

## ANALYSIS OF ELECTROMAGNETIC PROCESSES IN A TURBOGENERATOR WITH EQUIVALENT ROTOR TOOTH ZONES AT NO-LOAD

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**Abstract:** For the purpose of providing a much fuller analysis of electromagnetic processes in electrodynamic devices, it is necessary to carry out the calculation of electromagnetic field in movable and immovable elements of their structure taking into account coordinate systems of moving and stationary items. Besides, another important problem while developing mathematical field models of electric devices consists in the reproduction of the mode of given voltages rather than currents, as the currents in windings are the result rather than the cause of the general electromagnetic phenomenon. In most scientific works dedicated to the calculation of electromagnetic field in movable media the difference between reference systems in which the process is considered is not taken into account while the mode of given currents is used. That fact causes errors in the results obtained from the analysis of real physical phenomena in the devices. In this paper, 2-D mathematical field models for calculating a quasi-stationary electromagnetic field in the cross-section of a turbogenerator at no-load with an equivalent rotor zone at the given voltage of the excitation winding have been developed. The models suggested provide the calculation of an electromagnetic field in all zones of the device on the basis of a vector potential in both a single system of coordinates of the moving rotor, and coordinate systems of the rotor and stator simultaneously. The results of computer simulation of the electromagnetic processes on the basis of the developed models show that while developing mathematical models of electrodynamic devices, it is necessary to consider the coordinate systems in which the calculation of electromagnetic phenomena is performed. In addition, the character of electromagnetic processes occurring in real structures of the object cardinally differs from those obtained on the basis of equivalent schemes of the device zones.

**Key words:** electromagnetic field, vector potential, turbogenerator, movable media, system of coordinates.

### 1. Introduction

The materials of the article are the evolution of and logical sequel to the earlier publications [1-3].

The development of mathematical field models of electrical devices, which provide the fullest description of their construction and most exact reproduction of electromagnetic processes, is a difficult problem not only from the perspective of practice but also in terms of theory. In scientific calculations, to simplify the models alternate structures are applied or the equivalent substitution of the heterogeneous structures of different electrical objects by continuous anisotropic media is performed [4-7]. This always leads to the distorted representation of electromagnetic phenomena in the devices. Besides, the method of considering the real physical electromagnetic characteristics of the construction elements of the device in the mode under simulation affects the conformity of the process reproduction.

The mathematical field models of a turbogenerator with equivalent stator and rotor tooth zones at no-load are given in [3]. The next step towards the improvement of the models developed in [3] is dividing a stator zone into two separate constructive elements – the laminated part of the stator body which forms a tooth structure of the stator, and a zone of the stator slots with a winding enclosed. Such detailing of the stator tooth zone is not sufficiently complete as, in a real turbogenerator, the stator winding consists of separate isolated parallel copper buses with internal bores for cooling, while in the models, phase windings in the stator slots are represented by a continuous medium with electromagnetic copper properties. Accepting such an assumption leads to some inaccuracy which is important while analyzing transitional electromagnetic processes in the zones of electric windings of the device. Since at no-load the currents of conductivity are absent in the stator windings, the chosen way, in which the models represent the level of detailing of the stator tooth zone in the models, doesn't significantly affect on the character of reproduction of the real electromagnetic phenomena in the device at the given mode.

Taking into account the coordinate system in which the main unknown variables are formed is the important

factor for the development of mathematical field models. This is a vital issue for the calculation of an electromagnetic field in the movable and immovable elements of the device on the basis of Maxwell's equations in a time domain.

To reproduce the turbogenerator construction as full as possible, considering the main disadvantages of the existing approaches to the development of mathematical field models of electrodynamic devices [3-7], let's consider the development of two field models of a turbogenerator with an equivalent rotor tooth zone: in the coordinate system of the moving rotor, as well as in the coordinate systems of both the stator and the rotor simultaneously. The models are developed on the basis of Maxwell's equations with respect to potentials in the conditions of the given voltage in the rotor winding. They are intended for the direct time integration of electromagnetic field equations.

## 2. Statement of the problem

Most of the existing mathematical models of electrodynamic devices are designed to calculate electromagnetic fields using pre-specified currents of the winding. The currents in the windings of electrical objects are the constituent components of electromagnetic phenomena, and so they must be determined directly from the process and not given from the outside. The proposed mathematical field models of the turbogenerator imply that calculation of an electromagnetic field at no-load is done with the given voltage of the excitation rotor winding. This is the very way in which power supply of the real turbogenerator is provided.

Considering the absence of currents in the stator windings at no-load and taking into account the relation of the length of the turbogenerator to its diameter ( $\approx 3$ ), the electromagnetic process in all zones of its cross-section at no-load can be considered as plane-parallel. It is necessary to emphasize that this class of models

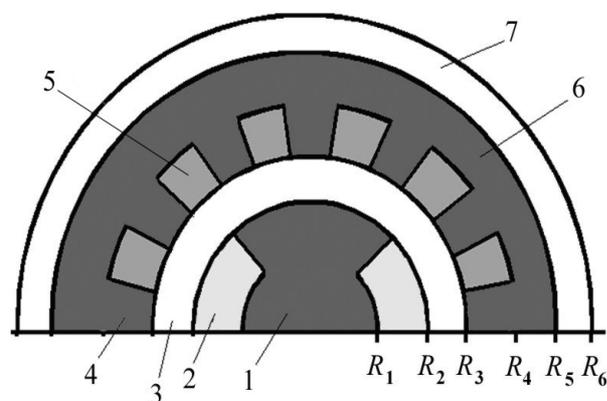


Fig. 1. Calculated zones of the turbogenerator cross-section.

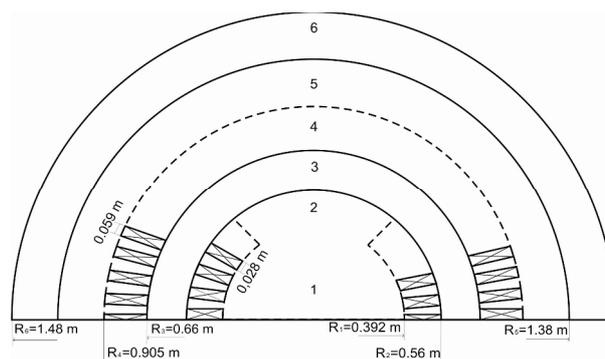


Fig. 2. Main geometrical zone dimensions of the turbogenerator cross-section.

cannot be used to analyze electromagnetic phenomena in end zones of the turbogenerator. But since the end effects do not have a significant influence on the character of electromagnetic processes in the device at no-load, the accepted simplification of the task statement may be considered admissible.

The gradual (step-by-step) and more complete particularization of the turbogenerator construction enables to reveal the influence of different assumptions accepted in the models on the character of the obtained processes, and to compare them with physical phenomena in the real object. That is why, taking into account the structure of the stator tooth zone is the next logical step concerning the turbogenerator models described in the work [3]. Besides, the rotor tooth zone, like in the mentioned article [3], is replaced by an equivalent anisotropic medium possessing the electromagnetic characteristics given above, on the basis of the familiar (used in the theory of electromagnetic circuits) methods of the equivalent replacement of series and parallel connection of electric and magnetic resistances.

The calculation of electromagnetic fields in the proposed mathematical field models of the turbogenerator at no-load at the specified voltage of the rotor winding is performed:

- in all movable and immovable elements of the device in one coordinate system connected with the moving rotor;
- in movable and immovable elements of the device in the respective system of coordinates of the moving rotor or stationary stator simultaneously.

The turbogenerator TGV-500, whose pole division of the stator tooth zone consists of 24 slots (8 slots for each phase of the stator winding (A, B, C)) has been chosen as a prototype of the mathematical model. The turbogenerator cross-section is shown as a diagram in Fig. 1, where 1 stands for the massive rotor body; 2 represents the equivalent rotor tooth zone; 3 is the air gap between the stator and rotor; 4 is the stator tooth; 5 stands for the stator winding zone; 6 represents the stator

## *Calculation of Electromagnetic Processes for a Turbogenerator With Equivalent*

body zone; 7 is the air zone outside the turbogenerator.

Geometrical cross-sectional dimensions of the real turbogenerator TGV-500 corresponding to Fig. 1 are represented in Fig. 2.

### **3. Mathematical model of the turbogenerator with an equivalent rotor tooth zone in the coordinate system of the moving rotor**

To decrease the number of main equations and to simplify the task of determining boundary and edge conditions when developing the models, the authors employed Maxwell's equations of electromagnetic field written with respect to potentials.

Such an approach means that in order to reach unambiguity of the solution, it is necessary to define the way of calibration for vector  $\mathbf{A}$  and scalar  $\varphi$  potentials of the electromagnetic field. When calculating electromagnetic transition processes for the major part of electric devices by means of potentials, it is reasonable to apply the calibration  $\varphi = 0$ . Such calibration provides an unambiguous solution, maximally simplifies the system of equations, and does not contradict a mathematical description of the physical nature of real electromagnetic phenomena in the way, chosen to analyze electromagnetic processes in movable and immovable, linear and nonlinear, constructionally isotropic and anisotropic zones of the devices. Consequently, the given calibration of potentials was applied when developing the proposed mathematical field models of the turbogenerator.

Taking into consideration the periodicity of electromagnetic processes on a certain angular segment in the cross-section of turbogenerator zones (on the pole division of the turbogenerator), it is not necessary to calculate an electromagnetic field across the whole area of the device cross-section.

The pole division of the turbogenerator TGV-500 along the angular coordinate is  $180^\circ$ . In the proposed mathematical field models, the calculation of an electromagnetic field is done in the zones of the device cross-section only on the defined segment (pole division). At the same time, if necessary, electromagnetic processes on another section of the turbogenerator (another pole division) are obtained automatically.

Equation (1) is the initial basic equation of a mathematical field model for the calculation of a turbogenerator no-load mode [1-3]

$$\frac{\partial \mathbf{A}}{\partial t} = -\Gamma^{-1}(\nabla \times (\mathbf{N} \nabla \times \mathbf{A}) \pm \delta); \quad (1)$$

where  $\mathbf{A}$  is the vector potential of electromagnetic field in quasi-stationary (stable) approximation;  $\Gamma$  is the matrix of static electrical conductivities;  $\mathbf{N}$  is the matrix of static inverse magnetic penetrability of the medium;  $\delta$  is the extraneous current density vector;  $\nabla$  stands for Hamiltonian.

Expression (1) is used to calculate electromagnetic processes in the replaced equivalent zones of the rotor winding with an electric current induced by an external voltage source, connected to the excitation winding of the turbogenerator.

An electromagnetic field in the rotor body can be calculated by using the following expression

$$\frac{\partial \mathbf{A}}{\partial t} = -\Gamma^{-1}(\nabla \times (\mathbf{N} \nabla \times \mathbf{A})). \quad (2)$$

Electromagnetic processes in the air gaps between the stator and rotor are simulated on the basis of the given ratio

$$0 = v_0 \nabla \times \nabla \times \mathbf{A}, \quad (3)$$

where  $v_0$  is the inverse magnetic air penetrability.

The electromagnetic phenomena in the body and equivalent tooth zone of the rotor, as well as in the air gaps between the stator and rotor are analyzed in the reference system related to the moving rotor. Consequently, the function of vector potential  $\mathbf{A}$  in equations (1)-(3) belongs to the mentioned system of coordinates.

In the given mathematical field model, the calculation of an electromagnetic field in the stator teeth, slots and body is carried out in the stator coordinate system which is reduced to the coordinate system of the moving rotor. Thereby, in the model the reciprocal mechanical movement of linear and nonlinear, isotropic and anisotropic media is eliminated. This considerably simplifies the model, but the electromagnetic processes, obtained in the given zones, do not correspond to the processes in the stator. To simulate the character of real electromagnetic phenomena in the stator, it is necessary to convert the electromagnetic quantities to the physical coordinate system of the stator.

The electromagnetic process in the stator's teeth and body is described by the following equation

$$0 = \nabla \times \mathbf{N} \nabla \times \mathbf{A}', \quad (4)$$

where  $\mathbf{A}'$  is the vector potential of electromagnetic field in the coordinate system reduced to the moving rotor.

The practical use of equation (2) in order to calculate electromagnetic phenomena in the slots of the tooth zone of the stator at no-load leads to the appearance of induced conduction currents in the stator windings. This contradicts real physical processes in the turbogenerator in the regime under investigation.

As conductivity currents are absent in stator windings when the turbogenerator operates at no-load, therefore, to provide this property in the models, the calculation of electromagnetic processes in the stator slot zones with a copper winding enclosed is carried out by means of the following expression

$$0 = v_0 \nabla \times \nabla \times \mathbf{A}'. \quad (5)$$

The value of the vector potential of electromagnetic field in an air gap zone outside the turbogenerator can also be found from equation (5).

Considering natural periodicity of electromagnetic processes on the pole division of the turbogenerator using the first and higher spatial derivatives of the vector potential function, boundary conditions on the external edges along the radii of the integration zone of the main system of equations can be obtained from the expression given below [2, 3]

$$\left. \frac{\partial H_r}{\partial a} \right|_{a=0} = - \left. \frac{\partial H_r}{\partial a} \right|_{a=180^\circ}, \quad (6)$$

where  $H_r$  is the radial component of the vector of magnetic field intensity.

Based on ratio (6), boundary conditions along the radii of the rotor body zone, equivalent rotor tooth zone and air gap between the stator and rotor in the moving rotor coordinate system can be represented as

$$A_{k=1} = 2A_{k=2} + 2A_{k=n-1} - A_{k=3} - A_{k=n-2} - A_{k=n};$$

$$A_{k=n+1} = A_{k=2} + A_{k=4} + A_{k=n-1} - 2A_{k=3} - 2A_{k=n}, \quad (7)$$

whereas those along the radii of the stator tooth zone, stator body and air gap outside the turbogenerator is given by

$$A'_{k=1} = 2A'_{k=2} + 2A'_{k=n-1} - A'_{k=3} - A'_{k=n-2} - A'_{k=n};$$

$$A'_{k=n+1} = A'_{k=2} + A'_{k=4} + A'_{k=n-1} - 2A'_{k=3} - 2A'_{k=n}, \quad (8)$$

where  $k$  is the index corresponding to the spatial grid nodes along the angular coordinate.

The value of the vector potential of the electromagnetic field outside the turbogenerator is determined from the following ratio

$$A'_{i=m+1} = 2A'_{i=m} - A'_{i=m-1}, \quad (9)$$

where  $i$  is the index corresponding to the spatial grid nodes in the cylindrical system of coordinates along the radius.

On all internal boundaries of the turbogenerator pole division between the zones of different electromagnetic characteristics, the boundary conditions are found in an appropriate coordinate system based on the known behaviour dependencies of electromagnetic field vectors on the line of media distribution [3]

$$A_i = \frac{v_{i-1} A_{i-1} + v_{i+1} A_{i+1}}{v_{i-1} + v_{i+1}};$$

$$A'_i = \frac{v_{i-1} A'_{i-1} + v_{i+1} A'_{i+1}}{v_{i-1} + v_{i+1}}, \quad (10)$$

where the indexes  $i$  represent the node numbers corresponding to the direction perpendicular to the media distribution line.

After obtaining the spatial distribution of the vector potential of electromagnetic field on the pole division of turbogenerator cross-section, the value of the module and magnetic induction vector components in the grid nodes of all zones in the cylindrical coordinate system is found from the following ratios

$$B_r = \frac{1}{r} \frac{\partial A}{\partial \alpha}; \quad B_\alpha = -\frac{\partial A}{\partial r}; \quad B = \sqrt{B_r^2 + B_\alpha^2}, \quad (11)$$

where  $B_r$ ,  $B_\alpha$ ,  $B$  are the radial, tangential components, and the module of a magnetic induction vector in the grid nodes both of the rotor's coordinate system and the stator's coordinate system reduced to the rotor's. The consideration of nonlinear electromagnetic characteristics of the stator and rotor materials in the models is realized by means of the following splines

$$v(B) = \sum_{m=1}^3 a_i^{(k)} (B_k - B)^m, \quad k = 1, 2, \dots, n, \quad (12)$$

where  $n$  is the number of segmentations of the material magnetization curve along the axis B.

Electromagnetic characteristics of the equivalent rotor tooth zone, as well as the zones of the laminated body and laminated stator teeth are recalculated by means of the known ratios [2, 3]

$$v_r = \frac{d_f + v_0 \cdot d_0 / v}{d_f + d_0} \cdot v; \quad v_r = \frac{d_f + d_0}{d_f + v \cdot d_0 / v_0} \cdot v;$$

$$\gamma = \frac{\gamma_f d_f + \gamma_{Cu} \cdot d_0}{d_f + d_0}, \quad (13)$$

where  $\gamma_f$  is the electrical conductivity of the rotor material;  $\gamma_{Cu}$  is the copper conductivity;  $d_f$ ,  $d_0$  are either the width of the tooth and slot, or the ferromagnetic sheet and isolation of the laminated stator.

To provide correlation and unambiguity between electrical and magnetic quantities of the rotor winding during the transition process of the turbogenerator at no-load, the excitation current value is determined by the equation bellow [3]

$$\frac{di_f}{dt} = \left( u_f - r_f i_f - \frac{d\psi_f}{dt} \right) / L_f, \quad (14)$$

where

$$\frac{d\psi_f}{dt} = w_f k_f l_r \sum_{i=1}^n \frac{\partial A_{Ri}}{\partial t}, \quad (15)$$

with  $w_f$  being the number of the rotor's windings;  $l_r$  being the axis winding length;  $A_{Ri}$  being the value of the function of vector potential in the coordinate system of the rotor in the grid nodes being located in the winding;  $k_f$  being the coefficient involving the number of the no-

## Calculation of Electromagnetic Processes for a Turbogenerator With Equivalent

des along the angular coordinate  $\alpha$  which get into the rotor's winding zone.

Zones of the rotor winding whose integral currents define a general pattern of the process occurring in the device are the source of field in the mathematical model for no-load calculations of the turbogenerator. The value of current density  $\delta$ , which is necessary for equation (1), is obtained from the ratio below

$$\delta = \frac{i_f \cdot w_f}{S}, \quad (16)$$

where  $i_f$ ,  $w_f$ ,  $S$  represent the current, the number of windings, and the area of the cross-sectional zone of the excitation winding.

Equations for the stator phase windings have the following form

$$u_i = \frac{d\Psi_i}{dt} = w_i k_i l \sum_{m=1}^n \frac{\partial A'_{Si}}{\partial t}, \quad i = A, B, C, \quad (17)$$

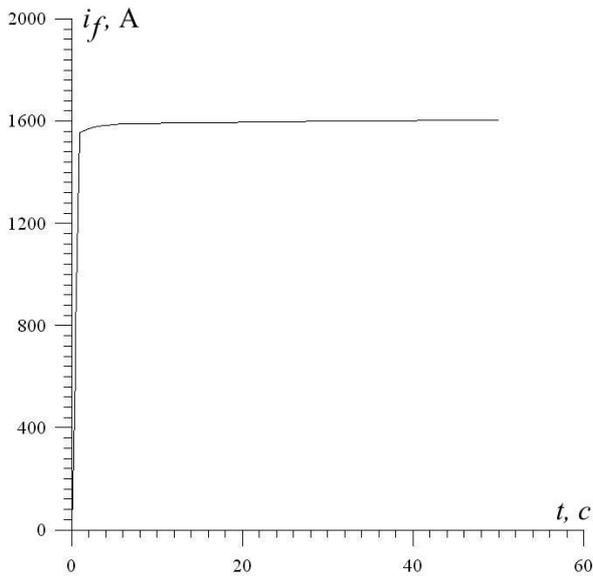


Fig. 3. Time values of an excitation current induced in the rotor's winding.

where  $u_i$  is the voltage in the stator's windings;  $w_i$  is the number of stator armature windings in each phase;  $l$  is the axial winding length;  $A'_{Si}$  is the value of the vector potential function in the nodes of a spatial discretization grid connected with the transferred stator coordinate system within the windings zone.

#### 4. Mathematical model of the turbogenerator with an equivalent rotor tooth zone in the rotor and stator systems of coordinates simultaneously

The given mathematical field model of the turbogenerator supposes electromagnetic processes in the rotor body and its equivalent tooth zone, and in the air gap

between the stator and rotor to be considered in the coordinate system of the moving rotor. The analysis of electromagnetic phenomena in the body and tooth zone of the stator, as well as in the air gap outside the turbogenerator is performed in the coordinate system of the stator. At the same time, the reciprocal displacement of constructional rotor zones relative to the stator zones along the angular coordinate can be noticed.

Thus, electromagnetic processes in the equivalent rotor tooth zone, the massive rotor body and the air gap between the stator and rotor are calculated with the help of the equations (1) - (3) mentioned above, respectively.

The electromagnetic processes in the stator's tooth zone and its body can be found in the coordinate system of the immovable stator from the ratio

$$0 = \nabla \times N \nabla \times \mathbf{A}. \quad (18)$$

The value of vector potential of electromagnetic field outside the turbogenerator we calculate in the same way as in the above expression (5)), but the stator system of coordinates

$$0 = v_0 \nabla \times \nabla \times \mathbf{A}. \quad (19)$$

Boundary conditions for electromagnetic field equations with respect to potentials along the perimeter of cross-sectional zones of the turbogenerator pole division in physical coordinate systems, similar to (7), (8) i (9), have the following form

$$\begin{aligned} A_{k=1} &= 2A_{k=2} + 2A_{k=n-1} - A_{k=3} - A_{k=n-2} - A_{k=n}; \\ A_{k=n+1} &= A_{k=2} + A_{k=4} + A_{k=n-1} - 2A_{k=3} - 2A_{k=n}; \\ A_{i=m+1} &= 2A_{i=m} - A_{i=m-1}. \end{aligned} \quad (20)$$

Respectively, on all the internal boundaries of the turbogenerator cross-section zones, the boundary conditions are to be found on the basis of ratio (10) in physical system of coordinates.

The calculations of the module and components of the magnetic induction vector in the nodes of the stator and rotor grids are performed by means of (11), and taking into consideration nonlinear properties of the magnetic materials of the stator and rotor is carried out by using (12). Recomputation of electromagnetic characteristics of the rotor tooth zone, laminated stator body and its teeth are done on the basis of (13).

The value of the current in the turbogenerator excitation winding is found from expression (14). The stator phase voltage is determined from expression (17), in which the function of the vector potential of the electromagnetic field belongs to the coordinate system of the stationary stator.

When calculating electromagnetic processes in turbogenerator cross-section zones at no-load on the basis of the above described model, there occurs the me-

chanical displacement of the node grid of the rotor system of coordinates relative to the grid of nodes of stator coordinate system fulfilling the following conditions

$$\Delta\alpha = \omega\Delta t, \quad (21)$$

where  $\Delta\alpha$  is the angular step of the discretization grid;  $\Delta t$  is the step of time integration of the system of differential equations;  $\omega$  is the angular rotary speed. So, in one step of time integration of the system of differential equations, the reciprocal location of the stator and rotor grids becomes one position displaced along the angular coordinate. This explains how physical rotation of the rotor takes place in the model relative to the stator.

### 5. Calculation results

By using the developed mathematical field models for the calculation of an electromagnetic field in the turbogenerator cross-section zones, the computer simulation of the transition process of the real turbogenerator TGV-500 at no-load has been carried out with the supply voltage of the excitation winding being  $u_f = 141$  V according to the device specifications. The results obtained are shown in Fig. 3 – 10.

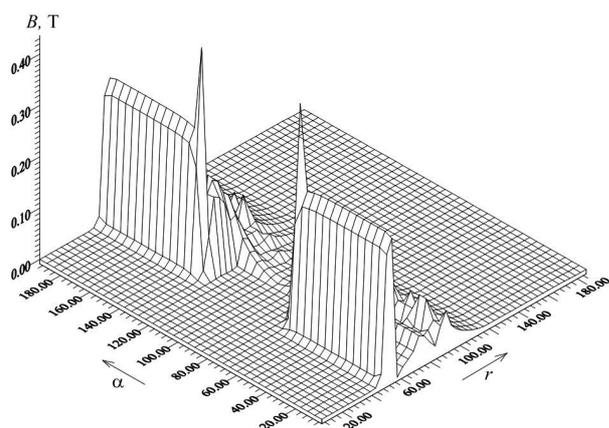


Fig. 4. Spatial distribution of the magnetic induction vector module in the rotor's coordinate system on the turbogenerator pole division at  $t = 1$  sec of the transition process at no-load.

The given spatial radius distributions of module of magnetic induction vector are shown in the coordinate system of rotor and stator grid nodes corresponding to the following zones of the device cross-section (according to Fig.1): 0÷54 is the ferromagnetic rotor body zone; 54÷75 is the equivalent rotor tooth zone; 75÷87 is the zone of an air gap between the stator and rotor; 87÷117 is the stator tooth zone; 117÷174 is the stator body zone; 174÷186 is the air gap outside the turbogenerator. The number of the discretization grid nodes along an angular coordinate on the pole division of the turbogenerator is equal to 192.

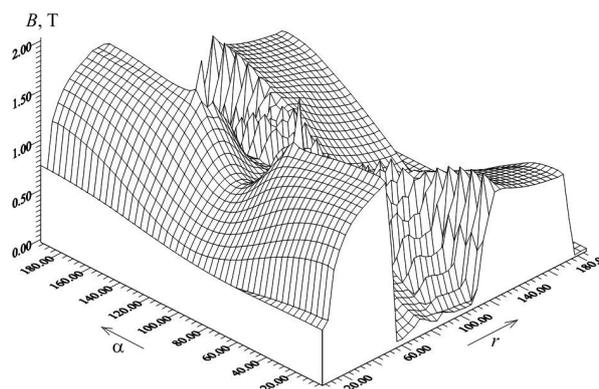


Fig. 5. Spatial distribution of the magnetic induction vector module in the rotor's coordinate system on the turbogenerator pole division at  $t = 100$  sec of the transition process at no-load.

Along the angular coordinate, the nodes of the coordinate grid are as follows: 2÷4; 8÷12; 16÷20; 24÷28; 32÷36; 40÷44; 48÷52; 56÷60; 64÷68; 72÷76; 80÷84; 88÷92; 96÷100; 104÷108; 112÷116; 120÷124; 128÷132; 136÷140; 144÷148; 152÷156; 160÷164; 168÷172; 176÷180; 184÷188; 192÷194 to the laminated teeth of the stator body; 4÷8; 12÷16; 20÷24; 28÷32; 36÷40; 44÷48; 52÷56; 60÷64; 68÷72; 76÷80; 84÷88; 92÷96; 100÷104; 108÷112; 116÷120; 124÷128; 132÷136; 140÷144; 148÷152; 156÷160; 164÷168; 172÷176; 180÷184; 188÷192 to the stator slots.

Figure 3 represents the time dependence of excitation winding current  $i_f$  of the turbogenerator in the transition process at no-load. As we can see in Fig. 3, the transition process in comparison with the excitation winding current is of short duration and lasts up to (under) 1 sec. Values of the index coincide with the respective parameter of the real turbogenerator at no-load.

Figure 4 depicts the spatial distribution of the magnetic induction vector module on the pole division of the turbogenerator at  $t = 1$  sec of the transition process at no-load obtained on the basis of the mathematical field model developed in the coordinate system of the moving rotor.

Figure 5 shows the spatial distribution of the magnetic induction vector module on the turbogenerator pole division at  $t = 100$  sec of the transition process at no-load obtained on the basis of the mathematical field model developed in the moving rotor coordinate system.

Figure 6 illustrates the spatial distribution of the module of a magnetic induction vector on the pole division of the turbogenerator at  $t = 650$  sec of the transition process at no-load obtained on the basis of the field mathematical model developed in the coordinate system of the moving rotor.

## Calculation of Electromagnetic Processes for a Turbogenerator With Equivalent

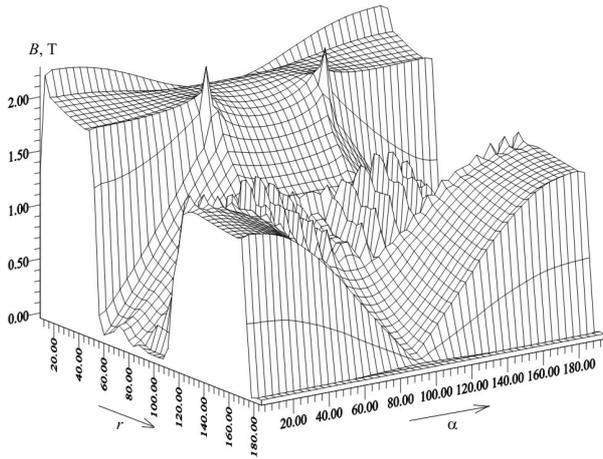


Fig. 6. Spatial distribution of the magnetic induction vector module in the rotor's coordinate system on the turbogenerator pole division at  $t = 650$  sec of the transition process at no-load.

Figure 7 represents the spatial distribution of the magnetic induction vector module on the pole division of the turbogenerator at  $t = 650,01543$  sec of the transition process at no-load obtained on the basis of the mathematical field model developed in the coordinate systems of the rotor and stator simultaneously.

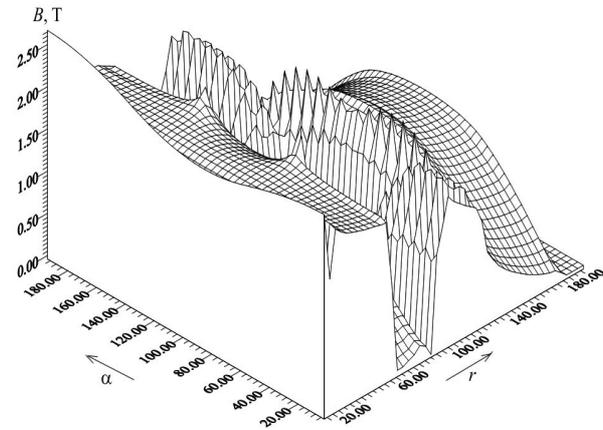


Fig. 7. Spatial distribution of the magnetic induction vector module in phase coordinate systems of rotor and stator simultaneously on the turbogenerator pole division at  $t = 650, 74482$  sec of the transition process at no-load.

Figure 8 shows the spatial distribution of the magnetic induction vector module on the pole division of the turbogenerator at  $t = 650, 74758$  sec of the transition process at no-load, obtained on the basis of the mathematical field model developed in the coordinate systems of the rotor and stator simultaneously. Figure 9 demonstrates the spatial distribution of the magnetic induction vector module on the pole division of the turbogenerator with a conductive tooth zone of the stator in the phase coordinate systems at the time when

$t = 650, 21045$  sec of the transition process at no-load obtained on the basis of the mathematical field model developed in the coordinate systems of the rotor and stator simultaneously.

Figure 10 shows the time-dependent values of A-phase voltage in the stator winding after the transition process at no-load in linear (1) and nonlinear (2) models with real tooth structure of the stator, as well as in linear (3) and nonlinear (4) models with equivalent tooth zones [3].

The results presented in Fig. 4, Fig. 5, and Fig. 6 demonstrate that the transitional magnetic process in the zones of turbogenerator cross-section is much longer than the electrical one in the turbogenerator excitation winding. This is caused by the fact that the electroconductive massive body of the rotor due to eddy currents of conductivity makes a damping influence, preventing an electromagnetic field from penetrating into the rotor.

The electromagnetic process in the stator tooth zone and its body shown in Fig. 3, Fig. 4 and Fig. 5 corresponds to the transformed coordinate system related to the moving rotor. In order to obtain the process in the physical stator coordinate system, it is necessary, in the defined zones, to make a inverse transformation. As we know, the relation of electromagnetic values in different inertial systems of reference is described by the Lorentz-Poincare ratio

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \quad (22)$$

where  $\mathbf{E}'$ ,  $\mathbf{E}$  are the vectors of electric field intensity in the moving and stationary system of coordinates respectively;  $\mathbf{v}$  is the vector of linear velocity of the moving coordinates system relative to the stationary one;  $\mathbf{B}$  is the vector of magnetic induction in the immovable reference frame.

Using the known expression which describes a ratio between vector and scalar potentials of the electromagnetic field and the vector of electric field intensity

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi, \quad (23)$$

under the condition of the accepted calibration  $\varphi = 0$ , in the given formulation of the task, equation (22) can be written in the simplified form

$$\frac{\partial A'_z}{\partial t} = \frac{\partial A_z}{\partial t} + \omega \frac{\partial A_z}{\partial \alpha}, \quad (24)$$

where  $A'_z$ ,  $A_z$  are the axial components of the vector potential of electromagnetic field both in the moving and stationary systems of coordinates respectively;  $\omega$  represents the angular velocity of the rotor.

Taking into consideration that

$$\omega = \frac{d\alpha}{dt}, \quad (25)$$

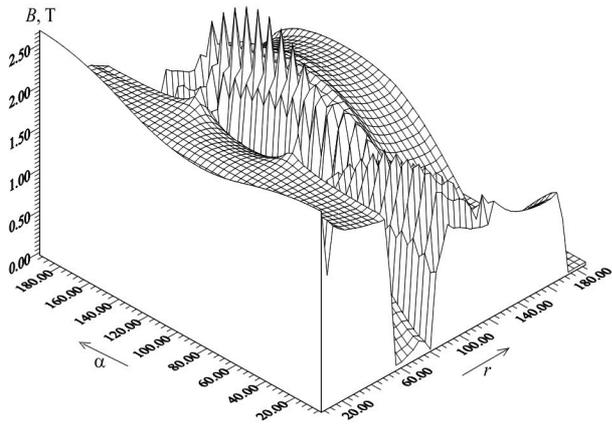


Fig. 8. Spatial distribution of the magnetic induction vector module in the rotor and stator's phase coordinate systems simultaneously on the turbogenerator pole division at  $t = 650, 74758$  sec of the transition process at no-load.

equation (24) will be given by

$$\frac{\partial A'_z}{\partial t} = \frac{\partial A_z}{\partial t} + \frac{d\alpha}{dt} \frac{\partial A_z}{\partial \alpha}, \quad (26)$$

Having integrated expression (26) with respect to time, we shall have

$$A'_z(t) = A_z(t) + \Delta A_{z\alpha}(t). \quad (27)$$

Expression (27) shows that the spatial distribution of the function of vector potential in the moving system of coordinates (or the stationary one) is obtained from the spatial distribution in the stationary system of coordinates by the  $\alpha$  displacement of the values along the angular coordinate which corresponds to reciprocal position of the rotor and stator at a given time  $t$ . Thus, to change from the transformed coordinate system in the tooth zone and body of the stator into physical coordinate systems of the stator, it is necessary that the obtained distribution of the vector potential be  $\alpha$ -angle displaced. This is verified by the results obtained on the basis of the mathematical field model of the turbogenerator developed simultaneously in the systems of coordinates of the rotor and stator depicted in Fig. 7 and Fig. 8. They differ from the distribution represented in Fig.6 only in a certain displacement along the angular coordinate. The range of this displacement in a certain case is determined by reciprocal position of the rotor and stator at a given time  $t$ .

Since the mathematical field model of a turbogenerator in the system of coordinates of the moving rotor is much simpler in implementation, it is reasonable to apply this very model to analyze electromagnetic phenomena in the zones of the turbogenerator cross-section at no-load.

Taking the initial assumptions into account, at no-load there are no currents in the stator which may influence a general pattern of the process. Therefore, to analyze electromagnetic phenomena in the turbogenera-

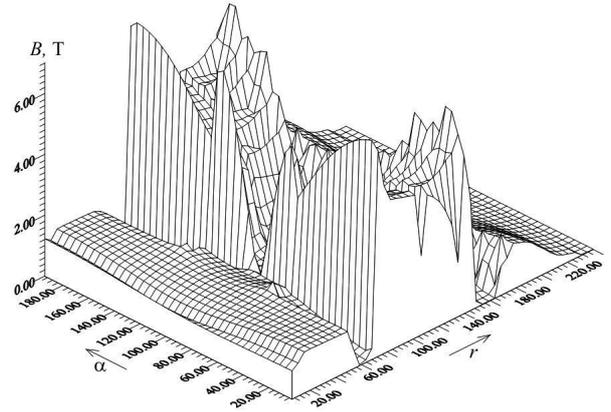


Fig. 9. Spatial distribution of the magnetic induction vector module on the turbogenerator pole division with a conductive stator tooth zone in phase coordinate systems at  $t = 650, 21045$  sec of the transition process at no-load.

tor in the given mode, it is sufficient to do calculation of an electromagnetic field in all the zones of the device cross-section for one fixed reciprocal position of the moving and stationary systems of coordinates. The conversion of the calculation results to the physical coordinate system is carried out by a simple angular coordinate (theta) displacement of the spatial distribution of an electromagnetic field. Exactness of such a transformation depends upon the number of nodes of a spatial discretization grid that are assigned for the slot division of the stator.

Obtaining maximum adequate results requires the calculation of the electromagnetic field for all cases of the reciprocal position of the stator and rotor coordinate systems along the angular coordinate on the slot division of the stator. Due to the tooth structure of the periodicity of the pole division of the device, the data obtained are comprehensive to represent an electromagnetic process in the stator at no-load at the given number of nodes in a discretization grid.

Reasonable number of nodes on the slot division of the stator in the model is chosen from the practical perspective and is equal to 8. Therefore, to obtain a full pattern of the process, it is necessary to calculate an electromagnetic field for 8 cases of the reciprocal position of the stator and rotor. As the mechanical displacement of the media is minimal, then, by using the calculated data obtained from any fixed reciprocal position of the coordinate systems as initial, it is easy to gain spatial distributions of the electromagnetic field for all other necessary grid positions. In such a way, on the basis of the developed model in the system of coordinates of the moving rotor, it is possible to have the most complete pattern of electromagnetic processes in the turbogenerator cross-section at no-load.

## Calculation of Electromagnetic Processes for a Turbogenerator With Equivalent

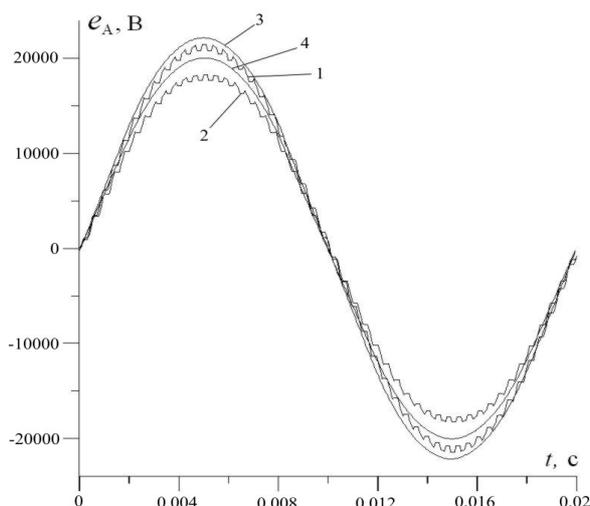


Fig. 10. Time values of A-phase voltage in the stator's winding after completing the transition process in linear (1) and nonlinear (2) models with a real tooth structure of the stator, as well as in linear (3) and nonlinear (4) models with equivalent tooth zones.

Much attention must be paid to spatial distribution of the module of magnetic induction vector given in Fig.9. This distribution has been obtained on the basis of the mathematical field model of a turbogenerator developed simultaneously in the coordinate systems of the rotor and stator. In this particular case (in this model) the calculation of electromagnetic processes in the slots of the stator tooth zone has been carried out by using equation (2), but not relation (5). Applying this equation in the no-load mode of the turbogenerator leads to the appearance of conductive currents in slots with the stator winding enclosed. In fact, the calculation of an electromagnetic field in the stator slots by means of expression (2) implements, in the model, a short-circuit mode in the stator winding, but not the no-load one. As we can see from Fig.9, the application of (5) has led to a great increase in magnetic induction on the rotor surface, on the internal surface of the stator tooth zone and in the air gap between the rotor and stator, as well as to a decrease in magnetic induction in the stator body (that stator is demagnetized). Such processes take place in the short circuit mode of the stator winding, and are characteristic for the turbogenerator at no-load. This has been considered in detail in the work [3].

The result obtained confirms a necessity of taking into account the peculiarities of electromagnetic characteristics of the device zones in accordance with the operation mode reproduced by the mathematical model.

Comparing the results given above and obtained in work [3] (Fig.10), we can state that an equivalent substitution of the stator tooth zone while developing the mathematical field model of the turbogenerator at no-

load is the cause of a 3% error in the linear variant, and about 11% error in the nonlinear one in relation to the models given above.

However, the general error of analysis of the turbogenerator at no-load on the basis of the models considered above equals 31,9 % in the linear variant, and 11,9 % in the nonlinear one. This result is better than that obtained in [3], but it is insufficiently acceptable for the class of the most exact mathematical field models of electrical devices. The nature of the errors, as in [3], is associated with the equivalent replacement of the tooth zones of the stator winding.

### 5. Conclusion

From the practical perspective, the development of mathematical field models of electrical devices on the basis of Maxwell's equations with respect to potentials is suitable to analyze the electromagnetic processes. Such an approach essentially decreases the system of main equations and simplifies the task of finding boundary and edge conditions.

When developing mathematical field models of electrodynamic devices, it is necessary that the systems of coordinates for calculating the electromagnetic field be taken into consideration. Neglecting this fact causes wrong results and lack of complete analysis of real physical phenomena in electrical objects.

To optimize the calculation of electromagnetic field in the devices, it is reasonable to use such property as periodicity of electromagnetic process in the device. This, on the one hand, shortens the system of equations, but on the other hand, provides comprehensive information on the electromagnetic phenomenon in the device.

The mathematical field models of a turbogenerator at no-load developed in the work, show that the character of electromagnetic processes in real structures of the object construction essentially differs from the processes obtained on the basis the equivalent schemes of substitution of the device zones ( as the example given in the work [3]). Hence, when developing mathematical field models of electrical objects, it is necessary to reproduce the existing structure of the device as accurately as possible.

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#### **РОЗРАХУНОК ЕЛЕКТРОМАГНІТНИХ ПРОЦЕСІВ ТУРБОГЕНЕРАТОРА З ЕКВІВАЛЕНТНОЮ ЗУБЦЕВОЮ ЗОНОЮ РОТОРА В РЕЖИМІ НЕРОБОЧОГО ХОДУ**

Ярослав Ковівчак

З метою повнішого аналізу електромагнітних процесів в електродинамічних пристроях необхідно здійснювати розрахунок електромагнітного поля в рухомих і нерухомих елементах їх конструкції з урахуванням систем координат рухомих і нерухомих тіл. Також важливим питанням при побудові польових математичних моделей електротехнічних пристроїв є відтворення режиму заданих напруг, а не струмів, оскільки струми в обвитках є одним із наслідків загального електромагнітного явища, а не його при-

чиною. У більшості наукових праць, присвячених розрахунку електромагнітного поля у рухомих середовищах, нехтують відмінністю між системами відліку, в яких розглядається процес, та використовують режим заданих струмів, що приводить до спотворення результатів аналізу реальних фізичних явищ у пристроях. У роботі розроблено двохвимірні польові математичні моделі розрахунку квазістационарного електромагнітного поля у поперечному перерізі турбогенератора в режимі неробочого ходу з еквівалентною зубцевою зоною ротора при заданій напрузі обмотки збудження. Запропоновані моделі передбачають розрахунок електромагнітного поля у всіх зонах пристрою на основі векторного потенціала як в одній системі координат рухомого ротора, так і в системах координат ротора та статора одночасно. Результати комп’ютерного симулювання електромагнітних процесів на основі розроблених моделей показують, що при побудові польових математичних моделей електродинамічних пристроїв необхідно враховувати системи координат, в яких здійснюється розрахунок електромагнітних явищ. Крім того, характер електромагнітних процесів у реальних структурах конструкції об’єктів кардинально відрізняється від процесів, отриманих на основі еквівалентних заступних схем для окремих зон пристроїв.



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