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Ihor TREVOHO[1a], Fedir ZABLOTSKYI[2a], Andrzej PISKOREK[3], Bohdan DZHUMAN[2b], Andriy VOVK[1b]

[1] Department of Geodesy, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, 79013, Ukraine; [1a] https://orcid.org/0000-0002-2368-9088; [1b] https://orcid.org/0000-0002-0445-1947

[2] Department of Higher Geodesy and Astronomy, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, 79013, Ukraine, ph. +38 (068) 7632139, e-mail: teojuman@gmail.com; [2b] https://orcid.org/0000-0003-2483-6833

[3] Geokart-International, Sp. z o.o., ul. Wita Stwosza 44, 35-113 Rzeszów, Poland.

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ABOUT MODERNIZATION OF UKRAINIAN HEIGHT SYSTEM

Purpose. The purpose of this work is obtaining connections between the Baltic and European height systems based on the I class leveling between the Ukrainian and Polish control points of the base vertical networks and construction of the quasigeoid surface on the border area. Method. Full integration of the hight system of Ukraine into the European vertical reference system (EVRS) consists of two stages: modernization of the height network of Ukraine through its integration into the United European leveling network UELN; construction and use as a regional vertical date the model of high-precision quasigeoid, which will be consistent with the European geoid EGG2015. The analysis of methods of high-precision leveling in Ukraine and Poland, and also the analysis of methods of construction of quasigeoid models in these countries is performed. Results. For integrating the Ukrainian hight system into the UELN/EVRS2000 system, the Ukrainian side performed I class geometric leveling along two lines: Lviv-Shehyni–Przemysl and Kovel–Yagodyn–Chelm with total length of 196 km. The root mean square systematic error on both lines of leveling was $s<0.01$ mm/km. In turn, the mean square random error along the line Lviv–Shehyni-Przemysl is $h=0.29$ mm/km, and along the line Kovel–Yagodyn–Chelm is $h=0.27$ mm/km. For double control on the cross-border part, the Polish side performed high-precision leveling with a length of 33 km. The differences between the Ukrainian and Polish leveling in all sections are within the tolerance. The analysis of influence of geodynamic phenomena on control of high-precision leveling is carried out. GNSS-leveling was performed on all fundamental and ground benchmarks, as well as horizontal marks. These measurements were used to build a quasigeoid model for the border area of Ukraine. The MSE of the obtained quasigeoid model is about 2 cm, which corresponds to the accuracy of the input information. Scientific novelty and practical significance. The connection of the Ukrainian and European height systems will ensure Ukraine's integration into the European economic system, participation in international research of global ecological and geodynamic processes, study of the Earth's shape and gravitational field and mapping of Ukraine using navigational and remote-sensing satellite technologies. Calculation of a high-precision model of a quasigeoid on the Ukraine area in relation to the European height system, agreed with the European geoid EGG2015, will allow to obtain gravity-dependent heights using modern satellite technologies.

Key words: height system, quasigeoid model, leveling.

Introduction

The Baltic Height System 1977 has been functioning on the Ukraine area since the times of USSR, the starting point of which is the zero of the Kronstadt mareograph. The implementation of this system is the height network of Ukraine. However, it is obsolete primarily due to the great distance from zero point of reference height (about 2 thousand km) and the inability to adapt for using GNSS-leveling methods. Therefore, the height network of Ukraine today does not correspond to the level of development of modern geospatial technologies and it needs to be modernized.

Obviously, the best way to modernize Ukraine's height network is to integrate it into the United European Leveling Network UELN [Sacher, et al., 2006]. To ensure the achievement of this goal, the State Geocadastre has leveled and cataloged the points of leveling networks of I, II classes as well as prepared and transferred to the Processing Center of the European integrated leveling network BKG (Federal Agency for Cartography and Geodesy) data for alignment of nodal points of a leveling network of the I class that, in turn, will provide definition in heights system UELN/EVRS2000 [Sacher, et al., 2007] heights of geodetic points and area objects during performing topographic, geodetic and cadastral works as well as their use in solving geographic information problems by central executive authorities, local governments, enterprises, institutions and organizations with high accuracy.

The current resolution of the Cabinet of Ministers of Ukraine "Some issues of application of the height system UELN/EVRS2000 (United European Leveling Network/European Vertical Reference System 2000)" states that "the implementation of topographic, geodetic and cartographic works starting from January 1, 2023 will be carried out using the UELN / EVRS2000 height system".

To use progressive space technologies, such as GNSS-technology, during determining gravity-dependent heights the basis of any height system must be high precision geoid/quasigeoid model. This approach should be applied in Ukraine, in addition to the fact that the European gravimetric geoid EGG15 [Denker, 2015] gives quite significant errors in this area.

Purpose

The purpose of this work is to make connections between the Baltic and European height systems based on the I class leveling between the Ukrainian and Polish leveling points and the construction of the quasigeoid surface on the border area.

Method

Complete integration of the height system of Ukraine into the European vertical reference system consists of two stages: a) modernization of the height network of Ukraine through its integration into the United European Leveling Network UELN; b) construction and use as a regional vertical datum the model of high-precision gravimetric quasigeoid, which will be consistent with the European geoid EGG2015. We will analyze the methods of high precision leveling and computing quasigeoid models in Ukraine and Poland areas.

Analysis of high-precision leveling methods in Ukraine and Poland

For today there are no modern instructions of leveling in Ukraine. Therefore, when performing this type of work it is necessary to use obsolete instructions that were written for optical levels many decades ago. [Instruction…, 1966; Instruction…, 1971]. Modern digital levels have a number of advantages over such levels, so the use of outdated instructions leads to significant excess economic costs. Since the connection between the Ukrainian and European height systems envisaged the I class leveling with a length of almost 200 km, there was a question of adapting existing instructions to modern digital levels.

The following compromise has been found to solve this problem. On the digital level Trimble DiNi 0.3 [Trimble…, 2017], which performed this work, was installed targeted updated software Trimble DiNi Update R 2.2.1, obtained from the official website of Trimble, which was specially designed for leveling according to the method of I class (Fig. 1).

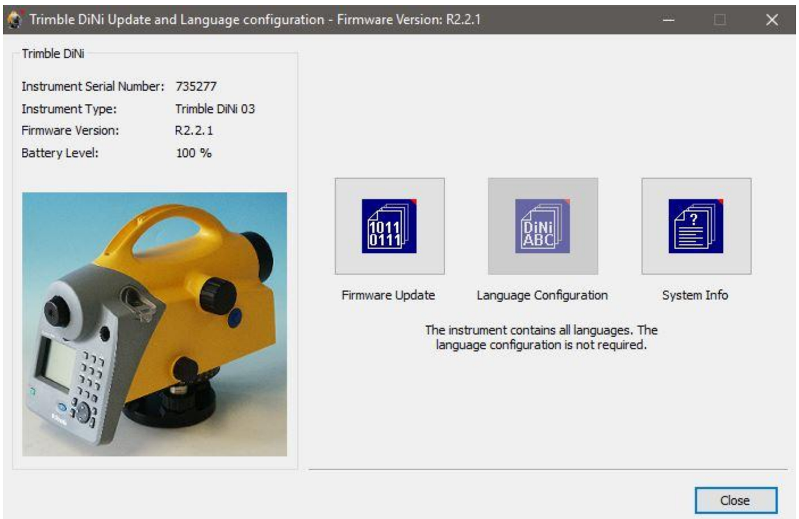

Fig. 1. Targeted software update Trimble DiNi Update R 2.2.1 for I class leveling by digital level Trimble DiNi 0.3, downloaded from the official website trimble.com

This software provides a program of measurements at the station in the following sequence: "1R, 1F, 1F, 1R, 2R, 2F, 2F, 2R", where "1" – left leveling line, "2" – right leveling line, "R" – rear rod, "F" – front rod (Fig. 2).

The main difference between the Ukrainian and Polish high-precision leveling technologies is that when the Polish side performs leveling, the left and right leveling lines coincide, ie the rods remain stationary during measurements at the station. Instead, on each line, measurements are made at different instrument horizons (Fig. 3).

Analysis of methods for constructing quasigeoid models in Ukraine and Poland

For the full functioning of the modern height system it is necessary to use a high-precision model of a geoid or quasigeoid. In recent years, several quasigeoid models UQGxxxx have been computed in Ukraine [Marchenko, et al., 2007; Marchenko, et al., 2013], and the last implementation is UQG2017. For comparison, let us consider the model of the geoid GDQM-PL13 [Szelachowska & Krynski, 2014], built for the Poland area.

Gravimetric data (Fig. 4) and data from 6059 GNSS-leveling points were used to compute the UQG2017 model.

About one million of gravitational anomalies located both in Poland and abroad were used to build the GDQM-PL13 model. Deviations of plumb lines, obtained at 171 astronomical points, and height anomalies from GNSS-leveling were also used (Fig. 5).

In general, in both cases for the construction of the model UQG2017 and GDQM-PL13 [Szelachowska & Krynski, 2014], the dominant input data were gravitational anomalies, which were compared with geometric data, obtained from GNSS-leveling at points of the State Geodetic Network. Also in both cases, the calculations of the quasigeoid model were performed within the procedure "Remove–Compute–Restore", and the global gravitational model EGM2008 was used as a systematic component.

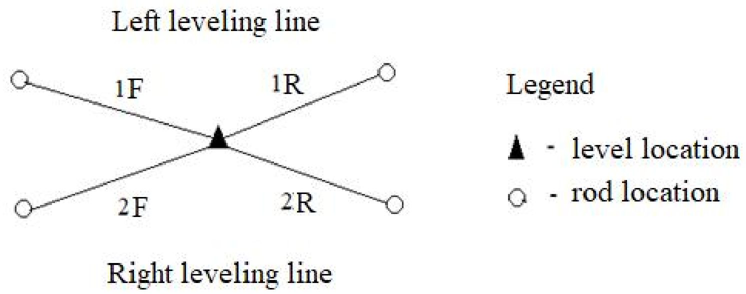

Fig. 2. Scheme of the program of measurements at the station by the Ukrainian side

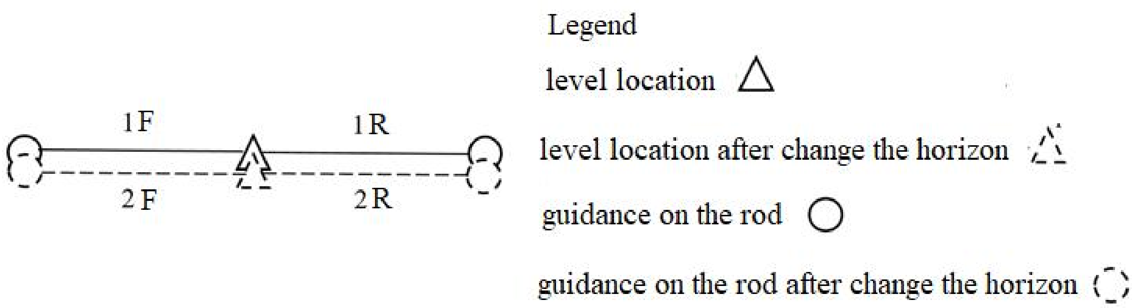

Fig. 3. Scheme of the program of measurements at the station by the Polish side

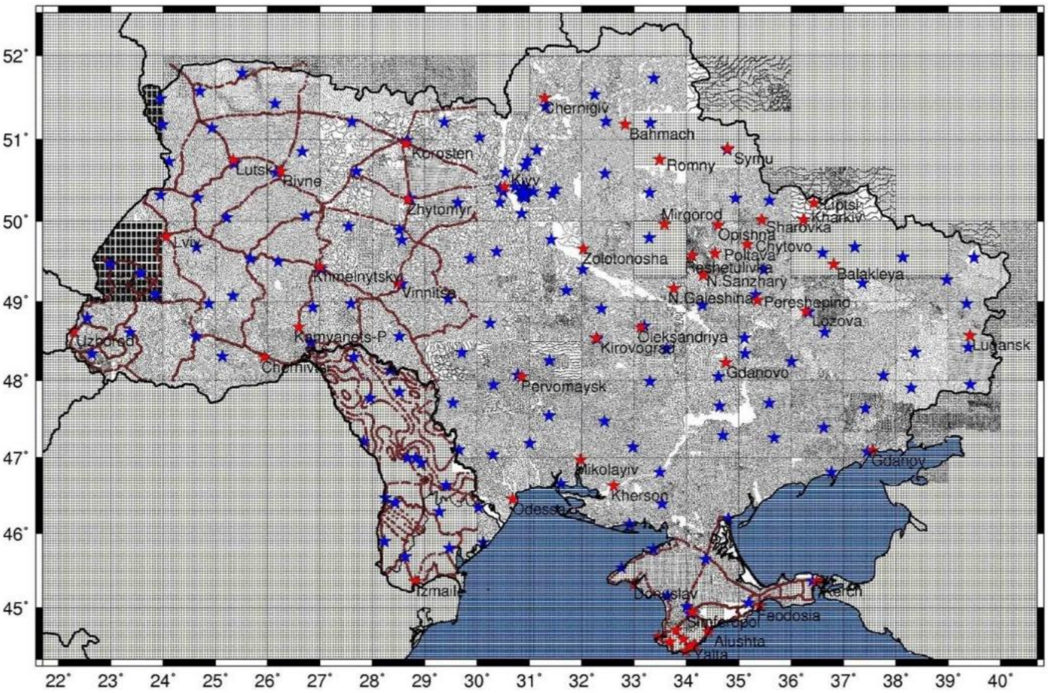

Fig. 4. Distribution of input gravity anomalies for the construction of UQG2017:
· - 201698 Bouguer anomalies obtained from digitization of maps with scale 1:200 000;
· - 20 844 Bouguer anomalies obtained from altimeter data on the grid 2¢ 2¢
· - 55 210 Bouguer anomalies calculated by the model EGM08 to 2190 degrees/order;
· - 2709, 1141 and 1465 Bougue anomalies in the course of leveling in Western Ukraine, the Crimean peninsula, Moldova and Odessa region; « - 53 I class gravimetric points,
« - 182 II class gravimetric points in the region of Ukraine and Moldova

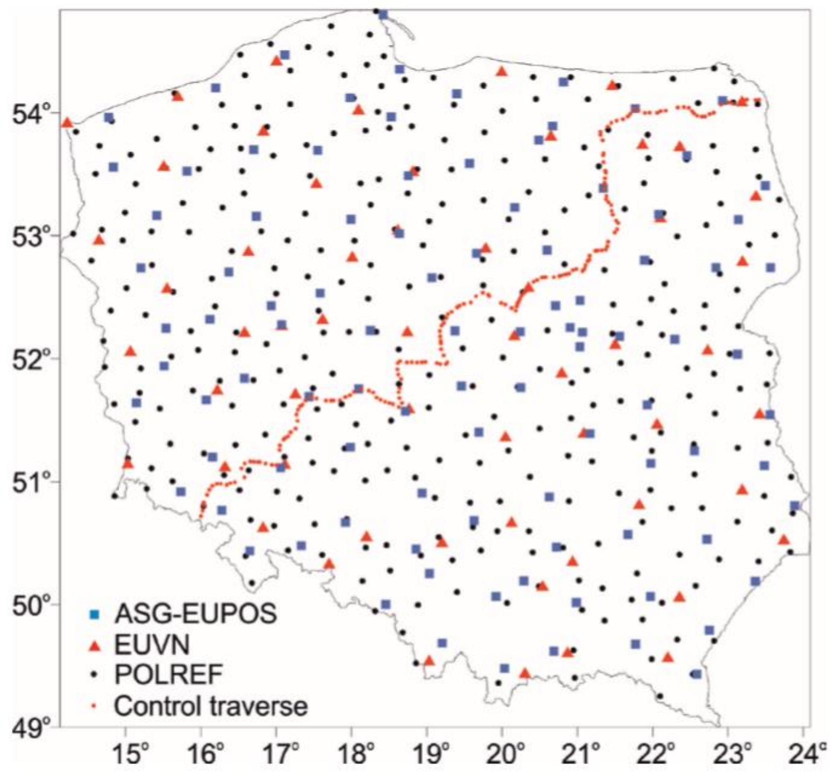

Fig. 5. Distribution of input height anomalies for construction

Calculation of UQG2017 and GDQM-PL13 models was performed by the method of least square collocation [Moritz, 1976]. In the case of UQG2017 analytical covariance function (ACF) [Knudsen, 1987; Tscherning & Rapp, 1974] consists of the family of potentials of first order radial multipoles, and in the case of GDQM-PL13 is approximated by a planar logarithmic function.

It should be noted that the UQG2017 model is built relative to the height of the Baltic Sea. Therefore, for the full integration of the Ukrainian height system into the European vertical reference system, it is necessary to build a high-precision quasigeoid model on the territory of Ukraine relative to the height of the Amsterdam footstock, which should be consistent with the European geoid EGG2015.

Results

Geodetic works to unite the Ukrainian and European height systems

To integrate the Ukrainian height system in the UELN/EVRS2000 system, the Ukrainian side performed I class geometric leveling along two lines: Lviv–Shehyni–Przemysl and Kovel–Yagodyn–Chelm with a total length of 196 km. This leveling was carried out in the period August-October 2020. Leveling on the line Lviv-Shehyni-Przemysl belongs to the third category of difficulty: (number of tripods per 1 km is 13–14), and on the line Kovel–Yagodyn–Chelm belongs to the fourth category of difficulty: (number of tripods on 1 km is 15–17). Mostly leveling lines took place along the railway track, and in some cases (≈10 %) and on the Polish side along highways.

Table 1

Comparison of excesses obtained by the Ukrainian and Polish sides along the line Lviv-Shehyni-Przemyśl

No.	Name	Length of section, *km*	h_{ukr}, m	h_{pol}, m	$h_{ukr} + h_{pol}$, mm
1	Wall mark B/№ 77.6 km				
		2.07	2.0757	-2.0752	0.5
2	Horizontal mark A0989				
		2.79	-9.1518	9.1521	0.3
3	Wall mark B/№ 82.4 km				
		1.41	-5.3907	5.3900	-0.7
4	Wall mark C0165				
		0.72	-2.3031	2.3024	-0.8
5	Soil benchmark A0871				
		0.12	-0.5019	0.5023	0.4
6	Fundamental mark A0739				
		0.02	-0.5775	0.5775	0.0
7	Control mark A0627				
		2.72	8.0162	-8.0160	0.2
8	Wall mark 40824100				
		1.62	-17.7964	17.7956	-0.8
9	Wall mark 408242101				
		1.36	-0.0777	0.0774	-0.3
10	Wall mark 17640103				
		2.60	0.604	-0.6054	-1.4
11	Wall mark 17630364				
		1.07	0.6618	-0.6624	-0.6
12	Wall mark 17630363				
		1.49	-0.1067	0.1051	-1.6
13	Wall mark 17630359				

Table 2

Comparison of excesses obtained by the Ukrainian and Polish sides along the line Kovel-Yagodyn-Chelm

No.	Name	Length of section, *km*	h_{ukr}, m	h_{pol}, m	$h_{ukr} + h_{pol}$, mm
1	Wall mark B/№, Yagodyn				
		3.39	1.5993	-1.6001	-0.8
2	Wall mark B/№, Starovoitove				
		1.96	-3.1342	3.1341	-0.1
3	Wall mark 1087				
		1.27	7.5432	-7.5444	-1.2
4	Control mark A2929				
		0.02	0.5131	-0.5128	0.3
5	Fundamental mark A0273				
		0.05	4.6768	-4.6765	0.3
6	Triangulation point Wolf Carriage, 2				
		3.29	-10.5307	10.5307	0.0
7	Wall mark 13730454				
		0.47	-0.1244	0.1242	-0.2
8	Wall mark 13730462				
		0.46	-1.6890	1.6898	0.8
9	Wall mark 13730460				
		1.26	2.7918	-2.7920	-0.2
10	Wall mark 13730452				
11	Wall mark 13730402	2.64	-0.9001	0.8968	-3.3

The root mean square systematic error on both leveling lines is s<0.01 mm/km. In turn, the mean square random error along the line Lviv-Shehyni-Przemysl is h=0.29 mm/km, and along the line Kovel–Yagodyn–Chelm is h=0.27 mm/km.

As we can see from Tables 1 and 2, all differences between the excesses are within the tolerance, calculated by the formula $3\sqrt{L}$, where L is section length in kilometers.

According to the results of I class leveling, the residuals in the UELN/EVRS2000 height system is +0.9 cm along the line Lviv–Shehyni–Przemysl and -2.4 cm along the line Kovel–Yagodyn–Helm, which is allowed. The permissible residual of I class leveling is calculated by the formula $3\sqrt{L}$ and is ±3.0 cm for the line Lviv–Shehyni–Przemysl and ±2.9 cm for the line Kovel–Yagodyn–Chelm.

Since the European height system is balanced by geopotential numbers [Sansò, et al., 2019], the geopotential numbers of leveling points in the UCS-2000 coordinate system on the plane in the Gauss–Krueger projection and in the UELN/EVRS2000 height system on both lines are calculated. Differences in geopotential numbers dC were calculated by the formula [S´anchez & Sideris, 2017]

$$dC = 10^{-6} g_m dh, \quad (1)$$

where dh is balanced excess between two points, g_m – the average value of the acceleration of free fall between these points, obtained from gravitational anomalies $g = \Delta g + \gamma$. In turn γ is the normal value of the acceleration of free fall, calculated according to [Instruction…, 1971]. Geopotential numbers of initial leveling points (fundamental marks 1712 and 5290) were obtained from the Administrator of the Geodetic Data Bank.

The residuals on the Lviv–Shehyni–Przemysl and Kovel–Yagodyn–Helm lines in the Baltic Height System 1977 is +4.1 cm and +5.8 cm respectively, which is outside the tolerance. It should be noted that the Baltic Height System 1986, which used in Poland, and the Baltic Height System 1977, which used in Ukraine, differ, so the calculations of the discrepancies in the heights of the marks, defined in different coordinate systems, have an incorrect result. Due to this, the alignment on the lines Lviv–Shehyni–Przemysl and Kovel–Yagodyn–Helm was not performed, the marks of the leveling points were obtained from the calculations.

Influence of geodynamic phenomena on control of high-precision leveling

During connecting state height networks, built at different times in different countries, there are certain difficulties of different nature. Differences in high-precision leveling technologies, instrumental and methodological support, the influence of atmospheric and geodynamic phenomena on the process and the obtained results, etc. All this imposes additional requirements to control the quality of this type of work. Due to geodynamic processes in different regions of Europe, the heights of geodetic points vary within 2–5 mm, and, for example, in mountainous areas with active geodynamics – up to 10 mm per year. In addition, a significant proportion of gravimetric measurements, which were performed in parallel with the performance of high-precision leveling, were carried out in the 50–60s of the XX century, sometimes outdated methods. According to the Resolution of the Cabinet of Ministers, the normal heights of the points of the state leveling network of Ukraine are determined in the Baltic Height System in 1977. But there are a number of factors that reduce the status of the network. This is primarily due to the difficulty of its adaptation to the European Vertical Reference Network EUVN, the vertical EVRS system and GNSS-leveling methods. The issue of quantitative differences between height networks is also unresolved [Melnik, 2014].

The peculiarity of the problem of improving the accuracy of state height networks is due to the long time of their creation. In most European countries, such networks have been established for decades. During this time, under the influence of geodynamic processes, the position of the network marks was constantly changing. Accordingly, measurements of excesses between marks performed for specific epochs are distorted by the influence of factors of endogenous nature. Therefore, the differentiation of the main sources of errors of measurement of excesses and their distortions by natural factors is necessary to take them into account and increase the accuracy of the final results of network balancing [Tretyak & Turuk, 2003].

In view of this, it is necessary to take into account corrections for the influence of geodynamic phenomena, in particular vertical movements of the earth's crust, and the diversity of height networks to improve the accuracy of the final results of balancing height networks.

The study of crustal movements caused by geodynamic processes is of great application in geodesy, in particular to solve such basic tasks as the study of the stability of points of the geodetic basis, predicting changes in coordinates and heights of points and studying the relationship between different coordinate systems and heights, their transition parameters . That is why it is important to conduct research on the geodynamics of the European continent, as due to its tectonic structure, Europe undergoes constant geodynamic processes, in addition, such processes are associated with seismic activity in the region and have a direct impact on virtually all engineering structures and geodetic works [Vovk, 2016].

The study of geodynamic processes and areas of both Ukraine [Tretyak et al., 2015] and the European continent [Tretyak & Vovk, 2016] in general is a relevant aspect of geodetic research, as well as establishing the magnitude of the annual change as horizontal [Tretyak & Vovk, 2014; Vovk, 2015] and vertical [Tretyak & Romaniuk, 2014; Tretyak & Romaniuk, 2018] movements of the earth's crust associated with age-related geodynamic processes, especially in man-made areas [Zayats et al., 2017; Mordvinov et al., 2018; Savchyn & Vaskovets, 2018; Savchyn & Pronyshyn, 2020]. The results of such geodetic measurements give information about the movement only at certain points on the earth's surface, and not for the entire earth's crust. However, if in a certain region there is a systematic nature of such movements, they can be interpreted as movements of the earth's crust, which allows to establish the influence of geodynamic processes on the results of geodetic works, including high-precision leveling, especially in combination with gravimetric studies and studies of the Earth's gravitational field [Marchenko & Dzhuman, 2015; Marchenko & Lopushansky, 2018].

Information of the movements of the earth's crust, collected on the basis of geological, geophysical, geodetic data, is taken into account in geodetic works on large engineering structures, in the construction of ports, dams, bridges, hydroelectric power plants, nuclear power plants, etc.

In practice, to study regional geodynamic phenomena, repeated high-precision leveling is used, which, in combination with regular observations of sea level, allows to determine modern vertical movements of the earth's crust within large areas. This method can be supplemented by geotechnical methods that allow to obtain permanent data on the slopes of the earth's surface within rigid plates, which should be a supplement to the “discrete”, ie obtained with a certain frequency of data, based on high-precision geometric leveling. The initial basis for monitoring vertical displacements in man-caused areas is a network of marks, the heights of which are determined by repeated high-precision geometric leveling. Such data are discrete and contain information about the state of the network only at the time of measurement. Therefore, when monitoring vertical displacements at the local and regional levels, much attention should be paid to the stability of network points, on the basis of which it is possible to predict geodynamic processes between observation cycles. This, in turn, will allow the zoning of the studied areas by common kinematic characteristics. The use of various geodetic (GNSS – observation, geometric, trigonometric, hydro-leveling) and geotechnical observations for the monitoring of man-caused areas requires the development of new legal documents (methods, instructions). Such documents should regulate monitoring in man-caused areas according to geodetic, geotechnical observations, the results of remote sensing of the earth's surface, which in turn will take into account the impact of all geodynamic processes in these areas [Petrov, 2019].

During such geodetic monitoring, considerable attention should be paid to the observation of vertical movements of the earth's surface and structures. Today, the most advanced quantitative method of studying vertical movements is high-precision geometric leveling. However, given that in Ukraine vertical movements are mostly in the range of 1–5 mm/year, it is possible to determine them on the basis of repeated measurements for a short period of time only if the measurements will be carried out with the highest accuracy. Since the measurements are not performed instantaneously, but over a period of time, there is also the question of the possible duration of a single cycle of measurements and their shift in time to minimize the impact on the accuracy of determining the speed of vertical movements [Chernyaga, et al., 2010].

Based on this, we see that comprehensive monitoring of geodynamic processes, in particular vertical displacements, at the local and regional levels, taking into account its results when performing high-precision geometric leveling will increase the accuracy of the results. Cyclic observation of geodynamically active zones, through which, or near which, the lines of I class leveling networks pass, will allow to assess the need for repeated high-precision leveling. And when working on the connection of leveling networks of different countries and the establishment of a single system of heights, will increase the accuracy of the final results. This is an important aspect for the study and consideration of the influence of geodynamic phenomena when performing high-precision leveling.

Construction of quasigeoid model for the border area

GNSS-leveling was performed on all fundamental and ground marks, as well as on horizontal marks. The measurements were performed by GNSS receivers from Leica and South and included continuous observations for at least 6 hours in static mode with frequency of 1 second. Data processing is performed in the Leica Geo Office software package for permanent GNSS stations of the GeoTerrace network, which are included in the SGN. The root mean square error in determining the geodetic height, obtained from GNSS leveling in static mode, did not exceed 20 mm. These measurements were used to build a quasigeoid model for the border area. In [Zablotskyi, et al., 2021] it is shown that the European geoid EGG2015 is best suited for this area. Since the quasigeoid model was calculated within the procedure “Remove-Compute-Restore”, the EGG2015 model was chosen as a systematic component.

Therefore, the residual values of the quasigeoid heights δN, obtained by subtracting from the heights N_{mes}, obtained from the measurements, the heights $N_{EGG2015}$, obtained from the model EGG2015, can be written in the form:

$$\delta N = N_{mes} - N_{EGG2015}. \qquad (2)$$

Let us extend the residual values of the quasigeoid heights δN in a series of STHA-functions [Dzhuman, 2018] up to 4 degrees/order:

$$\delta N_m = \\ = \frac{GM}{\gamma R}\sum_{k=1}^{4}\sum_{m=0}^{k}\left\{\bar{C}_{km}cos\left(2\pi m\frac{\lambda-\lambda_{min}}{\lambda_{\max}-\lambda_{\min}}\right)+ \overline{+\bar{S}_{km}sin\left(2\pi m\frac{\lambda-\lambda_{\min}}{\lambda_{max}-\lambda_{\min}}\right)}\right\}\bar{P}_{km}(\theta). \quad (3)$$

to find the eigenvalues n_k it is necessary first to find the coordinates of the vertices of trapezium. In this case $\varphi_{\min} = 49.7^0$, $\varphi_{\max} = 51.3^0$, $\lambda_{\min} = 22.9^0$, $\lambda_{\max} = 24.8^0$ ($\theta_{\min} = 38.7^0, \theta_{\max} = 40.3^0, \lambda_{\min} = 22.9^0, \lambda_{\max} = 24.8^0$), and $\theta_0 = 0.8^0$, $\theta_{mean} = 39.5^0$. Let us find eigen numbers n_k for $\theta_0 = 0.8^0$ up to 4 degrees. They are presented in Table 3.

Coefficients $\bar{C}_{km}$ and $\bar{S}_{km}$ we found using the least squares method and the Tikhonov regularization parameter to stabilize the solution:

$$V^TV + \alpha X^TX \to \min, \quad (4)$$

where V is vector of amendments, X is vector of unknown coefficients.

The Tikhonov regularization parameter was selected in such a way that the condition was fulfilled

$$m_N \approx m_{Nm}, \quad (5)$$

where m_N is RMS of quasigeoid heights obtained from measurements, m_{Nm} is RMS of model heights of the quasigeoid. Thus we accepted $\alpha = 1 \cdot 10^{-6}$.

The obtained values of the unknown coefficients are given in Table 4.

Table 3

Eigen numbers n_k for a spherical trapezium with vertex coordinates $\theta_{\min} = 38.7^0, \theta_{\max} = 40.3^0, \lambda_{\min} = 22.9^0, \lambda_{\max} = 24.8^0$

k/m	0	1	2	3	4
0	0				
1	171.7327	131.3678			
2	273.9262	273.9262	218.2478		
3	394.8464	381.3372	367.3136	300.394	
4	501.9546	501.9546	479.7931	456.4486	380.3496

Table 4

Coefficients $\bar{C}_{km}$ and $\bar{S}_{km}$

Coefficient	Value	Coefficient	Value
$\bar{C}_{10}$	-2.57E+00	$\bar{S}_{32}$	1.37E-01
$\bar{C}_{11}$	2.31E+00	$\bar{C}_{33}$	-5.63E-02
$\bar{S}_{11}$	-5.54E+00	$\bar{S}_{33}$	-1.97E-04
$\bar{C}_{20}$	-7.38E+00	$\bar{C}_{40}$	-7.72E-04
$\bar{C}_{21}$	3.36E+00	$\bar{C}_{41}$	-7.50E-01
$\bar{S}_{21}$	6.12E+00	$\bar{S}_{41}$	7.61E+00
$\bar{C}_{22}$	-7.17E+00	$\bar{C}_{42}$	-5.65E+00
$\bar{S}_{22}$	1.50E-02	$\bar{S}_{42}$	1.66E-02
$\bar{C}_{30}$	2.70E-01	$\bar{C}_{43}$	2.56E-01
$\bar{C}_{31}$	-2.36E+00	$\bar{S}_{43}$	1.60E-04
$\bar{S}_{31}$	-6.36E+00	$\bar{C}_{44}$	-1.99E-04
$\bar{C}_{32}$	-6.28E+00	$\bar{S}_{44}$	-3.61E-07

The variance σ_k of coefficients $\bar{C}_{km}$ and $\bar{S}_{km}$ of the obtained model can be calculated by the following formula:

$$\sigma_k = \sqrt{\frac{\sum_{m=0}^{k}(\bar{C}_{km}^2 + \bar{S}_{km}^2)}{2k+1}}. \quad (6)$$

The value of the variance is shown in Figure 6.

Map of the model of residual values of quasigeoid heights δN_m is shown in Fig. 7.

The differences between the residual values of the quasigeoid heights δN and the model of the residual values of the quasigeoid heights δN_m are also calculated:

$$\Delta\delta N = \delta N - \delta N_m. \quad (7)$$

A map of these differences is shown in Fig. 8.

Finally, the model of the quasigeoid N_m can be described by the formula

$$N_m = N_{EGG2015} + \delta N_m. \quad (8)$$

It is shown in Figure 9.

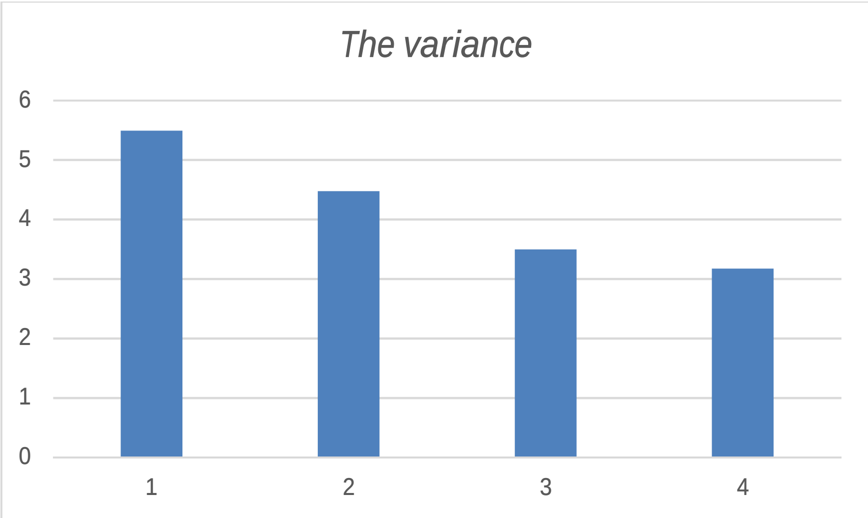

Fig. 6. The variance of coefficients $\bar{C}_{km}$ and $\bar{S}_{km}$

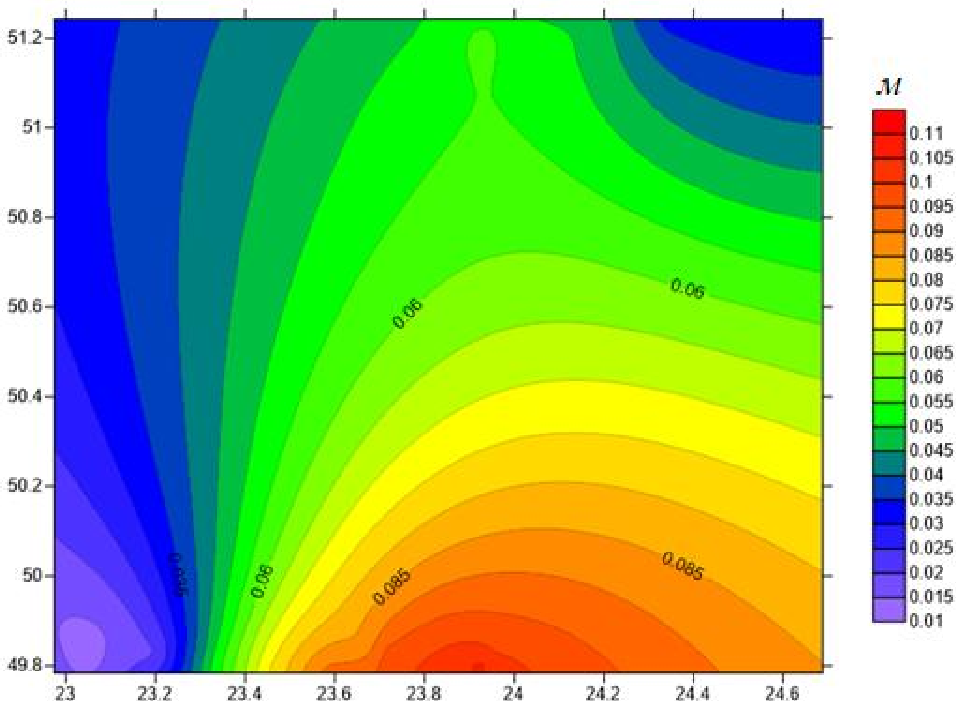

Fig. 7. Map of the model of residual values of quasigeoid heights δN_m

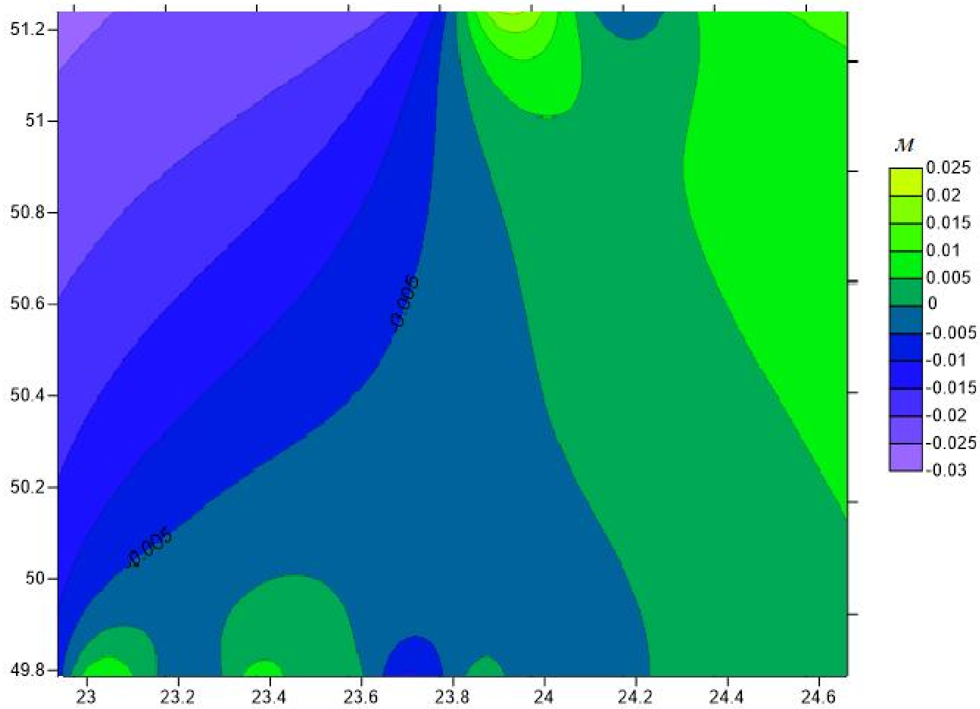

Fig. 8. Map of differences $\Delta\delta N$

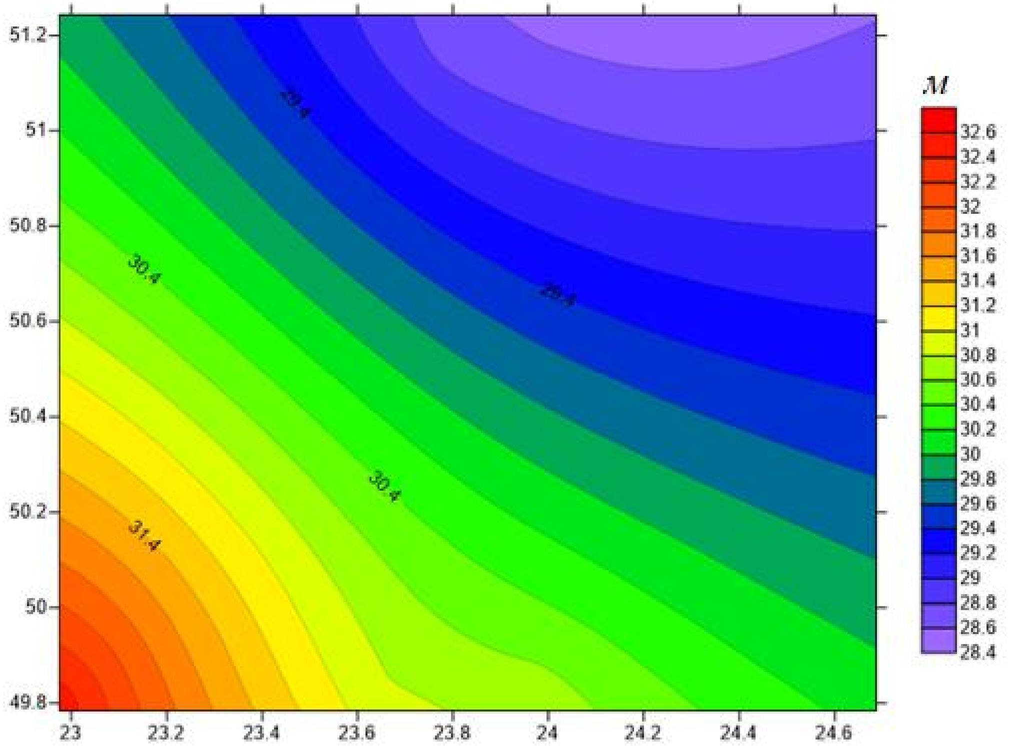

Fig. 9. Map of quasigeoid model N_m

To assess the accuracy of the obtained model, the root mean square error of the model m_{Nm} was calculated by the formula

$$m_{Nm} = \sqrt{\frac{\sum_{i=1}^{p} \Delta_i^2}{p}}, \qquad (9)$$

where Δ is the difference between the calculated from the measurements and the model value of the quasigeoid heights, p is the number of all values of these heights (p = 26). The root mean square error is m_{Nm} = 21 mm.

For all the fields described above, their main characteristics, such as minimum and maximum value and standard deviation, are calculated. These characteristics are shown in Table 5.

Table 5

The main characteristics of the considered fields

	The minimum value, m	The maximum value, m	Standard deviation, m
N_{mes}	28.48	32.51	30.44
N_m	28.48	32.52	30.44
$N_{EGG2015}$	28.45	32.50	30.38
δN	-0.01	0.12	0.07
δN_m	0.01	0.11	0.07
$\Delta\delta N$	-0.03	0.03	0.02

Scientific novelty and practical significance

The connection of the Ukrainian and European height systems will ensure Ukraine's integration into the European economic system, participation in international research of global ecological and geodynamic processes, study of the Earth's shape and gravitational field and mapping of Ukraine using satellite technologies. Calculation of a high-precision model of a quasigeoid on the Ukraine area in relation to the European height system, agreed with the European geoid EGG2015, will allow to obtain gravity-dependent heights using modern satellite technologies.

Conclusions

As a result of the performed work, the following conclusions can be drawn:

1. I class leveling was performed to connect the Ukrainian and European height systems. Discrepancies in the UELN/EVRS2000 height system are within the tolerance, at the same time in the Baltic height system they exceed the tolerance primarily due to the use of different implementations of this height system in Ukraine and Poland.

2. In order to correctly introduce the European height system on the territory of Ukraine, it is necessary to take additional steps, namely to close the leveling line Lviv-Shehyni-Przemysl and Kovel–Yagodyn–Chelm, performing I class leveling on the line Lviv–Kovel; to update the existing leveling connection between Ukraine and Poland, having performed I class leveling on the line Lviv–Rava-Ruska–Grebenne.

3. GNSS-measurements were performed at leveling points and a quasigeoid model for the border area of Ukraine was calculated, the accuracy of which is within the accuracy of the input information. For calculating a high-precision quasigeoid on the territory of Ukraine it is necessary to operate with a much larger amount of gravimetric and geometric data, which will evenly cover the entire territory of the state.

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Ігор ТРЕВОГО[1], Федір ЗАБЛОЦЬКИЙ[2], Анджей ПІСКОРЕК[3], Богдан ДЖУМАН[2], Андрій ВОВК[1]

[1] Кафедра геодезії, Національний університет “Львівська політехніка”, вул. С. Бандери, 12, Львів, 79013, Україна,.
[2] Кафедра вищої геодезії та астрономії, Національний університет “Львівська політехніка”, вул. С. Бандери, 12, Львів, 79013, Україна, тел. +38(068)7632139, ел. пошта: teojuman@gmail.com
[3] Geokart-International, Sp. z o.o., вул. Віта Ствоша, 44, 35-113, Жешув, Польща.

ПРО МОДЕРНІЗАЦІЮ УКРАЇНСЬКОЇ ВИСОТНОЇ СИСТЕМИ

Мета. Метою цієї роботи є встановлення зв’язків між Балтійською та Європейською системами висот на основі проведення нівелювання І класу між українськими та польськими контрольними пунктами базової висотної мережі та побудова поверхні квазігеоїда на прикордонну територію. Методика. Повноцінна інтеграція висотної системи України у Європейську вертикальну референцну систему складається з двох етапів: модернізації висотної мережі України шляхом її інтеграції в Об’єднану європейську нівелірну мережу UELN; побудови та використання в якості регіонального вертикального датуму моделі високоточного квазігеоїда, яка узгоджуватиметься з Європейським геоїдом EGG2015. Виконано аналіз методики нівелювання високих класів в Україні та Польщі, а також аналіз методик побудови моделей квазігеоїда в цих країнах. Результати. Для інтеграції української висотної системи в систему UELN/EVRS2000 українською стороною виконано геометричне нівелювання І класу за двома лініями: Львів–Шегині–Перемишль та Ковель–Ягодин-Хелм загальною протяжністю 196 км. Середня квадратична систематична похибка по обох лініях нівелювання становить $s<0.01$ мм/км. Своєю чергою, середня квадратична випадкова похибка по лінії Львів-Шегині–Перемишль рівна $h=0.29$ мм/км, а по лінії Ковель–Ягодин–Хелм – $h=0.27$ мм/км. Для подвійного контролю на транскордонній частині польською стороною виконано високоточне нівелювання протяжністю 33 км. Розходження між українським та польським нівелюванням по всіх секціях є в межах допуску. Проведено аналіз впливу геодинамічних явищ на контроль високоточного нівелювання. На всіх фундаментальних та ґрунтових реперах, а також горизонтальних марках виконано GNSS-нівелювання. Ці виміри використано для побудови моделі квазігеоїда на прикордонну територію України. СКП отриманої моделі квазігеоїда становить близько 2 см, що відповідає точності вхідної інформації. Наукова новизна і практична значущість. З’єднання української та європейської систем висот забезпечить інтеграцію України в європейську економічну систему, участь в міжнародних наукових дослідженнях глобальних екологічних і геодинамічних процесів, вивчення фігури Землі та гравітаційного поля, картографування території України з використанням навігаційних супутникових технологій та дистанційного зондування. Обчислення високоточної моделі квазігеоїда на територію України відносно європейської системи висот, узгодженої з європейським геоїдом EGG2015, дасть змогу отримувати гравітаційно залежні висоти з використанням сучасних супутникових технологій.

Ключові слова: система висот, модель квазігеоїда, нівелювання.

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