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THE METHODS OF CHOOSING THE WAVELETS FOR ONE DIMENSIONAL SIGNAL PROCESSING

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***Abstract*: The paper describes the problems of the effectiveness increasing in the selection of base functions for the processing of different types of one-dimensional signals in the wavelet domain. The efficiency of representing signals in the wavelet domain has been shown; their analysis and processing are related to the choice of base functions. The basic methods and algorithms for selecting base functions are defined, in which the choice of optimal wavelets has been carried out according to a particular criterion for certain types of signals. Methods have been presented for assessing the efficiency of the choice of base wavelets by the criterion for the ratio of the energy of the wavelet coefficients to the entropy of energy distribution of wavelet coefficients, the criterion for estimating the correlation coefficient, and the information criterion. The universal index of quality of the signal has been proposed and substantiated for the first time as a new criterion for choosing a wavelet and the method has been improved for the choice of base wavelets using a genetic algorithm according to the universal signal quality index criterion. The method of multi-criteria optimization of the choice of base wavelet for the processing one-dimensional non-periodic signals based on the tools of fuzzy logic has been proposed and developed, which made it possible to improve the efficiency of signal processing.**

***Index Terms*: wavelet, base wavelets, optimal wavelets, selection criteria, multi-criterion optimization**

I. INTRODUCTION

One of the development strategies of modern computer systems is the analysis, processing, storage, and transmission of information presented by different types of signals. However, for the analysis and processing of signals whose frequency varies in time, the usual spectrum is not very informative. Given that, the methods of time-frequency representation of signals are widely used. The wavelet transform is the most common among them. At the same time, the effectiveness of signal representation in the wavelet area depends on the choice of wavelet functions [1].

Mainly, the size of the support, the number of zero moments and the smoothness of the basic functions are taken into account in the selection of wavelet. These properties provide only a mathematical description of the wavelet functions and do not allow a practical recommendation for the analysis and processing of different types of signals.

Currently, there are many different wavelets. The general classification may be as follows [2]:

– "rough" – functions of the Gaussian type, Morlet and the "Mexican hat" (MHAT);

– infinite regular – continuous Meyer functions;

– orthogonal with compact support – wavelets of the Daubechies, Symlet and Coiflet families;

– bio-orthogonal with compact support – B-spline functions;

– complex with minimal properties – Gaussian, Morlet, Shannon functions, and frequency B-spline wavelet.

Overall, wavelets of the Daubechies, Symlet and Coiflet families and Haar wavelet have general properties such as orthogonality, having scaling function, reconstruction opportunity, and discrete transformations. Therefore, these functions are an effective tool for analyzing and processing discrete non-periodic signals.

The Wavelet transform uses the Daubechies functions allowing to save energy of signal more effectively and to redistribute this energy compactly. The negative property of the Daubechies functions is asymmetry.

The Symlet family includes orthogonal and almost symmetric wavelets whose properties are similar to those of the Daubechies functions. The symmetry property provides minimal phase distortions. Symlet's functions coincide with those of Daubechies' up to 4 orders.

The Coiflet functions are a family with some features. These functions are more symmetrical than the Daubechies functions that provide almost linear phase characteristics. However, the variation of the coefficients of the smoothness in the Coiflet functions is higher than of the Daubechies functions of the same order.

When using the Haar function, which has a compact carrier in the time domain [0,1], it is poorly localized in the frequency domain and ineffective in decomposing the signals represented by smooth functions. The advantages of the Haar feature are ease of use and speed of conversion.

Therefore, the actual task is to the choice of the optimal base function, the use of which provides the necessary accuracy of approximation of informative signals in the time-frequency domain. Also, this function allows to realize the qualitative decomposition and concentrate the energy of the signal in a small number of significant non-zero coefficients.

Currently, modern signal processing techniques employ approaches based on energy, correlation, and information criteria [3–8].

II. THE METHODS BASED ON ENERGY, CORRELATION, AND INFORMATION CRITERIA

It is known that signal energy is the main parameter that characterizes real signals. Parseval's theorem establishes the relationship between the energies of the signal in the time and frequency domains. For orthogonal wavelet functions, relationship between the energy of the signal and energy of its wavelet coefficients is represent as [3, 4]:

$$E_C = \sum_n |S(n)|^2 = \sum_m \sum_n |C_{m,n}|^2, \quad (1)$$

where E_s – the energy of the signal in the time-space; E_C – energy signal in wavelet space; $C_{m,n}$ – coefficients of the discrete wavelet transform.

It is also important to note that a spectral energy distribution of wavelet coefficients has a significant role in wavelet-based signal analysis and processing. The quantitative measure of the energy distribution is Shannon entropy

$$H = -\sum_n p_n \cdot \log p_n , \quad (2)$$

where $p_n = \frac{|C_{n,m}|^2}{E_C}$ is the probability distribution of the wavelet coefficients. The lower value is Shannon entropy, the higher one – the concentration of power.

Thus, the criterion for evaluating the effectiveness of base wavelet can be determined by the ratio between energy and Shannon entropy (Energy to Shannon Entropy ratio – EER).

$$EER = \frac{E_C}{En}, \quad (3)$$

In multilevel signal decomposition, the ratio of energy to Shannon entropy is determined in particular for each level m:

$$EER_m = \frac{E_{Cm}}{En_m}. \quad (4)$$

For some signals, when calculating expression (4), the maximum values of criterion differ by less than 1 % for most wavelet functions. In this case, it is advisable to use a modified criterion *EER*, which is determined by the following relation:

$$EER = \frac{\gamma_E}{En(d)}, \quad (5)$$

where γ_{E} – the fraction of the energy of the approximation coefficients in the total energy of the transformed signal, which is determined by the expression:

$$\gamma_E = \frac{\sum_n |a_n|^2}{\sum_n |a_n|^2 + \sum_n |d_n|^2}, \quad (6)$$

where a_n – approximation coefficients; d_n – detail coefficients of the discrete wavelet transform.

The closer the value is to 1, the more energy concentrates at the approximation coefficients. The detail coefficients contain some information about the features of the signal at all decomposition levels. Therefore, the smaller the entropy of the energy distribution of the detail coefficients is, the better they detect these features of the signal.

The correlation criterion is based on the similarity between the analyzed signal and the scaled version of the base wavelet [5]. An expression, used to calculate the correlation coefficient, is represented as:

$$CORR = \frac{COV_{S\psi}}{\sigma_S \cdot \sigma_\psi}, \quad (7)$$

where $CORR$ – the correlation coefficient between the analyzed signal and the base wavelet; $COV_{S\psi}$ – mutual covariance of sequences; σ_S and σ_ψ – standard deviations of sequences.

The higher the similarity between the signal and the base wavelet is, the closer the correlation coefficient is to 1.

However, the application of such a criterion is ineffective if the output signal is very noisy. In this case, it is advisable to determine the similarity between the signal and its wavelet coefficients, namely the coefficients of approximation by the following expression:

$$Cr = \frac{COV_{SW}}{\sigma_S \cdot \sigma_W}, \quad (8)$$

where Cr – the correlation coefficient between the analyzed signal and their wavelet coefficients; COV_{SW} – mutual covariance of sequences; σ_S and σ_W – standard deviations of sequences.

If the correlation describes only a linear relationship of variables, the information describes any relationship. The wavelet transform is mainly used for the analysis and processing of signals that have some information uncertainty, so an appropriate use of the entropy allows to communicate between the signal and the wavelet coefficients. The ratio of the energy to the Shannon entropy, which is defined by (2), evaluates the energy content of wavelet coefficients. To get information content of wavelet-coefficients and compare with the information content of the signal, it is proposed to use such information criteria as joint entropy, conditional entropy, and mutual information [6, 7, 8].

Mutual information is defined as the average amount of information about the signal which is included in the wavelet coefficients and represented as:

$$I(S,C) = -H(S,C) + H(S) + H(C), \quad (9)$$

where $H(S,C)$ – joint entropy of the signal and its wavelet coefficients; $H(S)$ – entropy signal; $H(C)$ – entropy wavelet coefficients.

Another characteristic of the theory of information is Kullback-Leibler divergence or relative entropy, which is a measure of the distance between two probability distributions defined on the same alphabet. In contrast, mutual information is a measure of the distance of two variables within a distribution of relative entropy which determines the distance between the distributions [6]:

$$D(S \parallel C) = \sum_{i \in S,C} p(S_i) \ln \frac{p(S_i)}{p(C_i)}. \quad (10)$$

Thus, taking into account the need to ensure the maximum mutual information and the minimum relative entropy, a criterion for evaluating the effectiveness of selected basis wavelet can be determined using the ratio:

$$IER = \frac{I(S,C)}{D(S \parallel C)}, \quad (11)$$

where $I(S,C)$, $D(S \parallel C)$ – mutual information and relative entropy between the signal and its wavelet coefficients, respectively.

Therefore, the base wavelet, which ensures the maximum relation of the mutual information to the relative entropy of the test signal, most suitable for further processing of the signal.

III. COMPARATIVE ANALYSIS OF THE CRITERIA EFFICIENCY OF THE CHOICE OF BASE FUNCTIONS

For the evaluation of the developed models, families of orthogonal functions were selected with a compact carrier Daubechies (db1 ... db20), Coiflets (coif1 ... coif5), Symlets (sym1 ...sym20) and test signals from the Matlab package: blocks, bumps, doppler, heavy sine, sumlichr, trsin, wcantor [9]. Test signals are shown in Fig. 1.

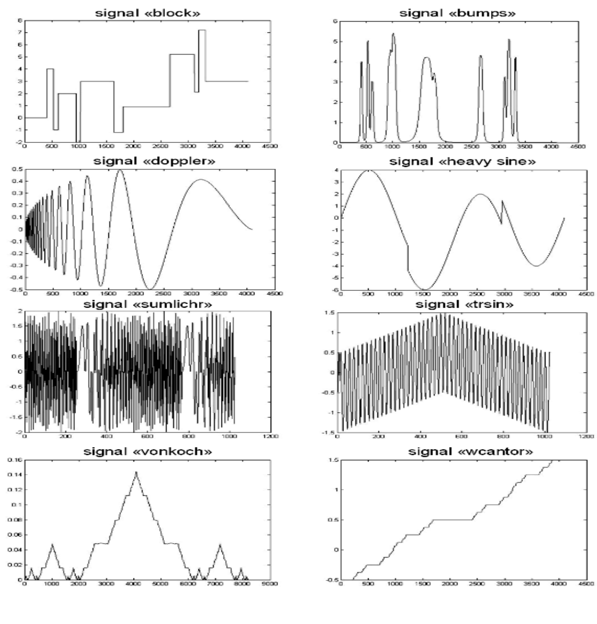

Fig. 1. The test signals to evaluate the efficiency of the choice of base wavelet

For each of the test signals and the chosen base functions, the Matlab package evaluates the values for the energy, information, and correlation criteria by expressions (3, 7, 11). As a result, arrays of values for each of the criteria are formed. Table 1 represents the results of this research, namely, the base functions, which are determined to be optimal by each criterion [10].

Table 1

The optimal base functions by the energy, information and correlation criteria

Test signals	Base functions		
	EER	Cr	IER
"blocks"	sym20	db2, sym2, sym14, sym16, coif1	db19
"bumps"	sym12	db1, db2, sym, coif1,coif2	db20
"doppler"	db19, db20	db2, sym2, sym7, sym11-sym14, sym20, coif1	sym14, coif1
"heavy sine"	sym7, sym11	db1, db2, sym1, sym2, sym12-sym14	db2, sym2
"sumlichr"	db19, db20	coif1	db11
"trsin"	sym19, sym20	sym2, coif1, sym7, sym11, sym14, sym16, sym20	db19
"vonkoch"	db4	db1, db2, db3, sym, coif1, coif2	db18
"wcantor"	sym20	db1, db2, sym1, sym2, coif1	coif1

The efficiency of the criteria was determined based on further studies:

1. The noise was applied to each test signal;
2. Base functions, defined as optimal by each of the criteria (Table 1), were used for denoising;
3. The SNR estimation determines the efficiency of DWT-based denoising of the signal according to the expression:

$$SNR = 10 \cdot \log_{10} \left(\sum_{i=1}^{N} s_i^2 \Big/ \sum_{i=1}^{N} \left| s_i - \tilde{s}_i \right|^2 \right), \quad (12)$$

where S – output signal; $\tilde{S}$ – denoise signal.

The result of the denoising of test signals using the base functions represented in Table 1 is shown in Fig. 2.

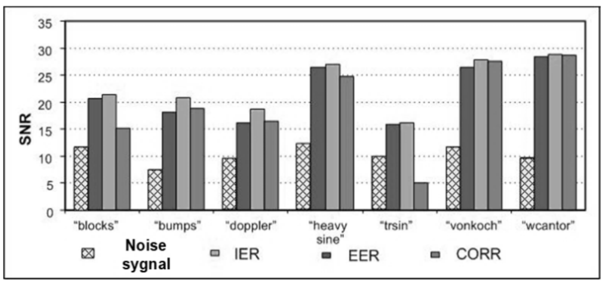

Fig. 2. Diagram of the obtained SNR values as a result of denoising of test signals using the base functions represented in Table 1

Based on the SNR values shown in Fig. 2, the best results were obtained using base functions defined by the criterion of the relation of mutual information to relative entropy.

IV. THE METHOD OF MULTICRITERIAL OPTIMIZATION OF CHOICE OF BASE WAVELET

The analysis showed that it is not always easy to reach the uniqueness of the choice of the base functions by the above criteria [10]. Therefore, a more generalized criterion is required. To solve the problem, when it is impossible to optimize all the conflict criteria by 100% and only each of them to a certain extent, it was proposed a new method based on the use of multicriteria optimization using fuzzy set theory. It is therefore needed to build a model of a multi-criteria optimization model for choosing base wavelet under uncertainty [11].

Suppose X = {x1, x2,…, xk} be the set of base functions, one of whose elements $x^* \in C$, and $C \subseteq X$, optimizes (maximizes) the given EER, IER, and Cr criteria, and G = {G1, G2, G3} is the set of criteria for evaluating efficiency of the base functions, where G1 is the EER criterion; G2 – IER criterion, G3 – Cr criterion.

Based on the analysis of methods for constructing the fuzzy set membership function, the direct method with one expert was chosen for the implementation of the multicriterial model of the choice of the optimal base function.

Since the purpose of optimization is to maximize the specified criteria, the corresponding membership functions are represented as a parametric sigmoid function with parameters a = 12 and c = 0.6:

$$\mu_G(x_i) = \frac{1}{1+\exp\left(-12\cdot(x_i-0.6)\right)}. \qquad (13)$$

As a result, the following fuzzy solution for equivalent criteria is obtained:

$$\tilde{D} = \left\{ \begin{array}{l} \dfrac{\min\left(\mu_{G_1}(x_1), \mu_{G_2}(x_1), \mu_{G_3}(x_1)\right)}{x_1}, \\ \dfrac{\min\left(\mu_{G_1}(x_2), \mu_{G_2}(x_2), \mu_{G_3}(x_2)\right)}{x_2}, \\ \ldots \\ \dfrac{\min\left(\mu_{G_1}(x_k), \mu_{G_2}(x_k), \mu_{G_3}(x_k)\right)}{x_k} \end{array} \right\}, \qquad (14)$$

The use of unequal criteria requires determining their relative weight. Based on the research of the effectiveness of the criteria (in term of the results of processing different types of signals), the following expert paired comparisons were made: 1) weak advantage of G2 over G1; 2) a significant advantage of G2 over G3; 3) significant advantage of G1 over G3. The generated matrix of paired comparisons on the Saati [12] scale is shown in Table.2.

The matrix of pairwise comparisons is diagonal and inversely symmetric. The degrees of belonging to the fuzzy set correspond to the coordinates of the eigenvector W = (w1, w2, ..., wk)^T of the matrix A. The weighting coefficients of the criteria G1 ÷ G3:

$$w_1 = 0,3;\ w_2 = 0,6;\ w_3 = 0,1.$$

Table 2

Matrix of paired comparisons of criteria

$[A]$	G_1	G_2	G_3	Priorities vector	Eigenvector
G_1	1	$1/3$	5	1.186	0.295
G_2	3	1	5	2.5	0.620
G_3	$1/5$	$1/5$	1	0.34	0.084
Sum:				4.026	1

Therefore, the fuzzy set required to select an effective base function is defined by the following expression:

$$\tilde{D} = \left\{ \begin{array}{l} \dfrac{\min\left(\mu_{G_1}(x_1)^{0.3}, \mu_{G_2}(x_1)^{0.6}, \mu_{G_3}(x_1)^{0.1}\right)}{x_1}, \\ \dfrac{\min\left(\mu_{G_1}(x_2)^{0.3}, \mu_{G_2}(x_2)^{0.6}, \mu_{G_3}(x_2)^{0.1}\right)}{x_2}, \\ \ldots \\ \dfrac{\min\left(\mu_{G_1}(x_k)^{0.3}, \mu_{G_2}(x_k)^{0.6}, \mu_{G_3}(x_k)^{0.1}\right)}{x_k} \end{array} \right\} \qquad (15)$$

As a result, the best base function should be considered to be the one with the highest degree of membership:

$$\mu_D\left(x^*\right) = \max_{i=1,2,3,\ldots,n} \mu_D(x_i). \qquad (16)$$

The set of criteria selected is open and can be supplemented by more detailed requirements for the choice of a particular wavelet, as each criterion can be considered as a convolution of local indicators at the lower levels of the hierarchy.

V. IMPLEMENTATION OF THE METHOD OF MULTICRITERIAL OPTIMIZATION OF CHOICE OF BASE WAVELET

The practical realization of the multi-criteria optimization problem of choosing a base wavelet function is performed based on the FIS-editor of systems of fuzzy output from the Fuzzy Logic Toolbox, which is a part of the package of applied mathematical modelling software Matlab R2011b [13].

It is known that in the process of constructing fuzzy-out systems, the methods of Mamdani and Sugeno have become the most commonly used. An analysis of both methods showed the feasibility of using a fuzzy model based on the Mamdani method.

The levels of efficiency of the criterion for choosing base wavelet functions can be described as follows: low (L), middle (M), high (H). At the fuzzification stage, membership functions for the sets of input and output linguistic variables are given. Three linguistic variables represent the set of criteria G = {G1, G2, G3}:

EER={EER_L, EER_M, EER_H};
IER={IER_L, IER_M, IER_H};
Cr={Cr_L, Cr_M, Cr_H};

Next, the sigmoid function is selected as a membership function for each input linguistic variable, and the trapezoidal function is selected as a membership function for the output linguistic variable. In the next stage, a rule base is formed in the form of a structure with three inputs and one output based on expert pair comparisons (Table 3).

Table 3

Rules for forming a multicriteria optimization model for choosing a base wavelet function

№	Input			Output
rule	G_1(EER)	G_2(IER)	G_3(Cr)	Y(WV)
1	Low	Low	Low	Low
2	Low	Low	Medium	Low
3	Low	Low	High	Low
4	Low	Medium	Low	Low
5	Low	Medium	Medium	Medium
6	Low	Medium	High	Medium
7	Low	High	Low	Medium
8	Low	High	Medium	Medium
9	Low	High	High	Medium
10	Medium	Low	Low	Low
11	Medium	Low	Medium	Low
12	Medium	Low	High	Medium
13	Medium	Medium	Low	Medium
14	Medium	Medium	Medium	Medium
15	Medium	Medium	High	Medium
16	Medium	High	Low	Medium
17	Medium	High	Medium	Medium
18	Medium	High	High	High
19	High	Low	Low	Low
20	High	Low	Medium	Medium
21	High	Low	High	Medium
22	High	Medium	Low	Medium
23	High	Medium	Medium	Medium
24	High	Medium	High	Medium
25	High	High	Low	High
26	High	High	Medium	High
27	High	High	High	High

Finally, in the process of defuzzification, to find x^* the centroid method is being used.

VI. GRAPHS OF THE OUTPUT VALUE OF EACH OF THE INPUT VARIABLES ARE OBTAINED AND SHOWN IN FIG. 3.THE METHODS BASED ON THE MAXIMUM EFFICIENCY OF WAVELET SIGNAL PROCESSING

The vast majority of existing methods for choice base wavelets are based on the principle of similarity of signal and base function. Studies in this field have shown that such a principle does not always allow to determine the optimal base function for signal processing. Therefore, it is advisable to make this choice based on the maximum efficiency of wavelet signal processing. Currently, the mean square error (MSE) criterion is most commonly used to evaluate the effectiveness of the wavelet transform. In most cases, the MSE criterion is used to evaluate the quality of the wavelet function and determine the accuracy of signal reconstruction. It is represented as:

$$MSE_t = \frac{\sum_{i=1}^{N}\left(S-\tilde{S}\right)^2}{N}, \quad (17)$$

where S – output signal; $\tilde{S}$ – reconstructed signal.

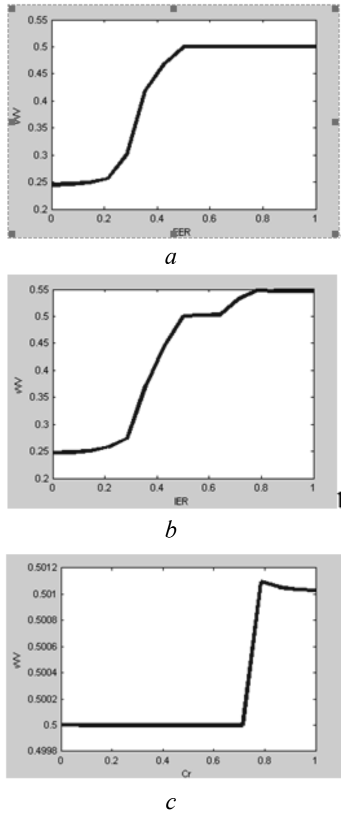

Fig. 3. Dependencies of the level of efficiency of the base functions on the value of each of the criteria: a) EER criterion; b) IER criterion; c) Cr criterion

Also, the MSE criterion can be used to estimate the contribution of the detail wavelet coefficients and is presented as:

$$MSE_f = \frac{\sum_{i=1}^{N}\left(c-c_{Noise}\right)^2}{N}, \quad (18)$$

where c – wavelet coefficients of an output signal; c_{Noise} – wavelet coefficients of the noise component of the signal.

The wavelet transform can be considered the most efficient when the MSE value is minimal.

As well, to evaluate the effectiveness of the results of the wavelet transform of one-dimensional signals, it is possible to use a universal index quality (UIQ) of the signal [15]:

$$Q = Q_{corr} \cdot Q_{dyn} \cdot Q_{mean} = \\ = \frac{\sigma_{S\tilde{S}}}{\sigma_S \cdot \sigma_{\tilde{S}}} \cdot \frac{2 \cdot S \cdot \tilde{S}}{\left(\bar{S}\right)^2 + \left(\bar{\tilde{S}}\right)^2} \cdot \frac{2 \cdot \sigma_S \cdot \sigma_{\tilde{S}}}{\sigma_S{}^2 + \sigma_{\tilde{S}}{}^2}, \qquad (19)$$

where Q_{corr} is the correlation coefficient between S and $\tilde{S}$, its dynamic range is [-1;1]; Q_{dyn} is the estimation of change of average value $\tilde{S}$ as to S, with a value range of [0,1]; Q_{mean} – estimation of change of dynamic range of signals S *i* $\tilde{S}$, its range of values is also [0,1], where the best value 1. Such an estimate is particularly useful when processing low-power and high-noise of signals.

Sufficiently effective in the case of signal denoising is the use of a genetic algorithm (GA) to determine the optimal wavelet functions. The right choice is not only in base wavelet but also in the basic parameters for the denoising signals, such as the level of decomposition, the type of threshold function, the threshold estimation rule and the threshold processing rule provides a comprehensive approach to the choice of the wavelet function and to improving the process of denoising signal [17].

Fig. 4 shows a block diagram of the search for optimal base wavelets and denoising parameters using GA.

The first input of the circuit receives a noisy signal S(N), and the second forms a set of noise compensation components.

Numerical results indicate that the GA-based method ensures optimal denoising in terms of successful restoration of the original signal with a significant reduction in the noise level. The method enables automatic and fast determination of parameters of denoising signals. Optimization of characteristics realizes with the help of GA. The result of the optimization is an array of denoising parameters.

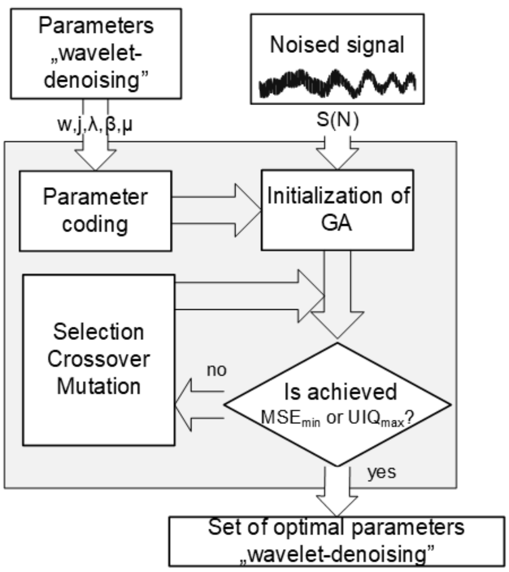

Fig. 4. Block diagram of the search for optimal base wavelets and denoising parameters using GA

The right choice of not only base wavelet but also of the basic parameters for the denoising signals, such as the level of decomposition, the type of threshold function, the threshold estimation rule and the threshold processing rule provides a comprehensive approach to the choice of the wavelet function and its parameters and plays an important role in improving the process of denoising signal.

The set of parameters for noise removal is presented in binary form, namely encoded by a sequence consisting of 15 binary digits, as it is shown in Fig. 5.

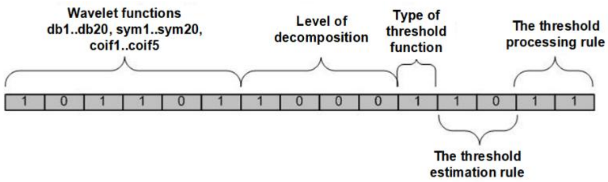

Fig. 5. Set of parameters for denoising signal

Basic GA parameters such as population size (Np), crossover probability (Pk) and mutation probability (Pm) have a significant impact on GA performance and its implementation results.

Another problem associated with GA optimization is the criteria used to stop the GA program. These criteria set is as follows:

1. If the maximum number of generations (Ns) is achieved, the program stops.
2. When fitness function has reached its most practical value within the next generations, GA stops.

It is advisable to use the MSE value calculated according to expression (17) and the UQI of signal calculated according to expression (19) to implement the fitness function of the genetic algorithm.

The research of presented methods of choice of base wavelets was carried out using the test signals of the Matlab package: blocks, bumps, doppler, heavy sine, sumlichr, trsin, vonkoch, wcantor. The test signals were selected so that they differed both in shape and spectral content.

VII. RESEARCH OF THE EFFICIENCY OF MSE AND UIQ CRITERIA FOR THE IMPLEMENTATION OF THE FITNESS FUNCTION OF GA

Noise signals with different noise levels were cleaned using wavelet functions and parameters obtained through GA to compare the efficiency of the MSE and UIQ criteria.

The signal-to-noise ratio (SNR) is used as evaluating the efficiency of signals denoising. Fig. 6 shows a diagram of SNR values of the cleared signals with different values of the initial noise level in the signals [18].

Based on the analysis in Fig. 6, it may be concluded that the best results were obtained using the parameters defined by the UQI for most signals with initial noise levels of 5 dB, 10 dB, and 20 dB. For a "trsin" signal with an initial SNR of 10 dB and 20 dB, as well as parameters defined by the MSE criterion. For "vonkoch" and "wcantor" fractal signals with initial noise levels of 5 dB and 10 dB are almost identical.

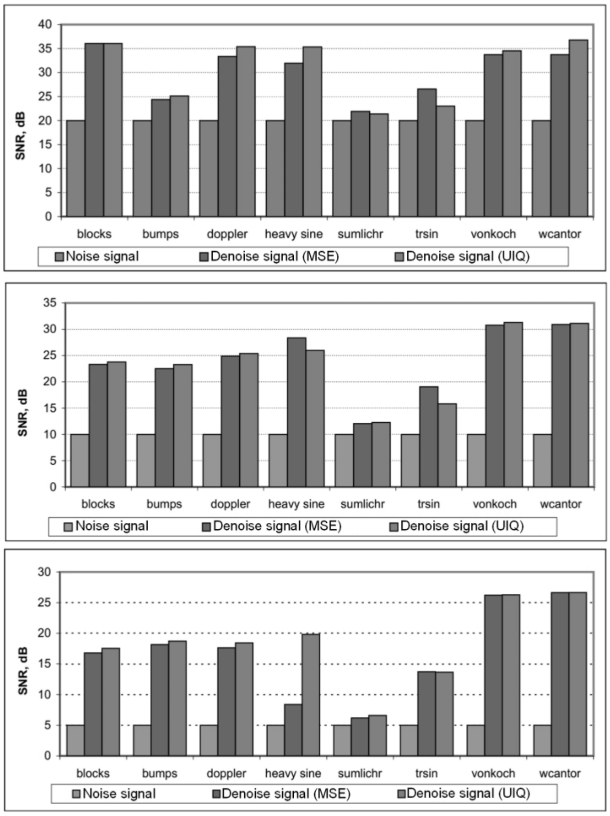

Fig. 6. Comparison of SNR values obtained by clearing noise from the initial SNR = 5dB, 10dB and 20dB when using the parameters defined by the MSE and UIQ criteria

VIII. CONCLUSIONS

New and developed known theoretical and practical methods for the choice of base wavelet functions for processing one-dimensional signals have been considered in the paper. Using these methods allows ensuring high accuracy of signals in the time-frequency domain, concentration of the signal energy in a small number of significant coefficients, and increases the conversion speed.

The obtained theoretical and experimental results are the basis for the improvement of existing and development of new methods for the choice of base wavelet functions for processing both one-dimensional and multidimensional signals in computer systems.

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