MATHEMATICAL MODELING AND COMPUTING, Vol. 6, No. 2, pp. 304–310 (2019)

Modeling Computing Mathematical

Economic and mathematical modeling of the distribution of financial resources for research and development

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(Received 2 June 2019; Accepted 1 October 2019)

The authors constructed an economic-mathematical model of optimal allocation of financial resources for research and development using the mechanism of dynamic programming. In the conditional example, the authors calculated the allocation of budget funds using the method of dynamic programming. In order to evaluate the prospects of applying the proposed model, the authors analyzed the current state of funding for research and development and considered the positive foreign experience. The expediency of using the proposed model in domestic practice is substantiated.

Keywords: *dynamic programming, scientific research, financial resources, budgetary resources, science development strategy.*

2000 MSC: 49N05

UDC: 336.027 **DOI:** 10.23939/mmc2019.02.304

1. Introduction

Cardinal changes in all spheres of public life of Ukraine induce functioning of many industries to critical rethinking of the current functioning models of many industries, including educational. Particular attention should be paid to the field of higher education, which is responsible for the formation of the intellectual capital of the country.

Conceptual bases of perspective development of the national higher education system are thoroughly revealed in many basic researches of Ukrainian scientists [1]. However, the implementation of any project is directly linked to the providing of the transformation process. It is the providing component that will focus our attention. The peculiarities of financing higher education institutions and the procedure of state regulation are disclosed in articles [1–3]. Considering the implementation of the model of financing of science at the present stage of socio-economic development of Ukraine, it is first of all advisable to determine with emphasis — what exactly should the state finance and on what basis should the budget allocation be based?

The conceptual decision on the allocation of budgetary funds was manifested in the following steps. In order to improve the current situation in the field of science, the National Council of Ukraine for the Development of Science and Technology (NCDST) was established in 2017, which adopted the Regulations on the formation of the National Research Foundation (NRF). The document provides for the transition to basic and competitive financing of science, namely: 60% will be basic and 40% will be allocated for competitive financing. Therefore, there is a need to distribute funds for scientific research. The study of the problem showed that the optimization of the distribution of funding can be achieved through the introduction of a dynamic programming mechanism [4–6].

2. Use of dynamic programming mechanism for optimal allocation of public funds

The prerequisites for applying the dynamic programming method to solving optimization problems are:

— the purpose function must be additive, that is, consist of functions, each of which depends only on the corresponding variable;
— the task must be interpreted as a multi-step decision-making process;
— the task must be defined for an arbitrary number of steps and have a structure that does not depend on their number.

Consider the use of dynamic programming mechanism for optimal allocation of public funds for the financing of university projects, which provides the maximum total expert evaluation (in points) of projects for which the funds are allocated. At the same time, we assume that the projects of each university are arranged in descending order (not increasing) of their expert evaluation, and no more than one loan can be allocated to each university, the size of which is k units of money, where $k = 0, 1, 2, \ldots, m$ and m is the total amount of public money. In this case, we will assume that the amount of public funds is less than the total need for university loans.

Let the state funds be distributed among n universities $Y_1, Y_2, \ldots, Y_n$. Suppose that the total expert evaluation of the projects of the i-th university for which $x_j = j$, $j = 0, 1, 2, \ldots, m$, monetary units are allocated, is $g_i(x_j)$. The task is to allocate budget resources, for which the overall expert evaluation of all projects, for which budget funds are allocated, would be the greatest.

If we denote by x_i, $i = 1, 2, \ldots, n$, the number of monetary units planned to be provided to the i-th university, then the mathematical model of the problem will have the form:

$$L = \sum_{i=1}^{n} g_i(x_i) \to \max$$

under conditions

$$\sum_{i=1}^{n} x_i = m, \quad x_i \in \{0, 1, 2, \ldots, m\}, \quad i = 1, 2, \ldots, n.$$

Denote by $P_i(x_j)$, $i = 1, 2, \ldots, n$, the total expert evaluation of the projects of the first i universities $Y_1, Y_2, \ldots, Y_i$, for which there are allocated $x_j = j$, $j = 0, 1, 2, \ldots, m$, units of funds and by $P_i^*(x_j)$, $i = 1, 2, \ldots, n$, the maximum total expert evaluation of projects for which $x_j = j$, $j = 0, 1, 2, \ldots, m$, units of funds are allocated.

The process of solving the task will be divided into n steps. At the first step, we determine the maximum total expert estimate from the allocation of $x_j = j$, $j = 0, 1, 2, \ldots, m$, monetary units of funds to the first university Y_1. In the second step, we determine the maximum total expert estimate from the allocation of $x_j = j$, $j = 0, 1, 2, \ldots, m$, monetary units of funds to the first two universities Y_1 and Y_2. At the s-th step, $s = 3, 4, \ldots, n-1$, we determine the maximum total expert estimate from the allocation $x_j = j$, $j = 0, 1, 2, \ldots, m$, monetary units of the first s Universities $Y_1, Y_2, \ldots, Y_s$. In the last step, we determine the maximum expert estimate from the allocation of m monetary units for all universities.

First step:

$$P_1(x_j) = P_1^*(x_j) = g_1(x_j), \quad j = 0, 1, 2, \ldots, m,$$

where $g_i(0) = 0$, $i = 1, 2, \ldots, n$.

The second step:

$$P_2(x_j) = \begin{cases} g_2(0) + P_1^*(x_j - 0), \\ g_2(1) + P_1^*(x_j - 1), \\ \ldots\ldots\ldots\ldots\ldots \\ g_2(x_j) + P_1^*(0), \end{cases}$$

$$P_2^*(x_j) = \max_{0 \leqslant k \leqslant j} \{g_2(k) + P_1^*(x_j - k)\}$$

for $j = 0, 1, 2, \ldots, m$.

s-th step ($s = 3, 4, \ldots, n-1$):

$$P_s(x_j) = \begin{cases} g_s(0) + P_{s-1}^*(x_j - 0), \\ g_s(1) + P_{s-1}^*(x_j - 1), \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ g_s(x_j) + P_{s-1}^*(0), \end{cases}$$

$$P_s^*(x_j) = \max_{0 \leqslant k \leqslant j} \left\{g_s(k) + P_{s-1}^*(x_j - k)\right\}$$

for $j = 0, 1, 2, \ldots, m$.

n-th step:

$$P_n(m) = \begin{cases} g_n(0) + P_{n-1}^*(m), \\ g_n(1) + P_{n-1}^*(m-1), \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ g_n(m) + P_{n-1}^*(0), \end{cases}$$

$$P_n^*(m) = \max_{0 \leqslant k \leqslant j} \left\{g_n(k) + P_{n-1}^*(m-k)\right\}.$$

We define the optimal plan for the allocation of public funds as follows. Let $P_n^*(m)$ reach its highest value at $k = l_1$. Then l_1 monetary units should be allocated to Y_n University. Next, it is necessary to distribute $m - l_1$ monetary units among universities $Y_1, Y_2, \ldots, Y_{n-1}$. If $P_{n-1}^*(m - l_1)$ reaches a maximum at $k = l_2$, then l_2 of the monetary units should be allocated for the university Y_{n-1}. Let $P_{n-2}^*(m - (l_1 + l_2))$ reach its highest value at $k = l_3$, then l_3 monetary units should be allocated to Y_{n-2} University. And so on. If $P_2^*(m - (l_1 + l_2 + \ldots + l_{n-2}))$ reaches a maximum at $k = l_{n-1}$, then l_{n-1} monetary units should be allocated to Y_2 University. Finally, $l_n = m - (l_1 + l_2 + \ldots + l_{n-1})$ monetary units should be allocated to Y_1 University.

The maximum total expert assessment of the distribution of budgetary funds to universities is $P_n^*(m)$ units.

3. Calculation of the distribution of budgetary funds using the dynamic programming method

Suppose that the state allocated 600 thousand UAH to finance three universities Y_1, Y_2, Y_3. Let the state allocate to the separate University k monetary units ($k = 0, 1, 2, \ldots, 6$), each of which is 100 thousand UAH. It is known that granting a loan in the amount $x_j = j$, $j = 0, 1, 2, \ldots, 6$, of monetary units for the i-th University of Y_i ($i = 1, 2, 3$) provides a total expert evaluation of the projects for which the funds are allocated, in the size of $g_i(x_j)$ units. It is necessary to distribute public funds in such a way that the total expert evaluation of all projects for which public funds are allocated is maximum. We will solve the task based on the following data (Table 1).

Table 1.

Total expert evaluation	x_j						
	0	1	2	3	4	5	6
$g_1(x_j)$	0	10	20	38	44	48	50
$g_2(x_j)$	0	9	17	23	28	32	35
$g_3(x_j)$	0	10	19	27	33	37	39

Note: to simplify the calculations, we have assigned an expert evaluation of projects from 0 to 10 points.

Solution. We divide the process of solving the task into three steps. In the first step, we will determine the total maximum expert estimate from the allocation of $x_j = j$, $j = 0, 1, 2, \ldots, 6$, monetary units for the first university Y_1. In the second step, we determine the total maximum expert estimate from the allocation of $x_j = j$, $j = 0, 1, 2, \ldots, 6$, monetary units for the first two universities Y_1 and Y_2. Finally, in the third step, we will determine the maximum total expert evaluation from allocation of 600 thousand UAH for three universities Y_1, Y_2, Y_3.

First step:

$$P_1(x_j) = P_1^*(x_j) = g_1(x_j), \quad j = 0, 1, 2, \ldots, 6.$$

Second step:

$$P_2(x_j) = \begin{cases} g_2(0) + P_1^*(x_j - 0), \\ g_2(1) + P_1^*(x_j - 1), \\ \ldots\ldots\ldots\ldots\ldots \\ g_2(x_j) + P_1^*(0), \end{cases}$$

$$P_2^*(x_j) = \max_{0 \leqslant k \leqslant j} \{g_2(k) + P_1^*(x_j - k)\}$$

for $j = 0, 1, 2, \ldots, 6$.

Third step:

$$P_3(6) = \begin{cases} g_3(0) + P_2^*(6), \\ g_3(1) + P_2^*(5), \\ \ldots\ldots\ldots\ldots \\ g_3(6) + P_2^*(0), \end{cases}$$

$$P_3^*(6) = \max_{0 \leqslant k \leqslant 6} \{g_3(k) + P_2^*(6 - k)\}.$$

Table 2.

x_j	k	$g_2(k)$	$P_1^*(x_j - k)$	$P_2(x_j)$
0	0	0	0	0*
1	1	9	0	9
	0	0	10	10*
2	2	17	0	17
	1	9	10	19
	0	0	20	20*
3	3	23	0	23
	2	17	10	27
	1	9	20	29
	0	0	38	38*
4	4	28	0	28
	3	23	10	33
	2	17	20	37
	1	9	38	47*
	0	0	44	44
5	5	32	0	32
	4	28	10	38
	3	23	20	43
	2	17	38	55*
	1	9	44	53
	0	0	48	48
6	6	35	0	35
	5	32	10	48
	4	28	20	48
	3	23	38	61*
	2	17	44	61*
	1	9	48	57
	0	0	50	50

Data of calculations $P_2(x_j)$, $j = 0, 1, 2, \ldots, 6$, we input in Table 2.

From Table 2 we see that $P_2^*(0) = 0$, $P_2^*(1) = 10$, $P_2^*(2) = 20$, $P_2^*(3) = 38$, $P_2^*(4) = 47$, $P_2^*(5) = 55$, $P_2^*(6) = 61$.

These calculations of $P_3^*(6)$ are written in Table 3. From Table 3 we see that $P_3^*(6) = 66$.

The optimal allocation plan for public funds is defined as follows. Since $P_3^*(6) = 66$ and is reached for $k = 2$, then the third university Y_3 should be allocated two units of money. Next, it is necessary to allocate four units of funds to the first two universities Y_1 and Y_2. From Table 2 we see that as $x_j = 4$ we have $P_2^*(4) = 47$ and is reached at $k = 1$. This means that the second university has to be allocated one unit of cash. So, three cash units remain to be allocated to the first university Y_1.

The maximum total expert evaluation of the projects for which funding is allocated is 66 units.

Table 3.

x_j	k	$g_3(k)$	$P_2^*(x_j - k)$	$P_3(x_j)$
6	6	39	0	39
	5	37	10	47
	4	33	20	53
	3	27	38	65
	2	19	47	66*
	1	10	55	65
	0	0	61	61

The proposed model can be used by budget managers spending units in the budgeting process. Providing an inverse model, you can use it, by the way, also for formulating a plan of allocation of general budget fund by the recipient of budgetary funds — to use for the monthly allocation of budget appropriations approved in the general budget of estimate, by the reduced economic classification of budget expenditures, which regulates during the budget period the budget commitments and payments in accordance with these commitments [7].

4. Prospects for the use of dynamic programming method for the allocation of budget funds for research and development in Ukraine

Today, we have a system of financial support for research and development at the expense of the state budget, proceeds from the implementation of research projects, as well as international grants. Competitive selection of research and development projects is carried out at the expense of the state budget in accordance with the Law of Ukraine "On Scientific and Technical Activities" [8] and the Procedure for Forming the Subjects of Scientific Research and Scientific and Technical (Experimental) Developments [9]. Financing of university research is carried out according to the results of competitive selection of projects on the basis of their scientific and expert evaluation.

Establishment of the National Research Fund in Ukraine aims at support fundamental and applied scientific research in the field of natural, technical, social and human sciences and developments in priority areas of science and technology development. The Fund provides financing under concluded contracts with the winners of competitions and control over their implementation; assesses the quality and effectiveness of the projects by it supports [10].

However, the question of combining the interests of science and business remains open. The simple financing of research and development (R&D) does not guarantee a proper return on investment. So, Russia is among the ten world leaders in terms of funding for research, but the contribution of its knowledge-intensive production to the world economy does not exceed 0.3%. This indicates a poor efficiency and misuse of the investments made. In this regard, it is appropriate to study positive foreign experience.

Leaders in research funding are countries such as the United States, China, Japan, Germany, South Korea, France, India, England, Brazil, Russia, which account for almost 80% of global investment [11]. International experience is rich in examples that confirm the effectiveness of relationship of science and business. An illustrative example is the formation of Singapore's scientific potential. Scientific activities are carried out in research parks, which are a union of state research institutes and industrial corporations. About 2.7% of GDP is allocated for scientific and educational purposes in the country. In general, public investment in research and development is under certain programs and in areas that bring good returns. The United States annually receives about $ 150 billion in revenue from the sale of intellectual property, which is 12% of the country's GDP. For comparison, Russia's annual sales of oil are $ 90 billion. Patent revenue in Finland is 20% of GDP.

In world practice, commercial structures play an important role in financing R&D. Israel is the leader by the business investment in research, with 85% of total business expenses. In China, the private sector's share of R&D expenditures is 77.3%, in the US — 70.6%, Russia — 59.6%, Singapore — 59%, India — 35.5%. At the same time, research funding in Brazil is exclusively for public funds [12].

In most countries of the world, the corporate sector accounts for the major part of the cost of new developments. The world's leaders in terms of investment in research and development are included Volkswagen, Samsung, a South Korean group of companies, maker of electronic devices and computer components, Intel, a multinational software company, Swiss biopharmaceutical company Roche, pharmaceutical corporation Novartis International AG, Google, Toyota, Johnson & Johnson Auto Corporation, Merc. Their annual R&D expenditures reach $ 8 – 14 billion USA [13].

The share of research funding in the GDP of Ukraine in 2016 amounted to about 15% of the share of GDP financing in 1990, with GDP contracting almost twice during the period. This situation

is connected with the change of country's socio-economic structure. Given the link between science and the state economy, funding for research and development was justified by their introduction into production. The change in the socio-economic structure that has led to the denationalization of the economy requires the introduction of a new financing model that takes into account the link between research and market economy.

Information on the amount of R&D expenditures and the contingent of scientists indicates a rapid decline in both volumes. Whereas in 1991 there were about 450,000 scientists in Ukraine, of whom 295,000 were researchers, then in 2015, their contingent in the national science decreased to 97.8 thousand. It was 23 times reduction of specialists employed at the enterprises of the research and production base [14]. The result of this situation was a complete breakdown of science and production. There are a number of reasons for this situation and trends, but the biggest manifestation was caused by the following:

— lack of a strategy for state support for science;
— low indicators of financing of scientific activity in relation to GDP;
— ineffective use of budget funds for research.

So, the use of dynamic programming method in the process of allocation of budget funds for research and development allows us to ensure the optimization of the use of budget funds, but the prerequisite of such allocation should be first of all the development of a strategy of state support for science and legislative regulation of the process of its implementation.

5. Conclusions

As a result of the review and comparative analysis of research and development financing in Ukraine and abroad, we conclude that the basis for the effective use of funds for certain purposes is the introduction of optimization economic and mathematical models, which combine the interests of the state, corporate sector, and social and economic functions of the state. Implementation of the proposed model of allocation of public funds using the mechanism of dynamic programming allows us to optimize the specified process.

At the same time, the lack of a model of management and financing of science in Ukraine raises the problem of ways to meet its needs. Therefore, in the process of developing a strategy of state support for science, an important place should be given to the model of allocation of public funds using the mechanism of dynamic programming.

Speaking about the sources of funding for research and development, it is necessary to state the multifactorial process, which goes beyond a single country, exemplified by transnational corporations. Yet the role of the state in regulating this process is indisputable. The fact that government funding is appropriate and necessary is a well-known truth. In the conditions of market relations, when it is necessary to answer the question: what to allocate funds, in what volumes and what is the order of evaluating the return of invested funds, resort to the tools of economic and mathematical modeling. A model of optimizing the allocation of financial resources to research and development helps to prevent unreasonable spending of budgetary resources and helps them to channel appropriately for specified purposes.

[1] Yastrubskyy M. Harmonious Development and State Regulation of the Activity of Higher Educational Institutions of Ukraine. Lviv (2018), (in Ukrainian).

[2] Kuzmin O., Yastrubskyy M. Peculiarities of higher educational establishments funding accounting: national and foreign experience. Economics, Entrepreneurship, Management. **3** (2), 1–6 (2016).

[3] Kuzmin O. Ye., Tsisar N. Z., Yastrubskyy M. Ya. University policy as an object of harmonious development and state regulation. Economics, Entrepreneurship, Management. **4** (1), 91–97 (2017).

[4] Bellman R. Dynamic Programming. Moscow, Publishing House of Foreign Literature (1960), (in Russian).

[5] Bellman R. Dreyfus S. Applied problems of dynamic programming. Moscow, Science (1965), (in Russian).

[6] Tsehelyk G. G. Mathematical Programming: Textbook. Lviv, LNU named after Ivan Franko (2011), (in Ukrainian).

[7] The procedure for drafting, review, approval and basic requirements for the implementation of budgets budget estimates. Resolution of the Cabinet of Ministers of Ukraine No. 228 (2002). https://zakon.rada.gov.ua/laws/show/228-2002-п.

[8] On scientific and scientific and technical activities. Law of Ukraine, Bulletin of the Verkhovna Rada of Ukraine. No. 3, p. 25, (2016), (in Ukrainian).

[9] On Approval of the Procedure for the Formation of the Subject of Scientific Research and Scientific and Technological (Experimental) Development, Financed by the State Budget, and the Recognition as Decommitted, of Some Resolutions of the Cabinet of Ministers of Ukraine: CMU Resolution No. 13 (2018), (in Ukrainian).

[10] Regulation on the National Fund for Researches of Ukraine. Resolution of the Cabinet of Ministers of Ukraine No. 528 (2018). https://zakon.rada.gov.ua/laws/show/ru/528-2018-п.

[11] R&D Spending by Country (in PPP$). Howmuch.net understanding money. https://howmuch.net/articles/reseach-development-spending-by-country.

[12] How much do government and business spend on R&D. http://ect-center.com/blog/rashodi-niokr.

[13] The 10 biggest R&D spenders worldwide. http://fortune.com/2014/11/17/top-10-research-development/.

[14] Ukrstat.org — publication of documents of the State Service of Statistics of Ukraine. http://ukrstat.org/uk/operativ/operativ2017/ni/kpno_kp/kpno_kp_u.htm.

Економіко-математичне моделювання розподілу фінансових ресурсів на наукові дослідження і розробки

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Побудовано економіко-математичну модель оптимального розподілу фінансових ресурсів на наукові дослідження і розробки з використанням механізму динамічного програмування. На умовному прикладі здійснено розрахунок розподілу бюджетних коштів з використанням методу динамічного програмування. З метою оцінки перспектив застосування запропонованої моделі проаналізовано стан фінансування наукових досліджень і розробок та розглянуто позитивний зарубіжний досвід. Обґрунтовано доцільність використання запропонованої моделі у вітчизняній практиці.

Ключові слова: *динамічне програмування, наукові дослідження, фінансові ресурси, бюджетні кошти, стратегія розвитку науки.*

2000 MSC: 49N05

УДК: 336.027