Economic and mathematical modeling of management processes and financing the training of specialists by higher educational institutions

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In the article, an economic and mathematical model is constructed for the optimal distribution of the state order for the training of specialists, based on the ranking of specialties of universities. By the hypothetical example, the distribution of the government procurement for the training of specialists has been figured on using the proposed mathematical model. According to the results of the analysis, the expediency of using the proposed model in domestic practice is substantiated.

Keywords: *training management of specialists, government procurement, higher education institution, optimization model, specialty rating.*

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1. Introduction

Nowadays challenges at the global and regional levels encourage the search for optimal ways to organize the higher education system. Forms of education are changing, new areas of training are emerging, the national economy and the labor market need specialists in new specialties. All this takes place against a background of limited resources, which, in turn, requires the rational use of available resources, as well as those expected in the future.

Today, there are highly required both the optimal distribution of funds between higher education institutions that train specialists and the optimization of the training of specialists in terms of specialties demanded for the current period and in the future.

We have proposed a model for the distribution of financial resources between public universities for research and development [1]. The algorithm of distribution of the government procurement for training of experts used by the Ministry of Education and Science is focused on the choice of entrants and popularity of higher education institution among potential consumers of educational services. However, it does not take into account the real rating of specialties, which is based on the calculated indicators.

The Deputy Minister of Education and Science of Ukraine Yegor Stadnyi has been stressed the need to take this criterion into account in the process of allocating the government procurement for the training of specialists. In his opinion, he declared the intention of the Ministry of Education and Science to take this indicator into account in the formula for calculating the distribution of government procurements, starting in 2021. The government plans to monitor the employment of graduates through the interaction of electronic databases to do. It will allow calculating the rating of the specialty of a particular university [2].

Today, the government procurement for the training of bachelor's degree specialists on the basis of complete general secondary education in all specialties and forms of higher education is formed depending on the number of applications submitted by entrants with the highest ratings and priorities to a higher education institution [3]. However, it does not provide the desired result for the national economy.

There is a need to construct a model of optimal distribution of the government procurement for the training of specialists, which is based on the rating of the university for the training of specialists in a particular specialty.

2. Use of the mechanism of mathematical modeling for optimal distribution of the government procurement for training of specialists

To construct a model of optimal distribution of the government procurement for the training of specialists, based on the ratings of university specialties, we introduce the following notation.

Let n be a number of higher education institutions capable of training specialists; m be a number of specialties; r_{ij} be a rating of the i-th higher education institutions for the training of a specialist in the j-th specialty (determined by the Ministry of Education and Science); b_j be the need for specialists of the j-th specialty; c_{ij} be the cost of specialist training in the j-th specialty in the i-th higher education institution; v_i be funds for training of specialists allocated for the i-th higher education institution ; x_{ij} be a number of specialists of the j-th specialty, which are planned for training in the i-th higher education institution (required values).

The task is to plan training of such a number of specialists in higher education institutions to provide achievements of the maximum rating of training specialists.

The mathematical model of the problem is as follows:

$$L = \sum_{i=1}^{n} \sum_{j=1}^{m} r_{ij} x_{ij} \to \max \tag{1}$$

under the conditions

$$\sum_{i=1}^{n} x_{ij} = b_j, \quad j = 1, 2, \ldots, m; \tag{2}$$

$$\sum_{j=1}^{m} c_{ij} x_{ij} \leqslant v_i, \quad i = 1, 2, \ldots, n; \tag{3}$$

$$x_{ij} \geqslant 0, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m. \tag{4}$$

Assuming that $c_{ij} = \alpha_i \beta_j$, where c_i depends on the higher education institution and β_j depends on the specialty, then the task can be reduced to an open model of the transport task. In fact, from (3) we obtain

$$\sum_{j=1}^{m} \beta_j x_{ij} \leqslant \frac{v_i}{\alpha_i}. \tag{5}$$

If we denote $\beta_j x_{ij} = y_{ij}$, $\frac{v_i}{\alpha_i} = R_i$, then inequality (5) is rewritten as

$$\sum_{j=1}^{m} y_{ij} \leqslant R_i.$$

The target function will have the form

$$L = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{r_{ij}}{\beta_j} y_{ij} \to \max.$$

Since

$$\sum_{i=1}^{n} y_{ij} = \sum_{i=1}^{n} \beta_j x_{ij} = \beta_j \sum_{i=1}^{n} x_{ij} = \beta_j b_j = d_j,$$

then the mathematical model of the problem is as follows

$$L = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{r_{ij}}{\beta_j} y_{ij} \to \max$$

in the conditions

$$\sum_{i=1}^{n} y_{ij} = d_j, \quad j = 1, 2, \quad , m;$$

$$\sum_{j=1}^{m} y_{ij} \leqslant R_i, \quad i = 1, 2, \ldots, n;$$

$$y_{ij} \geqslant 0, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m.$$

We will show how the problem can be solved without reducing it to an open model of the transport task.

For every $i = 1, 2, \ldots, n$ denote

$$\max_j c_{ij} b_j = c_i.$$

Here, we can present an algorithm for solving the task if the condition is satisfied

$$v_i \geqslant c_i, \quad i = 1, 2, \ldots, n.$$

The algorithm consists of n steps. In the first step, we look for the largest value of r_{ij}.

Let

$$\max_{ij} r_{ij} = r_{i_1 j_1}.$$

Then we put $x_{i_1 j_1} = b_{j_1}$.

In the second step we are looking for $\max\limits_{i, j \neq j_1} r_{ij}$.

Let

$$\max_{i, j \neq j_1} r_{ij} = r_{i_2 j_2}.$$

Then we put $x_{i_2 j_2} = b_{j_2}$.

In the third step we are looking for

$$\max_{i, j \neq j_1, j_2} r_{ij}.$$

If this maximum is reached for $i_3 j_3$, then we put $x_{i_3 j_3} = b_{j_3}$. And etc. In the n steps the plan will be found.

Consider an example. Let

$$b_1 = 50, \quad b_2 = 25, \quad b_3 = 25, \quad b_4 = 50;$$

$$V_1 = 260, \quad V_2 = 150, \quad V_3 = 210, \quad V_4 = 300;$$

$$\{r_{ij}\} = \begin{pmatrix} 4 & 2 & 3 & 1 \\ 3 & 6 & 4 & 4 \\ 3 & 4 & 5 & 4 \\ 2 & 3 & 5 & 4 \end{pmatrix}, \quad \{c_{ij}\} = \begin{pmatrix} 5 & 2 & 3 & 2 \\ 4 & 6 & 5 & 5 \\ 4 & 5 & 6 & 5 \\ 3 & 3 & 5 & 5 \end{pmatrix}.$$

First step

$$\max_j r_{1j} = r_{11} = 4, \quad x_{11} = b_1; \quad c_{11} x_{11} = 250 < 260 = V_1.$$

The second step

$$\max_{j \neq 1} r_{2j} = r_{22} = 6, \quad x_{22} = b_2; \quad c_{22} x_{22} = 150 = V_2.$$

The third step

$$\max_{j \neq 1,2} r_{3j} = r_{33} = 5, \quad x_{33} = b_3; \quad c_{33} x_{33} = 150 < 210 = V_3.$$

The fourth step

$$\max_{j\neq 1,2,3} r_{4j} = r_{44} = 4, \quad x_{44} = b_4; \quad c_{44}x_{44} = 250 < 300 = V_4.$$

The algorithm for model implementation in the case $\sum_{i=1}^{n} c_i \leqslant \sum_{i=1}^{n} V_i$.

The algorithm for implementing the model consists of a number of steps. In the first step, we look for the largest value of r_{ij}. Let

$$\max_{i,j} r_{ij} = r_{i_1 j_1}.$$

Then such cases are possible:

— if $c_{i_1 j_1} b_{j_1} = V_{i_1}$, then $x_{i_1 j_1} = b_{j_1}$ and we put $b_{j_1} = 0$, $V_{i_1} = 0$;
— if $c_{i_1 j_1} b_{j_1} > V_{i_1}$, then $x_{i_1 j_1} = b_{j_1} - \frac{V_{i_1}}{c_{i_1 j_1}}$, we put $V_{i_1} = 0$ and replace b_{j_1} with $b_{j_1} - \frac{V_{i_1}}{c_{i_1 j_1}}$;
— if $c_{i_1 j_1} b_{j_1} < V_{i_1}$, then $x_{i_1 j_1} = b_{j_1}$, we put $b_{j_1} = 0$ and replace V_{i_1} with $V_{i_1} - c_{i_1 j_1} b_{j_1}$.

In the second step, in the matrix $\{r_{ij}\}$ we cross out either the j_1 column, when $b_{j_1} = 0$, or the i_1 line, when $V_{i_1} = 0$, or j_1 column and i_1 line, when $b_{j_1} = 0$ and $V_{i_1} = 0$. After that, among the remaining values of r_{ij}, we look for the largest. Let

$$\max_{\substack{i\neq i_1 \\ j\neq j_1}} r_{ij} = r_{i_2 j_2}.$$

Then such cases are possible:

— if $c_{i_2 j_2} b_{j_2} = V_{i_2}$, then $x_{i_2 j_2} = b_{j_2}$ and we put $b_{j_2} = 0$, $V_{i_2} = 0$;
— if $c_{i_2 j_2} b_{j_2} > V_{i_2}$, then $x_{i_2 j_2} = b_{j_2} - \frac{V_{i_2}}{c_{i_2 j_2}}$, we put $V_{i_2} = 0$ and replace b_{j_2} with $b_{j_2} - \frac{V_{i_2}}{c_{i_2 j_2}}$;
— if $c_{i_2 j_2} b_{j_2} < V_{i_2}$, then $x_{i_2 j_2} = b_{j_2}$, we put $b_{j_2} = 0$ and replace V_{i_2} with $V_{i_2} - c_{i_2 j_2} b_{j_2}$.

We carry out similar steps until after some step all values of b_j $(j = 1, 2, \ldots, m)$ become equal to zero. Obviously, the number of steps is not more than $m + n$.

3. Calculation of the distribution of the government procurement for the training of specialists using the optimization model

Using the proposed model, give an example of calculating the government procurement, based on the ranking of specialties of universities.

Let

$$b_1 = 50, \quad b_2 = 25, \quad b_3 = 25, \quad b_4 = 50;$$

$$V_1 = 200, \quad V_2 = 150, \quad V_3 = 200, \quad V_4 = 300;$$

$$\{r_{ij}\} = \begin{pmatrix} 4 & 2 & 3 & 1 \\ 3 & 6 & 4 & 4 \\ 3 & 4 & 5 & 4 \\ 2 & 3 & 5 & 4 \end{pmatrix}, \quad \{c_{ij}\} = \begin{pmatrix} 5 & 2 & 3 & 2 \\ 4 & 6 & 5 & 5 \\ 4 & 5 & 6 & 5 \\ 3 & 3 & 5 & 5 \end{pmatrix}.$$

First step

$$\max_{i,j} r_{ij} = r_{22} = 6.$$

Because $c_{22}b_2 = 150 = V_2$, then $x_{22} = b_2 = 25$. We put $b_2 = 0$, $V_2 = 0$.

The second step

$$\max_{\substack{i\neq 2 \\ j\neq 2}} r_{ij} = r_{33} = 5.$$

Since $c_{33}b_3 = 150 < V_3 = 200$, then $x_{33} = b_3 = 25$, we put $b_3 = 0$ and replace V_3 with $V_3 - c_{33}b_3 = 50$.

The third step

$$\max_{\substack{i\neq 2,3\\ j\neq 2,3}} r_{ij} = r_{11} = 4.$$

Since $c_{11}b_1 = 250 > V_1 = 200$, then $x_{11} = \frac{200}{5} = 40$, we put $V_1 = 0$ and replace b_1 with $b_1 - 40 = 10$.

The fourth step

$$\max_{\substack{i\neq 1,2\\ j\neq 2,3}} r_{ij} = r_{34} = 4.$$

Since $c_{34}b_4 = 250 > V_3 = 50$, then $x_{34} = \frac{50}{5} = 10$, $V_3 = 0$ and replace b_4 with $50 - 10 = 40$.

The fifth step

$$\max_{\substack{i\neq 1,2,3\\ j\neq 2,3}} r_{ij} = r_{44} = 5.$$

Since $c_{44}b_4 = 200 < V_4$, then $x_{44} = 40$, we put $b_4 = 0$ and replace V_4 with $V_4 - c_{44}b_4 = 100$.

The sixth step

$$\max_{\substack{i=4\\ j=1}} r_{ij} = r_{41} = 3.$$

Since $c_{41}b_1 = 30 < V_4$, then $x_{41} = 10$.

The final plan

$$X = \begin{pmatrix} 40 & 0 & 0 & 0 \\ 0 & 25 & 0 & 0 \\ 0 & 0 & 25 & 10 \\ 10 & 0 & 0 & 40 \end{pmatrix}$$

$$L(X) = 655 \text{ units.}$$

As a result of the calculations, we obtained a plan of training of specialists, in which the overall rating of training is the highest.

4. Prospects of application of the optimization model for the distribution of the governmental procurement for specialists training in Ukraine

Introduction of the optimization model of formation and distribution of the governmental procurement for specialists training on the basis of the rating of the specialties being provided by the university is of relevant importance nowadays.

It is justified when the state makes targeted investments in those universities, which provide training in the required specialties at a high level and in predefined quantities, that meets the requirements of the labor market.

Today, we have a misbalance between the number of specialties and the real needs of the market. Applicants enter Institutions of higher education without questioning themselves why and where to work afterwards. The current mechanism of allocation of state-funded places in accordance with the principle “money per a student” is focused on the market of educational services, rather than on the real needs of the economy and employers. Although this approach has a number of advantages, it is not without drawbacks. As a result, we have the following situation, for example, when 13 specialists-candidates apply for one vacancy of a lawyer. At the same time, about a third of students admit during their studies that they would like to study another specialty. Such belated understanding of the incorrect choice of specialty and even the university, comprehension of the necessity to make more elaborate chose of a profession in the past, usually result in budget funds wasting on untapped opportunities. In fact, it is directly or indirectly related to the existing system of government procurement for specialists training.

The current situation testifies a shift in the direction of choice from universities choosing students to the students choosing institutions of higher education, which is caused by the demographic situation as well as a market situation in higher education. This imposes additional restrictions on institutions of higher education and increases the regulatory function of the state. The suggested model is adaptive to the model of the development and state regulation of institutions of higher education activity [4].

Rapid growth of internationalization of higher education has led to the development of the global knowledge economy. The international market of educational services has turned into a rapidly developing sector of the economy [5]. A precondition for raising the ranking of the university at the international level is to ensure high-quality educational services. It is obvious, that the state is interested in financing those institutions of higher education that are attractive both on the domestic market and for foreign students.

Nowadays, universities face the task of training specialists for future professions, or in other words — those professions that will be relevant and in demand in the near future. This is motivated by the aging process of knowledge, which today is quite rapid and is about from 2 to 5 years, depending on the field of science [6]. Determining the need for specialists of future professions is the prerogative power of the state, which can implement procurements for their training through the Ministry of Education and Science, based on the results of research of trends of the national economy development.

Choosing a strategy of state regulation of universities activities as to their financial and investment support, it is necessary to determine the priorities, which should be based on strategic significance.

The proposed model takes into account such an important factor as the competitive environment of higher education, which encourages the struggle for applicants who focus on the demand of the gained profession in the future.

5. Conclusions

According to the results of the conducted study of management and financial support of the specialists training by institutions of higher education, we observe imperfection of the existing system of public procurement forming. The consequences of this imperfection are imbalance between the number of graduators of various specialties and their demand by employers. The result of this imbalance is the inefficient use of budget funds for the training of a part of the specialists who are not in demand by the national economy.

The offered model is focused on the inclusion of an indicator of the specialty rating of a certain university into the existing system of government procurement of specialists training ; its introduction does ensure the maximum value of the overall rating of specialists training by all universities which have governmental procurement.

[1] Kuzmin O., Tsehelyk G., Yastrubskyy M. Economic and mathematical modeling of the distribution of financial resources for research and development. Mathematical modeling and computing. **6** (2), 304–310 (2019).

[2] Stadny Y. Higher education should be affordable. But it should be received by a person who has realized that it is her need, not just a tribute to the trend, because everyone does.
`https://www.unian.ua/society/10904138-yegor-stadniy-vishcha-osvita-povinna-buti-dostupnoyu-ale-otrimuvati-jiji-slid-lyudini-yaka-usvidomila-shcho-ce-jiji-potreba-a-ne-prosto-danina-tendenciji-tomu-shcho-vsi-tak-roblyat.html` (2020), (in Ukrainian).

[3] Terms of admission to higher education institutions of Ukraine in 2020. Order of the Ministry of Education and Science of Ukraine dated 11.10.2019 No.1285.
`http://osvita.ua/legislation/Vishya\_osvita/9990/` (2020), (in Ukrainian).

[4] Kuzmin O., Yastrubskyy M., Tsegelyk G. Model of development and state regulation of activity of higher educational institution (HEI): using experience in management of leading hei of East Asia. EEM. **6** (2), 1–11 (2019).

[5] Yastrubskyy M. Activities of higher educational institutions of Ukraine: harmonious development, state regulation and investment and innovation support. Lviv, Lviv Polytechnic Publishing House (2018), (in Ukrainian).

[6] Yastrubskyy M. Harmonious development and state regulation of higher educational institutions of Ukraine. Dissertation for the degree of Doctor of Economics. Lviv, Lviv Polytechnic National University (2018).

Економіко-математичне моделювання процесів управління та фінансування підготовки фахівців закладами вищої освіти

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Побудовано економіко-математичну модель оптимального розподілу державного замовлення на підготовку фахівців, виходячи з рейтингу спеціальностей університетів. На умовному прикладі проведено розрахунок розподілу державного замовлення на підготовку фахівців з використанням запропонованої математичної моделі. За результатами проведеного аналізу обґрунтовано доцільність використання у вітчизняній практиці запропонованої моделі.

Ключові слова: *менеджмент підготовки фахівців, державне замовлення, заклад вищої освіти, оптимізаційна модель, рейтинг спеціальності.*