total radiated power. In other words, the best waveform almost 50 times reduces OOB power compared to "average" and little less than a thousand times – compared to the "worst" case. For the frequency offset $\Delta f = \Delta F$ (from the edge of the necessary band) the decrease in signal power spectral density is about 35 dB for P=2, 50 dB for P=3 and almost 60 dB for P=4. On the contrary, the least spectral effective waveform increases the OOB emission relative to the "average" waveform by 15 dB or even more.

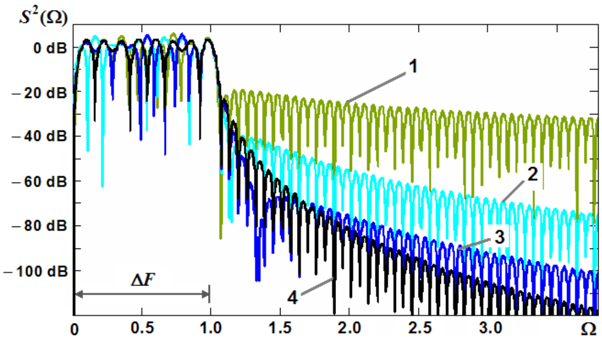

Fig. 3 MC-CDMA spectra for N=16.

4. Spectrally efficient coding for OFDM systems

The basic idea of spectrally-efficient encoding is similar to the CCs method [7], where a few subcarriers which do not carry any data themselves are inserted on both sides of the OFDM spectrum to cancel out a certain part of the OOB emissions. If the source data rate is equal to or less than $N_{dataMAX} = \log_2(Q_N)$ symbols per OFDM signal interval, then the transmitted data block can always be mapped onto some SE MC waveform. However when N is large, random nature of the transmitted data flow causes high complexity of the mapping algorithm.

Let us consider a simple OFDM mapping scheme where a fixed number of frequency positions are reserved for subcarriers which are not used for data allocation. Let us assume that:

N be the FFT window size;

K is the number of data subcarriers (equal to the number of information bits in the data block to be transmitted);

S is the number of the cancellation subcarriers in the data block, $S = N - K$;

$M = K/2$ is the number of data subcarriers in the even/odd “semiblock” to be transmitted;

$L = S/2$ is the number of the cancellation subcarriers in any semiblock, $L = N/2 - M$.

Suppose, α and β are the weights of odd and even semiblocks consisted of additive group symbols $c_n \in \{0;1\}$. The suppression of the first OOB emissions component is impossible if $|a - b| > L$ (or, $|a - b| \geq L+1$). It’s easy to calculate the total number of such combinations:

$$\sum_{\alpha=0}^{M} \mathbf{C}_M^{\alpha} \left(\sum_{\beta=\alpha+L+1}^{M} \mathbf{C}_M^{\beta} \right) = \sum_{\alpha=0}^{M-L-1} \mathbf{C}_M^{\alpha} \left(\sum_{\beta=\alpha+L+1}^{M} \mathbf{C}_M^{\beta} \right) +$$

$$+ \sum_{\alpha=L+1}^{M} \mathbf{C}_M^{\alpha} \left(\sum_{\beta=0}^{\alpha-L-1} \mathbf{C}_M^{\beta} \right) = 2 \sum_{\alpha=0}^{M-L-1} \mathbf{C}_M^{\alpha} \left(\sum_{\beta=\alpha+L+1}^{M} \mathbf{C}_M^{\beta} \right)$$

As far as the total number of possible data blocks is 2^{2M}, the probability of the situation where OOB emissions are not compensated is

$$P_{\text{OOB}} = \frac{1}{2^{2M-1}} \sum_{\alpha=0}^{M-L-1} \mathbf{C}_M^{\alpha} \left(\sum_{\beta=\alpha+L+1}^{M} \mathbf{C}_M^{\beta} \right) =$$

$$= \frac{2}{2^K} \sum_{\alpha=0}^{M-L-1} \mathbf{C}_M^{\alpha} \left(\sum_{\beta=\alpha+L+1}^{M} \mathbf{C}_M^{\beta} \right) =$$

$$= \frac{2}{2^{N-S}} \sum_{\alpha=0}^{M-L-1} \mathbf{C}_M^{\alpha} \left(\sum_{\beta=\alpha+L+1}^{M} \mathbf{C}_M^{\beta} \right)$$

There always will be a loss in data rate caused by the fact that a certain amount of the signal power is spent on the CCs and is not available for data transmission. So a rational balance between the data throughput and interference mitigation must be considered. We illustrate performances of a some flexible data allocation scheme by plotting P_{OOB} versus the number of data subcarriers K and a total number of subcarriers N (so $m = 2,3...P$ R=K/N is the relative information data rate). The instant transmission rate depends on the current data configuration. Fig. 4 shows that the cancellation scheme provides the OOB power level control at the expense of reasonable reduction of data rate.

For comparison, in a WiMAX system with 256 subcarriers 55 are disabled (28 subcarriers on the left side and 27 on the right side) in order to provide guard bands and limit OOB radiations [4].

The performance of the proposed method was investigated. For N=2048 the average transmission rate in a system with the 1st order of OOB spectrum suppression is about 0,93. Using more sophisticated waveform selection and mapping techniques, the required number of cancellation carriers can be reduced causing an increase in system complexity (instead of that, additional complexity can be used to further reduce a sidelobe power). An optimal CCs scheme achieves R≈0,995.

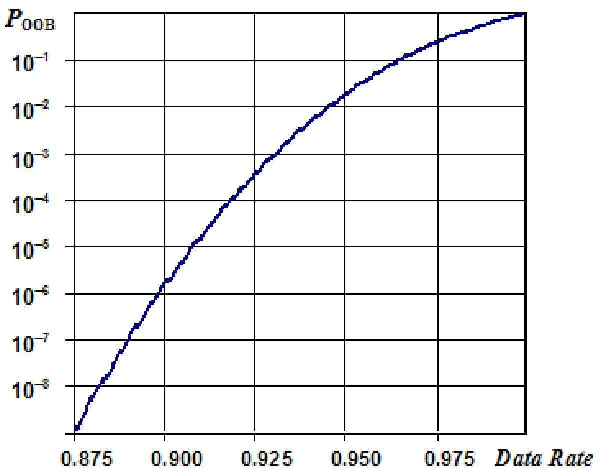

Fig. 4. P_{OOB} versus R, N=2048.

The system performance is compared with a conventional OFDM system without any suppression scheme. Fig. 5 shows the effect of the proposed technique in the OFDM power spectrum. It depicts the OOB emissions in OFDM spectra with and without inserting cancellation carriers.

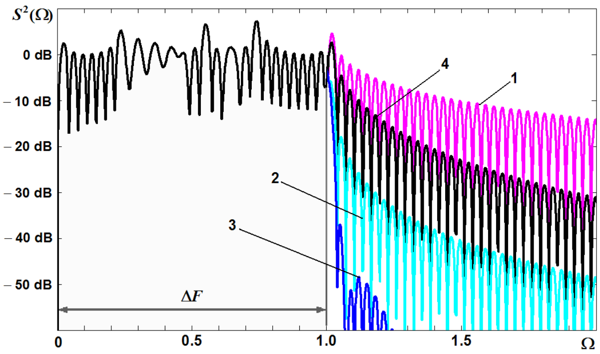

Fig. 4 OFDM spectra with and without inserting cancellation carriers.

The curve no. 1 represents the original spectrum with an uncontrolled OOB radiation level, while curves 2, 3 and 4 indicate similar spectra after CCs insertion. Curve no. 2 depicts OOB emissions of a typical "first order" spectrum, no.3 – the minimal one (achieved by using the selection stage), no.4 – the highest OOB level for spectra after the CC insertion. The sidelobe reduction produced by the proposed CCs algorithm is within 10 – 20 dB.

5. Synthesis and the number of SE-codes

According to (6) the weight of even symbols of the SE-code (which is assumed to be any code with $P \geq 1$) is equal to weight of its odd symbols. Actually this is a simple rule to construct a complete set of codes having the 1st order spectrum compactness. The number of SE-codes of length N is

$$Q_N = \binom{N/2}{N}^2 = \frac{N!}{[(N/2)!]^2}. \qquad (7)$$

The total number of SE-codes for some of $N \leq 1024$ defined according to (7) is given in the 3rd column of Table 2.

Table 2

Numbers of "spectrum-efficient" codes

N	2^N	Q_N, P³1	$Q_N / 2^N$	N_{dataMAX}	N_{dataMAX}/N
16	65536	12 870	0,1964	13	0,8125
32	$4.295 \cdot 10^{9}$	$6.011 \cdot 10^{8}$	0,1400	29	0,9063
64	$1.845 \cdot 10^{19}$	$1.833 \cdot 10^{18}$	0,0993	60	0,9375
128	$3.403 \cdot 10^{38}$	$2.395 \cdot 10^{37}$	0,0704	124	0,9688
256	$1.158 \cdot 10^{77}$	$5.769 \cdot 10^{75}$	0,0498	251	0,9805
512	$1.341 \cdot 10^{154}$	$4.726 \cdot 10^{152}$	0,0352	507	0,9902
1024	$1.798 \cdot 10^{308}$	$4.481 \cdot 10^{306}$	0,0249	1018	0,9941

From the 4th column of the table it's clear that the SE-code family is an essential part of the binary code set (2^N different codewords). For data transmission one can use SE-waveforms only at the expense of reducing the insignificant data rate. It is supposed, that if we use the 1/64 part of a total set of 1024 subcarrier binary waveforms (so reducing the number of data carriers for 6), the power level of OOB emissions is decreased tenfold.

To synthesize selectively SE-codes for $P \geq 2$, let us solve a system of $P-1$ equations

$$\sum_{j=1}^{N/2} i_j^{m-1} = \frac{1}{2m} \sum_{k=0}^{m-1} \binom{m}{k} B_k (n+1)^{m-k}, \; m = 2,3 \ldots P \qquad (8)$$

where $i_1, i_2, \ldots, i_{N/2}$ is a set of indexes of all positive symbols in the waveform and B_k denote Bernoulli numbers [10] which for integer $k \geq 1$ can be found by recursion

$$B_k = -\frac{1}{k+1} \sum_{j=0}^{k-1} \binom{k}{j} B_j, \; B_0 = 1.$$

Within this work, we obtain the complete solution of (8) when P=2. For $P \geq 3$ the common approach hasn't been found yet except the direct computer search. Instead, we have obtained some partial solutions for several special cases for $N=2^P \cdot k$ as well as $N=2^{P-1} \cdot (2k+1)$, $k=1,2,\ldots$. The total numbers of SE-codes for $N \leq 36$ for different P are given in the Table 3.

Table 3

The total numbers of SE-codes for different order P

N	S$Q_N(P)$	$Q_N(1)$	$Q_N(2)$	$Q_N(3)$	$Q_N(4)$	$Q_N(5)$
4	6	4	2	—	—	—
6	20	20	—	—	—	—
8	70	62	6	2	—	—
10	252	252	—	—	—	—
12	924	866	56	2	—	—
14	3432	3432	—	—	—	—
16	12870	12344	512	12	2	—
18	48620	48620	—	—	—	—
20	184756	179308	5400	48	—	—
22	705432	705432	—	—	—	—
24	2704156	2643048	60516	576	16	—
26	10400600	10400600	—	—	—	—
28	40116600	39393246	720468	2886	—	—
30	155117520	155117520	—	—	—	—
32	601080390	592171844	8873658	34810	76	2
36	9075135300	8962042278	112815212	277810	—	—

For any natural number s, equations (8) have a solution for the each pair of numbers (N, P), with

$N=2^P\cdot s$, where $P=1$, 2; as well as with $N=2^P\cdot s$ or $N=2^{P-1}\cdot(2s+1)$, where $P\geq 3$. At the same time for $P\geq 5$ the absence of solutions for other N has not been proven yet.

6. Conclusion

In this paper, a new method for shaping spectra of OFDM and MC-CDMA-based systems is presented. In MC-CDMA, a set of codes distributing their subsets between users is selected. Depending on the size of subsets each user applies them in binary or M-ary transmission (that is, multiplies a fixed sequence by incoming data bit or selects one of M pre-mapped sequences for transmission of the M-ary symbol). For OFDM, for each symbol time data allocated at $K=N_{\text{data}}$ subcarriers are mapped to the FFT symbol, which is some SE sequence of length $N=N_{\text{FFT}}$ by inserting CCs on $S=N_{\text{FFT}}-N_{\text{data}}$ reserved positions. This can be done directly (CCs method), or on preliminary calculated basis (MCS which is to produce a set of sequences from the original data sequence and select (from the MCS set for transmission) one sequence with the highest order of spectrum efficiency). Both CCs and MCS are easy to implement due to the low complexity of (6) compared to any of the FFT-based approach. MCS is useful for obtaining the output waveform of $P\geq 2$, so the implementation needs more reserved subcarrier positions (with greater redundancy).

A clear correlation between the data rate and the probability of successful suppression is shown. Numerical results show that proposed schemes achieve large suppression of sidelobe power in MC spectra and a significant advantage over the conventional MC system without suppression providing their successful co-existence with wireless systems in adjacent channels. CCs are orthogonal to subcarriers used for data transmission and do not impact the BER performance: the only downside is extra transmission power needed to transmit them.

References

[1] L. Hanzo, M. Munster, T. Keller, and B-J. Choi, *OFDM and MC-CDMA for Broadband Multi-User Communications, WLANs and Broadcasting,* John-Wiley and IEEE Press, 2003.

[2] Z. Yuan, S. Pagadarai, and A. Wyglinski, "Sidelobe suppression of OFDM transmissions using genetic algorithm optimization," in *Proc. IEEE Military Commun. Conf*., vol. 4, pp. 1–5, 2008.

[3] *IEEE Std 802.20 for Local and metropolitan area networks, Part 20*: "Air Interface for Mobile Broadband Wireless Access Systems Supporting Vehicular Mobility Specification: Physical and Media Access Control Layer," IEEE, 2008.

[4] A. Hisham A. Mahmoud, and Hüseyin Arslan, "Sidelobe suppression in OFDM-based spectrum sharing systems using adaptive symbol transition," *IEEE Commun. Lett*., vol. 12, no. 2, pp. 133–135, Feb. 2008.

[5] T. Magesacher, "Spectral compensation for multicarrier communicati-on", *IEEE Trans. Signal Proc*., vol. 55, no. 7, pp. 3366-3379, July 2007.

[6] I. Cosovic, S. Brandes, and M. Schnell, "Subcarrier weighting: a method for sidelobe suppression in OFDM systems," *IEEE Commun. Lett*., vol. 10, no. 6, pp. 444–446, June 2006.

[7] S. Brandes, I. Cosovic, and M. Schnell, "Reduction of out-of-band radiation in OFDM systems by insertion of cancellation carriers," *IEEE Commun. Lett.*, vol. 10, no. 6, pp. 420–422, June 2006.

[8] I. Cosovic and T. Mazzoni, "Suppression of sidelobes in OFDM systems by multiple-choice sequences," *Eur. Trans. Telecomms*., vol. 17, pp. 623–630, June 2006.

[9] T.F. Ho and V.K. Wei, "Construction of Spectrally Efficient Low-Crest Waveforms for Multicarrier CDMA System", in *Proc. 4th IEEE Int. Conf. on Universal Personal Communications Record*, vol. 4, Nov. 2006, pp. 522–526.Tokyo, 1995

[10] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products,* translated from Russian, Elsevier Academic Press, USA, 2007.

МЕТОД КОДУВАННЯ ДЛЯ ЗНИЖЕННЯ РІВНЯ ПОЗАСМУГОВИХ ВИПРОМІНЮВАНЬ У ТЕЛЕКОМУНІКАЦІЙНИХ СИСТЕМАХ З ТЕХНОЛОГІЯМИ MC-CDMA ТА OFDM

Ігор Гепко

У цій статті ми пропонуємо метод кодування для зниження рівня позасмугових випромінювань у комунікаційних системах із сигналами з багатьма носіями. Останнім часом системи з багатьма носіями домінують у бездротовому доступі завдяки їхній здатності досягати високих швидкостей пересилання інформації водночас із високою стійкістю до багатопроменевості та згасання. Незважаючи на ці переваги, системи з багатьма носіями продукують значний рівень позасмугових випромінювань. Позасмугове випромінювання веде до марнування небагатих спектральних ресурсів і серйозних загроз для суміжних бездротових каналів. Ми пропонуємо новий метод для зниження позасмугового випромінювання для систем із технологією мультиплексування з ортогональним частотним розділенням (OFDM) та технологією мульти-доступу при розділенні коду багатьма носіями (MC-CDMA). Для зниження поза смугового випромінювання у MC-CDMA системах ми пропонуємо аналітичний критерій оцінки ефективності використання спектру, а також простий алгебричний алгоритм для правильного

підбору форми сигналу. Структура обраної форми сигналу забезпечує тамування випромінювання за межами необхідної смуги пропускання сигналу. При застосуванні в системі OFDM запропонований алгоритм використовується для розрахунку фази при компенсації носіїв, затамовуючи найпотужніші побічні максимум позасмугового випромінювання передаваного сигналу. У заключній частині статті ми розглянули приклад простої процедури попереднього кодування для системи OFDM, що зменшує потужність поза смугового випромінювання на 10 дБ та більше за рахунок незначного зниження швидкості пересилання інформації.

Igor Gepko – DSc., Prof., Head of Scientific Support Department of the Ukrainian State Centre of Radio Frequencies. He received his M.Sc. degree in Electrical Engineering in 1989 at the Kyiv Air Force Institute, Ukraine. He is IEEE Member since 1999.

For several years he worked as a reviewer for IEEE Transactions on Communications journal. His current research interests include signal processing and code sequences design, CDMA, spread spectrum and multicarrier transmission technique, Cognitive Radio, electromagnetic compatibility and spectrum management as well as applied research in the field of information security and authentification.