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## THE PROCESSING OF GNSS OBSERVATION BY NON-CLASSICAL ERROR THEORY OF MEASUREMENTS

**The main goal of our research** is to show the need to use modern methods of processing GNSS observations time series by non-classical error theory of measurements (NETM), which is characterized by large sample sizes  $n > 500$ . The errors of high-precision observations, for the most part, cannot be represented by the classical law of Gaussian distribution. With the increase in sample size, the empirical error distribution will increasingly deviate from the classical Gaussian error theory of measurements (CETM). **Methods.** For this research we pre-processed GNSS observation at five permanent stations in Ukraine (SULP, GLSV, POLV, MIKL and CRAO). After applying the “clean” procedures based on the iGPS software package, we obtained the GNSS observation time series for 2018-2020. The verification of empirical error distributions was ensured by the procedure of non-classical error theory of measurements, based on the recommendations offered by G. Jeffreys and on the principles of hypothesis testing according to Pearson criteria. **Results.** It has been established that the coordinate time series of permanent stations obtained from precision GNSS observations do not confirm the hypothesis of their conformity to normal Gaussian distribution law. NETM diagnostics of the accuracy of high-precision GNSS measurements, which is based on the use of confidence intervals for estimates of asymmetry and kurtosis of a large sample, followed by the Pearson test, confirms the presence of weak, non-GNSS-treated sources of systematic errors. **Scientific novelty.** The authors use the possibility of NETM to improve the method of processing high-precision GNSS measurements and necessity to take into account sources of systematic errors. The failure to account for individual factors creates the effect of shifting the coordinate time series, which, in turn, leads to subjective estimates of station movement velocities, their geodynamic interpretation. **Practical significance** is based on the application of NETM diagnostics of probabilistic form of permanent stations topocentric coordinates distribution and improvement of the method of their determination. Research of the causes of the error distribution deviations from the established norms ensures the metrological literacy of large amount high-precision GNSS measurements.

**Key words:** Gaussian distribution law, Pearson-Jeffreys, non-classical error theory of measurements (NETM), Global Navigation Satellite System (GNSS), topocentric coordinates, GNSS observations, permanent stations.

### Introduction

The modern velocities of Earth's tectonic plates are an important subject for many studies, including geology, geophysics, and geodesy. Accurate information, which is regarding the surface movements of the Earth's surface is necessary for the analysis of earthquakes, the detection of local deformations, tectonic activity, displacements, as well as for the establishment of reference coordinate systems. GNSS technologies have been developing since the late 1970s. By improving the accuracy and development of receivers and antennas, GNSS is regularly used for monitoring high-accuracy tectonic movement. For determination the precision three-dimensional velocities of a GNSS observation station, we need minimum 2.5-year time interval of the coordinate time series [Savchuk, Dockich, 2017].

Global Navigation Satellite System (GNSS) – is a modern term used to describe various satellite navigation systems, such as GPS, GLONASS, Beidou and Galileo [Ray, J. et al., 2013]. At the end of XX century the Global Positioning System (GPS) with unprecedented accuracy has made a significant

contribution to navigation, positioning and scientific issues related to precise positioning on the Earth's surface. With the use of GPS and, in part, GLONASS was successfully explored a number of Earth science issues, including the establishment of a high-precision International Terrestrial Reference System (ITRS), Earth rotation, geocenter movement, time change in gravitational field, orbit determination, and remote, hydrology and oceans sensing. With the development of the next generation of multi-frequency and multi-system GNSS constellations, including upgraded GPS-IIF and GPS-III USA, updated Russian GLONASS, European Galileo System and Chinese Beidou System, additional areas and capabilities are implemented in exploring Earth by using GNSS [Hofmann-Wellenhof, et al., 2007].

Traditionally, the coordinates of GNSS stations were determined by two methods. First, the DD (double-differencing) method was used – classic difference networking, when GNSS observations are processed by base vectors, which connecting two stations and anchored to selected “reference” stations using high-quality observations. Second, the PPP

(Precision Point Positioning) method was used, when the coordinates of one station are determined directly from all available GNSS observations, which is carried out at that station only with the help of accurate orbits and accurate satellite clock corrections [Leandro, et al., 2011]. The Bernese GNSS Software is most often used to implement the DD method, and the GipsyX software is using for PPP method[<https://gipsy-oasis.jpl.nasa.gov>]. All calculations are performed in the latest implementation of the ITRF system.

In the general case, GNSS measurements obtained by estimating the propagation delay time of the carrier phase propagation signal resulting from measurements of the current navigational parameters, code and phase pseudogames, must be used for the precise determination of coordinates.

The major GNSS measurement errors are related to:

- the difference in time scales between the signal of user receiver and the specific GNSS;
- the difference in time scales between a particular navigation satellite and its navigation system;
- the delay of the radio signal propagation in the ionosphere of each individual satellite to the user receiver in the operating frequency range, such as L1 and L2;
- the delay of radio signal propagation in the Earth's troposphere;
- integer ambiguity of pseudo phase measurements [Karaim et al., 2018];

The discrepancy of time scales between the signal of user receiver and the specific GNSS is estimated from the GNSS observations as an unknown parameter. The divergence of time scales of satellite is fully offset by the DD method or determined by special programs of international centers by using PPP method [H  roux, Kouba, 1995].

In order to eliminate the ionospheric delay of signal propagation, a well-known linear combination of measurements is used in practice. This is usable only if these measurements are made at two or more frequencies. The tropospheric signal delay in GNSS measurements is eliminated by using the appropriate tropospheric model. Typically, in such models, the vertical tropospheric delay of the station signal is divided into dry and wet components. The value of the dry component is determined by some conventional model, and the uncompensated by model residual wet component of tropospheric delay is considered as an additional unknown parameter, which is determined from the processing of GNSS observations.

The integer ambiguity resolution procedure involves the use of pseudophase increments by using DD [Li et al., 2017]. In the case of good geometric factor PDOP, which can be achieved at long intervals (static mode), the problem of ambiguity of phase measurements is solved by their processing. Resolving ambiguity is one of the problems of high-precision absolute positioning (PPP). The presence in

the phase measurements of a number of non-simulated offsets, such as: hardware delays in the GNSS satellite and in the user receiver, the initial phases of the oscillation carrier radiation and the reference oscillation at the carrier frequency of the receiver, may be the reason that an integer ambiguity cannot be described by an integer and evaluated as the valid magnitude of the appropriate model of the non-ionospheric pseudophase combination. The fact that the integer nature of the ambiguity is not taken into account (the use of a pseudophase measurement model), does not limit the accuracy of the coordinates evaluated as a result of processing, but had an effects on the length of the convergence period. This period is only needed to obtain the specified accuracy of coordinates. As a rule, adaptive recursive type filters based on maximum posterior probability estimation are used to estimate the coordinates of the receiver. The Kalman filter and its modifications were of biggest use, among such filters. In this regard, a priority area for the development of PPP technologies was the development of integer-resolution procedures for the ambiguity of pseudophase measurements, which were called Integer PPP, for example, in the GipsyX software (see Table 1).

Thus, using the model of the non-ionospheric pseudophase combination in the PPP method, we assumed that GNSS measurements compensate for systematic displacements related with relativistic and gravitational effect, phase center shifts, tidal effects, windup effects, and atmospheric delays [Bogusz, J., Klos, A., 2016].

Table 1

**Methods of correction for various observation errors using PPP method for processing**

Errors sources	Method of Correction
Orbits and clocks	CODE precise clock and orbit product in RINEX format
Elevation cut-off	7°
Antenna phase center corrections	ANTEX14
Ionosphere model	Ionosphere-free combination and second order corrections
Troposphere model	Saastamoinen/GPT2/VM F1
Earth orientation modelling	IERS2010
Earth orientation parameters	EOP C04
Ocean loading effects	FES2012

The highest quality GNSS observations are made at permanent stations according to a number of

requirements for their operation. Such stations are integrated into the appropriate networks. IGS and EPN (European Permanent Network) networks belong to global / continental networks of permanent GNSS stations.

The reliable accuracy of determining the absolute coordinates of geodetic points, which is reached today from GNSS observations, is about  $\leq 1$  cm, and the velocities of coordinates change is coordinates 1-2 mm/year.

The results of regular GNSS observations in the form of time series are used in various applications, including the geodynamic interpretations. In the coordinate time series, we can detect linear or nonlinear trends, annual and semiannual signals, offsets, and measurement noise [Bos, M., et al., 2013]. Most of the analysis focuses on identifying annual signals, research time series breaks, and finally estimation of reliable changes in station coordinate velocities, for example, to determine tectonic movements. Depending on the nature of the signal and other factors that have an effect on time series, we need specific methods to distinguish between signals based on tectonic movement and other non-tectonic signals, such as seasonal variations. These methods can be used for visual interpretation and for time series pre-processing, as well as for statistical analysis of their accuracy and necessity to take into account a number of systematic errors sources [Jiang et al., 2017].

Visual interpretation and pre-processing of the obtained time series coordinates include the detection and removal of displacements and jumps, noise characteristics, trend and seasonal variations, and the analysis of residual errors. The most popular analysis tools for such purposes are GGM Matlab (TSView) [Herring, 2003], FODITS [Ostini et al., 2008], CATS, Hector, iGPS, etc. TSView is written in Matlab and complements the GAMIT/GLOBK package, FODITS is embedded into the Bernese GNSS software, whereas CATS [Williams, 2008], Hector and iGPS [Tian, 2011] are C/C++ and IDL (Interactive Language Data) written independent command line routines. The development of the iGPS package began with an attempt to overwrite the GGM Matlab software package on IDL, but resulted in a completely new graphical interface and many additional features for time series analysis. For the study described in this article, we used this software package.

It is possible to use a wide range of mathematical approaches for the need to take into account a number of sources of systematic errors for statistical analysis. One of them is non-classical error theory of measurements (NETM). This theory is defined as a modern theory of mathematical processing of time series data with a sufficiently large sample size (more than 500). In multiple GNSS measurements, the fundamental principles of the classical error theory of measurements (CETM). Therefore, the non-classical error theory of measurement is a modern mathematical instrument for the study of large arrays

of measurement information. NETM methods were used in astrometry, space research, geodetic tasks and geophysical experiments. Over the past 25 years, NETM ideas, approaches and methods have been tested in various fields of research: astronomical, cosmic gravimetric, geophysical, geodetic and other [Dvulit, Dzhun, 2017].

The NETM methods, mainly developed by F. Gauss, are based on two fundamental principles: a) observation errors submit to the normal law, and b) an absence the sources of systematic errors in measurement. However, from the second half of the XX century, there was an era of large samples, in which errors of observation could not be shorten within the bounds of normal law. The outstanding English scientist G. Jeffries has expressed three fundamentally important NETM concepts [Dzhun, 2015]:

1. Any hypothesis or theory that has a low probability must be replaced by a hypothesis or theory that must have a significantly higher probability because it is impossible to ensure the high practical certainty of our knowledge.
2. The normal error law for  $n > 500$  observations reveals its complete theoretical and practical failure.
3. Errors in the number of observations  $n > 500$  can be satisfactorily represented by a Pearson type distribution with a Fisher diagonal matrix.

### *Aim*

The main goal of our research was to show the need to use modern methods of processing GNSS observations time series by non-classical error theory of measurements (NETM), which is characterized by large sample sizes  $n > 500$ .

### *Methods*

For research of the high-precision GNSS measurements accuracy, we suggest to use the results of observations at stations of global and regional GNSS networks. The main feature of station selection was the presence of continuous long-term series of observations.

There are 19 permanent GNSS stations included in IGS / EPN networks, which are operated (was operated) on the territory of Ukraine. The continuance of observation at stations varies from 0.6 (IZRS station (Izmail, Odesa region) to 22.5 years. (GLSV (Holosiievo). As of early 2020, 7 EPN stations in Ukraine had a Category A (European Accuracy Classification) and could be used in the highest accuracy studies. These are the stations as 12371S001 ALCI (Alchevsk), 15501M001 CNIV (Chernihiv), 12344M001 EVPA (Yevpatoriia), 12335M001 MIKL (Mykolaiv), 12336M001 POLV (Poltava), 12366M001 SULP (Lviv), 12301M001 UZHL (Uzhhorod). It should be noted that the ALCI, EVPA and UZHL stations for now, for various reasons, do not work. The class B stations are 12: 12356M001 GLSV (Holosiievo), 12314M001 KHAR (Kharkiv),

12337M001 KTVL (Katsyveli), 15503M001 SMLA (Smila), 15502M001 PRYL (Pryluky), 15556M001 MARP (Mariupol), 15595M001 GDRS (Horodok, Kharkiv region), 15597M001 KRRS (Kropyvnytskyi), 15599M001 MKRS (Mukacheve), 18101M001 VNRS (Vinnytsia), 18102M001 ZPRS (Zaporizhzhia), 18115M002 IZRS (Izmail, Odesa region). Most of the class B stations still have relatively short observation times and are not included in the European analysis. The exceptions here are the GLSV, KHAR, KTVL and SMLA stations, which have a long time observations (from 15 to 23 years), but according to the centers of European analysis, the results of observations processing cannot be included in category A.

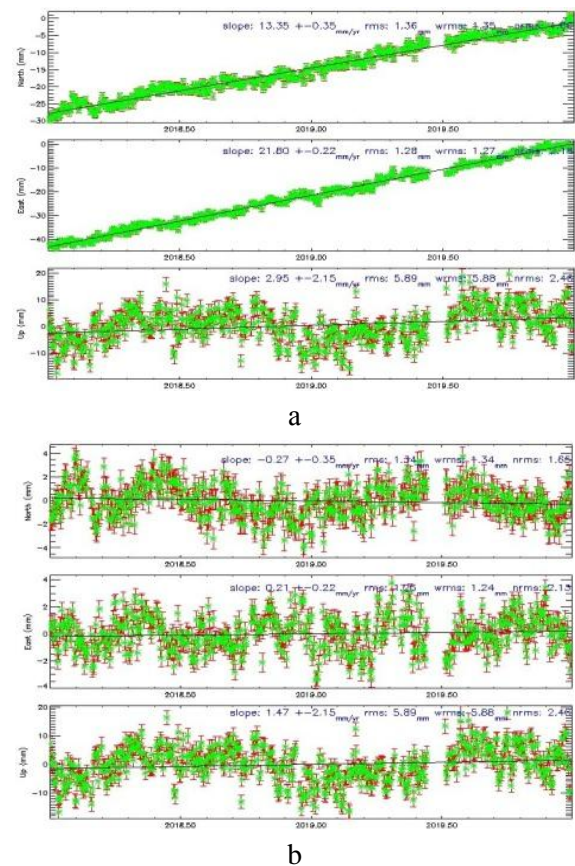
The Department of Higher Geodesy and Astronomy conducts regular processing of GNSS observations from the above-mentioned stations of the IGS / EPN networks and other reference stations of Ukraine. GNSS observations are processed in the GipsyX software package. A number of additional commands are used to create the coordinate time series, first to combine the daily files into one total file and then to use it for converting it to a new time series file in a topocentric coordinate system.

The application of a non-classical error theory of measurement for the diagnosis of multiple GNSS measurement results begins with the implementation of NETM to clarify the issue of residual error – comparing theory with experiment.

For this purpose, we have selected five permanent stations in Ukraine (SULP, GLSV, POLV, MIKL and CRAO), for which time series GNSS observations for 2018-2020 were used. This time series were downloaded from the Jet Propulsion Laboratory (JPL) server [<https://sideshow.jpl.nasa.gov/post/series.html>]. The spatial topocentric coordinates of the specified permanent stations of Ukraine with the amount of observations ranging from 582 to 722 were the initial data for the verification of the NETM of the empirical error distributions.

We used the iGPS software package to pre-check the stability of the observation station. Using this software we can determine the presence of a trend component of the time series, for example, the semi-annual or the annual, which is associated with the linear / non-linear rate of change of station coordinates. If the time series is characterized by significant nonlinear displacements, then we are using the utility <Outlier> to removed from processing. The software also determines and displays on the graphical interface the value of the RMS for each coordinate component separately. For automatic estimation of linear annual velocity and smaller ranges, we use the utility <Model>. If there are undetected displacements and shifts after using this utility, we can easily detect them by looking at the residual time series graph. We can then manually identify and remove them using the utility <Offset Selector>, while saving them in a special offset file.

We use a utility to account for these offsets <Model> again using the offset file. Figure 1.a shows a graphical example obtained from the GipsyX software package, the time series of the coordinates of the GNSS station SULP, and figure 1.b – the result obtained after applying the “clean” procedures based on iGPS.



**Fig. 1. a – The “raw” time series of SULP station  
b – The “clean” time series of SULP station**

A summary table of RMS before and after processing of permanent stations time series of Ukraine in the iGPS software package is presented below. This table also shows the percentage of this decreased values (Table 2).

*Table 2*

Comparison of RMS values							
Station	RMS, mm (before processing)			RMS, mm (processing by MODEL utility with an offset file)			
	N	E	U	N	E	U	%
SULP	1,36	1,28	5,89	1,34	1,25	5,89	2
POLV	1,53	1,15	6,39	1,51	1,14	6,24	2
MIKL	1,19	1,13	5,47	1,18	1,11	5,42	1
GLSV	3,14	1,53	6,15	3,11	1,50	6,11	1
CRAO	2,02	2,20	5,75	1,74	2,20	5,45	7

The next step was to calculate the mean values of the spatial topocentric coordinates N, E, U and the error of the deviations of each individual value from the sample mean. Thus, we obtained time-series empirical errors in determining the spatial topocentric coordinates of the appropriate stations.

### Results

Any deviation of their true distribution from the ideal mathematical form is caused by the action of systematic errors, which become noticeable in a large number of observations. These deviations are expressed by the values of asymmetry and kurtosis of true error distribution. If the weight function is non-singular, provided

$$A = 0; \text{ and } \varepsilon \geq 0, \quad (1)$$

then any deviation from these conditions will be evidence of the strong and unacceptable influence of the systematic error variables.

To verify that the obtained observation results fall within the permissible estimate A, it is necessary to construct confidential intervals for the asymmetry and the kurtosis values of errors, that can be obtained from unbiased moment estimates:

$$A = \frac{\sqrt{n(n-1)}}{n-2} \frac{m_3}{m_2^{1.5}}; \quad (2)$$

$$\varepsilon = \frac{(n-1)(n^2-2n+3)}{n(n-2)(n-3)} \frac{m_4}{m_2^2} - \frac{3(n-1)(2n-3)}{n(n-2)(n-3)} - 3, \quad (3)$$

Where  $n$  – sample,  $m_r$  – sample center moments of order  $r$ , calculated by measurement results of  $x_i$ :

$$m_r = n^{-1} \sum (x_i - \bar{x})^r; \quad \bar{x} = n^{-1} \sum x_i; \quad (4)$$

where  $x_i$  – station coordinates,  $\bar{x}$  – average coordinate value.

We use the standard errors of these statistics to construct confidential intervals for asymmetry and kurtosis:

$$\sigma_A = \sqrt{\frac{4\mu_2^2\mu_6 - 12\mu_2\mu_3\mu_4 - 24\mu_2^3\mu_4 + 9\mu_3^2\mu_4 + 35\mu_2^2\mu_3^2 + 36\mu_2^5}{4\mu_2^5n}}; \quad (5)$$

$$\sigma_\varepsilon = \sqrt{\frac{\mu_2^2\mu_6 - 4\mu_2\mu_3\mu_4 - 8\mu_2^3\mu_4 + 4\mu_3^2\mu_4 - \mu_2^2\mu_4^2 + 16\mu_2\mu_3^2\mu_4 + 16\mu_2^3\mu_3^2}{\mu_2^6n}} + \sqrt{\frac{16\mu_2\mu_3^2\mu_4 + 16\mu_2^3\mu_3^2}{\mu_2^6n}} \quad (6)$$

where  $\mu_r$  – center moments of order  $r$ ,  $n$  – sample.

Having received the value  $A$ ,  $\varepsilon$ ,  $\sigma A$ ,  $\sigma \varepsilon$  by formulas (2,3,5,6), defining confidential intervals for  $A$  and  $\varepsilon$ :

$$A \pm t_\alpha \cdot \sigma_A; \varepsilon \pm t_\alpha \cdot \sigma_\varepsilon, \quad (7)$$

where  $t_\alpha$  quantile, determined by the Laplace function for the significance level  $\alpha$ ;  $\sigma_A$  and  $\sigma_\varepsilon$  are calculated by the formulas (5) and (6).

If the confidential intervals cover zero, then it is possible to limit to the methods of estimating the NETM, during GNSS measurements. All other cases will indicate different pathologies in the device or a sharp, unacceptable deterioration of the observation conditions.

The results of our studies are shown in Tables 3, 4 and 5. They give a general description of the distribution samples and the empirical distributions of the errors of determining the spatial topocentric coordinates. Separately constructed histograms of the distribution of empirical errors for permanent stations (Fig. 2–6).

According to the theory of the Neumann-Pearson hypothesis test, if the confidence intervals (7) cover zero, this is a necessary and, as a rule, sufficient sign of the normality of measurement errors. If, however, at least one confidence interval does not cover zero, then the table should be used to solve the question of non-singularity or singularity of the weighting function, we need to use Table 3, bearing in mind that only the Gaussian laws and Pearson-Jeffries provide the possibility of obtaining non-degenerate estimates in mathematical data processing. Table 3 is essentially a program for metrological diagnostics of high-precision measurements  $g$  [Dvulit, Dzhun, 2019].

If the parameters estimation are from the general set of individual values of a random variable that obeys the normal distribution law, then this is not a guarantee that the estimations themselves have a normal distribution too. Therefore, it is necessary to find the

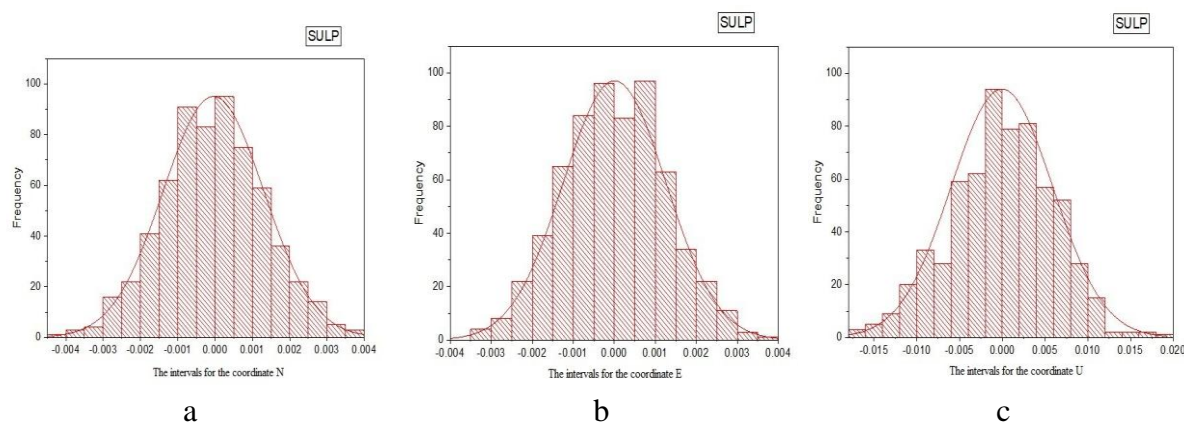
exact distribution laws of at least the main sample characteristics (Table 4). The law of distribution was used for this task  $\chi^2$ . Table 4 shows the intervals, the values of the empirical frequencies  $m_i$ , the calculated Gaussian frequencies  $m'_i$ , and their differences.

This distribution has a random variables that is the sum of squares of independent random variables that obey the normal law of distribution.

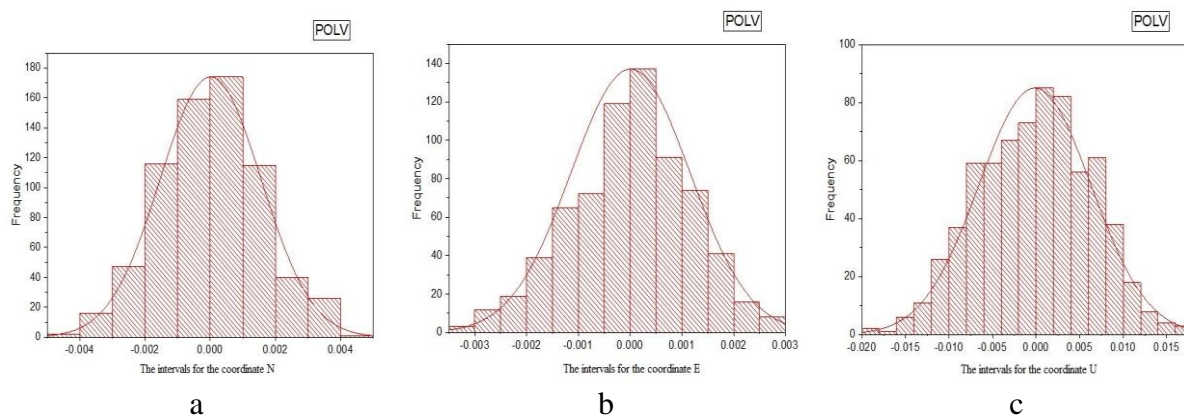
$$\chi^2 = X_1^2 + X_2^2 + \dots + X_n^2 = \sum_{i=1}^n X_i^2, \quad (8)$$

We found values of  $p(\chi^2)$  by the value of  $\chi^2$  and the number of degrees of arbitrariness  $r$  from the tables of  $\chi^2$ -distribution (Table 5).

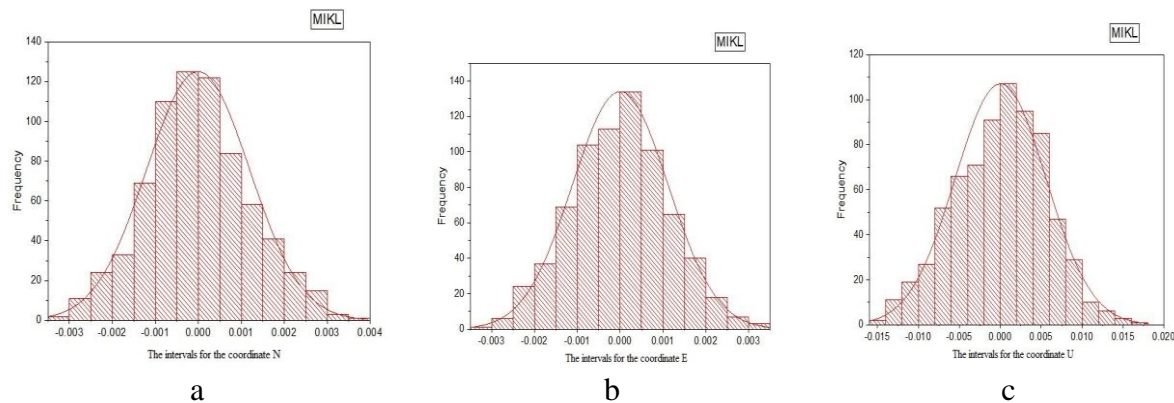




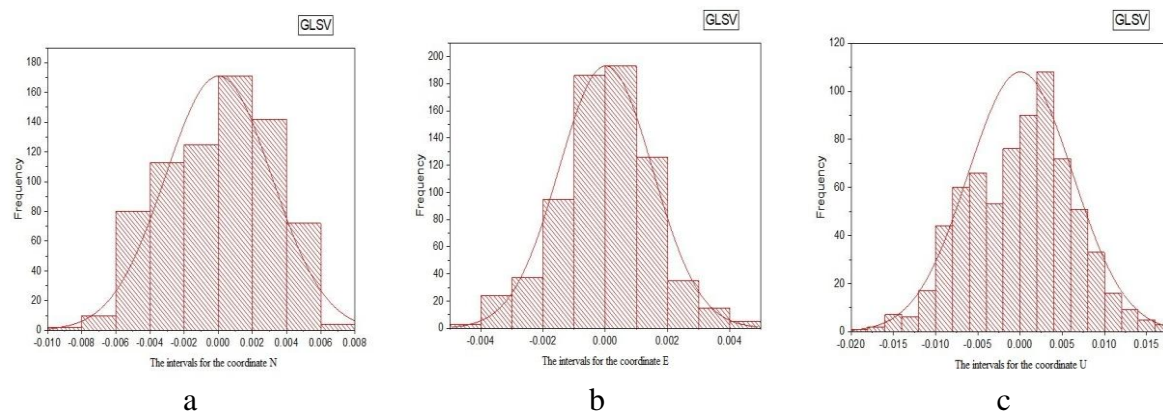
**Fig. 2. a, b, c.** Histograms for the distribution of empirical errors for the SULP permanent station



**Fig. 3. a, b, c.** Histograms for the distribution of empirical errors for the POLV permanent station



**Fig. 4. a, b, c.** Histograms for the distribution of empirical errors for the MIKL permanent station



**Fig. 5. a, b, c.** Histograms for the distribution of empirical errors for the GLSV permanent station

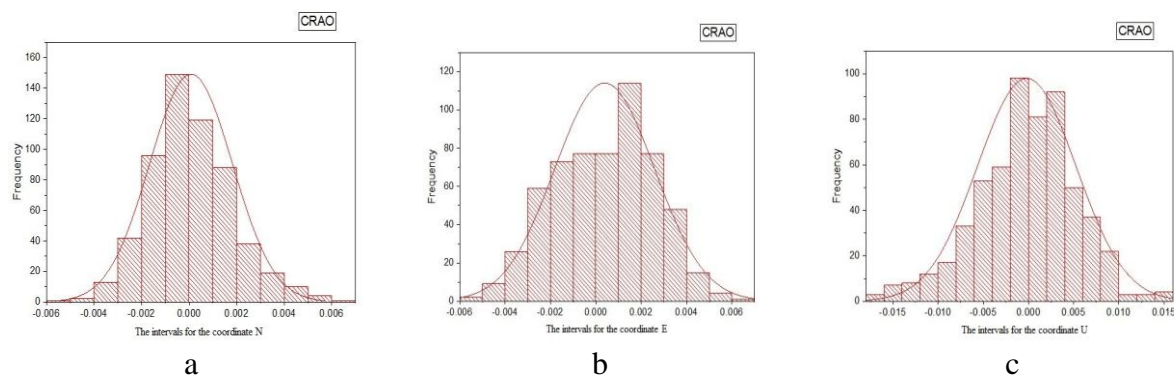


Fig. 6. a, b, c. Histograms for the distribution of empirical errors for the CRAO permanent station

Table 3

Characteristics of samples of spatial topocentric coordinates

Station		Volume of sample, $n$	Measurement results, average, mm	RMS, mm	Asymmetry and its standard: $A \pm \sigma_A$	Confidential interval for $A$	Kurtosis and its standard: $\varepsilon \pm \sigma_\varepsilon$	Confidential interval for $\varepsilon$
SULP	N	632	$-4,19 \cdot 10^{-2}$	1,34	$0,048 \pm 0,088$	$0,193 \pm 0,097$	$-0,043 \pm 0,140$	$0,188 \pm 0,275$
	E	632	$0,77 \cdot 10^{-2}$	1,26	$0,022 \pm 0,080$	$0,154 \pm 0,110$	$-0,236 \pm 0,127$	$-0,027 \pm 0,446$
	U	632	$-4,69 \cdot 10^{-2}$	5,89	$-0,148 \pm 0,087$	$-0,004 \pm 0,293$	$-0,110 \pm 0,153$	$0,143 \pm 0,363$
POLV	N	696	$1,29 \cdot 10^{-2}$	1,51	$0,027 \pm 0,075$	$0,151 \pm 0,097$	$-0,146 \pm 0,112$	$0,039 \pm 0,331$
	E	696	$0,74 \cdot 10^{-2}$	1,14	$-0,228 \pm 0,071$	$-0,110 \pm 0,346$	$-0,155 \pm 0,135$	$0,068 \pm 0,379$
	U	696	$-12,9 \cdot 10^{-2}$	6,24	$-0,090 \pm 0,075$	$0,031 \pm 0,212$	$-0,394 \pm 0,118$	$-0,199 \pm 0,588$
MIKL	N	722	$-0,49 \cdot 10^{-2}$	1,19	$0,158 \pm 0,076$	$0,283 \pm 0,034$	$-0,050 \pm 0,134$	$0,171 \pm 0,272$
	E	722	$0,18 \cdot 10^{-2}$	1,11	$0,019 \pm 0,074$	$0,141 \pm 0,103$	$-0,168 \pm 0,115$	$0,022 \pm 0,358$
	U	722	$4,01 \cdot 10^{-2}$	5,42	$-0,145 \pm 0,079$	$-0,014 \pm 0,275$	$-0,174 \pm 0,136$	$0,051 \pm 0,399$
GLSV	N	719	$1,82 \cdot 10^{-2}$	3,11	$-0,205 \pm 0,063$	$-0,100 \pm 0,309$	$-0,724 \pm 0,116$	$-0,532 \pm 0,916$
	E	719	$1,53 \cdot 10^{-2}$	1,50	$-0,152 \pm 0,090$	$-0,004 \pm 0,301$	$0,229 \pm 0,145$	$-0,499 \pm 0,020$
	U	719	$4,99 \cdot 10^{-2}$	6,11	$-0,096 \pm 0,080$	$0,035 \pm 0,226$	$-0,245 \pm 0,126$	$-0,037 \pm 0,453$
CRAO	N	582	$8,04 \cdot 10^{-2}$	1,74	$0,442 \pm 0,114$	$0,630 \pm 0,255$	$0,563 \pm 0,280$	$1,026 \pm 0,101$
	E	582	$39,9 \cdot 10^{-2}$	2,20	$-0,129 \pm 0,074$	$-0,006 \pm 0,252$	$-0,655 \pm 0,113$	$-0,468 \pm 0,841$
	U	582	$-19,6 \cdot 10^{-2}$	5,45	$-0,324 \pm 0,104$	$-0,151 \pm 0,496$	$0,375 \pm 0,208$	$0,719 \pm 0,031$

Table 4

Empirical distributions of topocentric spatial coordinates determination errors of Ukrainian permanent GNSS stations

SULP											
N				E				U			
1				2				3			
Intervals	$m_i$	$m_i$	$m_i - m_i$	Intervals	$m_i$	$m_i$	$m_i - m_i$	Intervals	$m_i$	$m_i$	$m_i - m_i$
-0,005—0,004	1	1.14	-0.14	-0,004—0,003	4	5.62	-1.63	-0,020—0,015	6	4.74	1.26
-0,004—0,003	7	8.47	-1.47	-0,003—0,002	30	31.47	-1.47	-0,015—0,010	31	29.45	1.55
-0,003—0,002	38	44.04	-6.04	-0,002—0,001	104	100.05	3.95	-0,010—0,005	91	101.12	-10.12
-0,002—0,001	103	119.56	-16.56	-0,001—0	180	178.41	1.59	-0,005—0	185	177.72	7.28
-0,001—0	174	180.5	-6.5	0—0,001	180	180.25	-0.25	0—0,005	187	183.22	3.78
0—0,001	170	191.62	-21.62	0,001—0,002	97	98.97	-1.97	0,005—0,010	110	97.01	12.99
0,001—0,002	95	112.49	-17.49	0,002—0,003	33	30.72	2.29	0,010—0,015	18	28.31	-10.31
0,002—0,003	36	41.08	-5.08	0,003—0,004	4	5.63	-1.63	0,015—0,020	4	4.49	-0.49
0,003—0,004	8	8.74	-0.74								
POLV											
N				E				U			
-0,005—0,004	2	3.13	-1.13	-0,004—0,003	3	3.62	-0.62	-0,020—0,015	5	7.38	-2.38

Continued Table

1				2				3			
-0.004–0.003	16	15.45	0.55	-0.003–0.002	31	27.21	3.79	-0.015–0.010	41	37.31	3.69
-0.003–0.002	47	49.49	-2.49	-0.002–0.001	104	106.56	-2.56	-0.010–0.005	121	114.28	6.72
-0.002–0.001	116	110.87	5.13	-0.001–0	191	207.65	-16.62	-0.005–0	174	182.56	-8.56
-0.001–0	159	163.00	-4.00	0–0.001	228	205.67	22.33	0–0.005	198	190.91	7.09
0–0.001	174	156.18	17.82	0.001–0.002	115	107.18	7.82	0.005–0.010	124	108.51	15.49
0.001–0.002	115	112.75	2.25	0.002–0.003	24	28.26	-4.26	0.010–0.015	28	35.57	-7.57
0.002–0.003	40	53.04	-13.04					0.015–0.020	5	6.54	-1.54
0.003–0.004	26	16.36	9.64								
0.004–0.005	1	3.62	-2.62								
MIKL											
N				E				U			
-0.004–0.003	2	5.13	-3.13	-0.004–0.003	1	3.32	-2.32	-0.020–0.015	2	3.18	-1.18
-0.003–0.002	36	32.35	3.65	-0.003–0.002	30	27.29	2.71	-0.015–0.010	30	26.14	3.86
-0.002–0.001	101	115.02	-14.02	-0.002–0.001	106	109.89	-3.89	-0.010–0.005	108	109.24	-1.24
-0.001–0	235	208.01	26.99	-0.001–0	217	220.28	-3.28	-0.005–0	199	222.23	-23.23
0–0.001	206	210.10	-4.10	0–0.001	235	218.26	16.74	0–0.005	252	222.23	29.77
0.001–0.002	99	112.92	-13.92	0.001–0.002	105	111.26	-6.26	0.005–0.010	111	109.24	1.76
0.002–0.003	39	32.35	6.65	0.002–0.003	25	27.94	-2.94	0.010–0.015	18	26.14	-8.14
0.003–0.004	4	5.13	-1.13	0.003–0.004	3	3.32	-0.32	0.015–0.020	2	3.18	-1.18
GLSV											
N				E				U			
-0.010–0.008	2	3.45	-1.45	-0.005–0.004	3	2.45	0.56	-0.020–0.015	7	6.04	0.96
-0.008–0.006	10	16.68	-6.68	-0.004–0.003	24	14.24	9.76	-0.015–0.010	26	34.51	-8.51
-0.006–0.004	80	52.70	27.30	-0.003–0.002	37	48.82	-11.82	-0.010–0.005	137	111.30	25.70
-0.004–0.002	113	114.40	-1.39	-0.002–0.001	95	114.75	-19.75	-0.005–0	162	204.27	-42.27
-0.002–0	125	168.89	-43.89	-0.001–0	186	172.99	13.01	0–0.005	231	198.01	32.99
0–0.002	171	161.85	9.15	0–0.001	193	163.72	29.28	0.005–0.010	123	115.90	7.10
0.002–0.004	142	117.49	24.52	0.001–0.002	126	119.21	6.79	0.010–0.015	28	35.81	-7.81
0.004–0.006	72	55.65	16.35	0.002–0.003	35	51.98	-16.98	0.015–0.020	5	6.40	-1.40
0.006–0.008	4	17.18	-13.18	0.003–0.004	15	15.46	-0.46				
				0.004–0.005	5	2.88	2.12				
CRAO											
N				E				U			
-0.006–0.004	3	7.51	-4.51	-0.006–0.004	11	16.99	-5.99	-0.020–0.015	4	2.27	1.73
-0.004–0.002	55	68.79	-13.79	-0.004–0.002	85	74.61	10.39	-0.015–0.010	26	21.42	4.58
-0.002–0	245	207.54	37.46	-0.002–0	150	162.55	-12.55	-0.010–0.005	71	91.26	-20.26
0–0.002	207	201.20	5.80	0–0.002	191	113.67	77.34	-0.005–0	189	166.63	22.37
0.002–0.004	57	73.86	-16.86	0.002–0.004	125	106.68	18.32	0–0.005	198	180.65	17.35
0.004–0.006	14	8.56	5.45	0.004–0.006	19	30.90	-11.90	0.005–0.010	84	81.48	2.52
0.006–0.008	1	0.41	0.59	0.006–0.008	1	3.55	-2.55	0.010–0.015	9	17.64	-8.64
								0.015–0.020	1	1.80	-0.80

Table 5

Values of  $p(\chi^2)$  for GNSS stations

		$\chi^2$	$r$	$p(\chi^2)$
SULP	N	9.48	6	0.17
	E	1.39	5	0.92
	U	7.35	5	0.22
POLV	N	13.75	7	0.06
	E	5.66	5	0.25
	U	6.38	5	0.28
MIKL	N	10.94	5	0.05
	E	4.05	5	0.54
	U	10.43	5	0.07
GLSV	N	49.40	6	0.001
	E	26.82	7	0.001
	U	24.87	5	0.001
CRAO	N	20.57	4	0.001
	E	66.71	4	0.001
	U	16.13	5	0.001



Let's now consider these errors from two points of view, one of which will be based on the CETM principles and the other on the thesis of the NETM. From the point of view of CETM, the measurements at the stations are satisfactory: the asymmetry in all cases is insignificant, and the confidential intervals cover zero only in 4 cases out of 15. For kurtosis, the most favorable situation is observed for SULP, POLV, MIKL stations, and the worst for GLSV and CRAO stations. Testing the Pearson criterion of the normal distribution of our empirical error series shows the following results: the probability that the measurements are selective from the normal general summation changing from 0.001 to 0.54. This means that the real measurements error distribution are not under normal law, but they are corresponding to the outdated classical concepts of the large-scale error distribution law.

### Conclusions

Based on our research, it can be stated that:

1. The most favorable situation regarding the accuracy of the empirical time series errors is observed for SULP, POLV, MIKL GNSS stations, and the worst for GLSV and CRAO stations.
2. The time series analysis of Ukrainian permanent stations based on high-precision GNSS measurements did not confirm the hypothesis of their subordination to the normal Gauss distribution law.
3. All the empirical time series of the sample is observed that the error distribution is not perfect, since the effect of weak, non-measured sources of systematic errors is confirmed.
4. The hard work of the researchers should be focused on identifying the causes that distort real distribution to bring it to an ideal, and the asymmetry and kurtosis to the proper boundaries of Pearson-Jeffreys distribution of type VII.

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#### ОПРАЦЮВАННЯ РЕЗУЛЬТАТІВ GNSS-СПОСТЕРЕЖЕНЬ НЕКЛАСИЧНОЮ ТЕОРІЄЮ ПОХИБОК ВИМІРІВ

**Мета дослідження:** показати необхідність використання сучасних методів опрацювання часових рядів GNSS–спостережень неklasичною теорією похибок вимірів (НТПВ), що характеризується великими обсягами вибірок  $n > 500$ . Такі похибки високоточних спостережень, здебільшого, не можуть бути представлені класичним законом розподілу Гаусса. Із збільшенням обсягу вибірок, емпіричний розподіл похибок все більше буде відхилятися від класичної теорії похибок вимірів (КТПВ) за Гауссом. **Методика досліджень.** Для проведення досліджень попередньо опрацьовані GNSS-спостереження на п'яти перманентних станціях України (SULP, GLSV, POLV, MIKL та CRAO). Після застосування “очищених” процедур на основі програмного пакету iGPS отримано часові ряди GNSS-спостережень за 2018–2020 роки. Перевірка емпіричних розподілів похибок забезпечувалася процедурою неklasичної теорії похибок вимірів на основі рекомендацій, запропонованих Г. Джеффрісом і на принципах теорії перевірок гіпотез за критерієм Пірсона. **Основний результат дослідження.** Встановлено, що отримані із високоточного опрацювання GNSS-спостережень часові ряди координат перманентних станцій не підтверджують гіпотезу про їх підпорядкування нормальному закону розподілу Гаусса. Проведення НТПВ-діагностики точності високоточних GNSS-вимірів, який ґрунтується на використанні довірчих інтервалів для оцінок асиметрії і ексцесу значної вибірки із наступним застосуванням – тесту Пірсона, підтверджує наявність слабких, не вилучених із GNSS-опрацювання, джерел систематичних похибок. **Наукова новизна.** Авторами задіяна можливість НТПВ для вдосконалення методики опрацювання високоточних GNSS-вимірів та необхідністю врахування джерел систематичних похибок. Неврахування окремих факторів породжують ефект зміщення часового координатного ряду, що, у свою чергу, зумовлює суб'єктивні оцінки швидкостей руху станції, тобто їх геодинамічну інтерпретацію. **Практична значущість** полягає в застосуванні НТПВ-діагностики ймовірнісної форми розподілу топоцентричних координат перманентних станцій та вдосконаленню методики їх визначень. Дослідження причин відхилень розподілу похибок від встановлених норм забезпечує метрологічну грамотність проведення високоточних GNSS-вимірів великого обсягу.

**Ключові слова:** закони похибок Гаусса, Пірсона-Джеффріса; неklasична теорія похибок вимірів (НТПВ); Глобальна навігаційна супутникова система (GNSS); топоцентричні координати; GNSS-виміри; перманентна станція.

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