ADVANCES IN CYBER-PHYSICAL SYSTEMS

Vol. 6, Num. 1, 2021

MODEL OF PROCESS SYNCHRONIZATION IN THROUGH ANALYSIS

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https://doi.org/10.23939/acps2021.01.___
Submitted on 09.05.2021

Abstract – Synchronization of parallel processes of distributed information systems (DIS) has been largely determined by decisions taken at the stages of their design. Having already been in structural and functional models, when determining cause-and-effect relationships for events and actions in DIS components, it becomes necessary to coordinate them. In the proposed multilevel systemic, structural and functional synchronization model, a hierarchy of such causal relationships with interlevel mappings, inheritance and encapsulation of events and actions have been formed. The model has been also based on hierarchical extended Petri nets, which make it possible to represent various aspects of a special analysis of technical diagnostics, in particular, analysis of correctness, verification, testing, for the adopted display of the asynchronous-behavioral nature of the multilevel interaction of DIS processes. Features of the synchronization model include mapping operations for cross-level inheritance and encapsulations that synchronize events and actions, as well as end-to-end synchronized quasi-order relationships and compatibility for them. The synchronization model is also distinguished by the possibility of specializing its objects, operations and relations for the tasks of check and recognition of behavioral properties set for analysis and verification, basic in technical diagnostics, including in online and offline testing. The synchronization model has allowed one to determine the formal conditions for methods of end-to-end asynchronous coordination of events and actions of multi-level models, that represent design solutions for DIS, in particular, for technical diagnostics methods, and also to reduce the computational complexity of a special synchronization analysis due to an end-to-end decomposition approach. The dimension of the synchronization model has been estimated using the representation of Petri net graphs and special graphs of reachable states using list structures. The above estimates determine the limits of applicability of the formal synchronization model.

***Index Terms*: network information system, multilevel behavioral model, synchronization, extended hierarchical Petri net**

I. INTRODUCTION

Existing and promising distributed information systems (DIS), rapidly penetrating into all spheres of human activity [1, 2], acquire the properties of super-complex systems in terms of structural, functional and informational organization approaching in these indicators to socio-biological systems, but remaining to a greater extent mathematically formalized [3-5]. The growing responsibility of tasks solved with the help of DIS, which often puts them in the rank of critical, significantly increases the requirements for the reliability of the functioning of DIS [6, 7]. The consequence of these trends is the high importance of the design and operational efficiency of DIS, the achievement of which is possible through comprehensive modeling and analysis of the results of formal-mathematical, software and physical experiments with projects and implementations of DIS [8, 9], in particular, their technical diagnostics [10, 11], taking into account the peculiarities of synchronization of events and actions [12, 13] of asynchronous parallel processes.

Investigation of models and methods of DIS synchronization for the levels of the system, structural and functional [13-15], logical, schematic [16-18], design and technological [19-21] levels of projects, for different technologies for manufacturing implementations show the use of mechanisms for coordinating the processes of functioning based on various synchronous and asynchronous models and methods [22-24] associated with explicit and indirect [25-27], linear and nonlinear [29-31] representation of the model of process time and cause-and-effect relationships for events and actions in behavior DIS.

In particular, to analyze the causal relationships of behavioral events and actions of DIS components at different levels of representation of their models and implementations, general relations and operations of synchronization of events and actions are used quite widely, both independently and in an integrated way, for example, relations and operations of network and multilevel ordering inherent in the logical clock formalism [32-34].

The synchronization of processes can take into account not only the features of models and implementations of DIS, but also the features of the choice of their behavior in the case of solving various problems, specializing the set of corresponding events and actions [35-37]. So, identification, check, recognition, verification, testing, controllability and observability of events and actions of behavior impose their own restrictions on intra-level and inter-level agreements associated with their definitions of atomicity and transactionality of events and actions [38-40].

The construction of a consistent hierarchy of such relations for through transitions of adjacent model levels in DIS projects and implementations can be provided by the operations of interlevel mapping of the properties and functions of components during their synchronized detailing or generalization. In the case of a structural-functional multilevel behavioral approach, the relations and operations of synchronization of events and actions in such a hierarchy can have a model commonality for the asynchronous formalism of cause-and-effect relationships of logical clocks. Such an opportunity for analyzing the synchronization of behavior, in particular, is provided by extended hierarchical Petri nets, which have asynchrony, eventfulness, parallelism, complex functional predicate and information markup, multi-level [41-43], with limited controllability and observability of latent behavior, supplemented by the analysis of internal events and actions, which is realized on the basis of their external manifestations.

At the same time, it can be noted that in the well-known works devoted to synchronization, the issues of multilevel behavioral synchronization have not been sufficiently studied, often in relation to specific implementation technologies [44-46] and much less often at the general structural-functional level [47]. This circumstance is even more pronounced in the case of a special behavior of the DIS, when solving problems that specialize the set of corresponding events and actions, in particular, during technical diagnostics.

In this regard, it is possible to draw a conclusion about the relevance of the study of multilevel models of behavioral synchronization of DIS, taking into account the analysis of hidden behavior for problems of recognition, verification and check based on extended hierarchical Petri nets.

II. PURPOSE, PROBLEM STATEMENT

The purpose of the work is to determine the conditions for achieving greater completeness and accuracy in the behavioral synchronization of DIS processes, performed in check and recognition experiments for extended hierarchical Petri nets, with the provision of manifestation of internal events and actions in external behavior.

To achieve this purpose, the work solves the problem of constructing a model of behavioral synchronization of multilevel DIS processes for check and recognition, based on hierarchical extended Petri nets, which has the features of a hierarchy of coordinated quasi-order and compatibility relations, operations of interlevel mapping of coordinated properties and functions of DIS components, during their translation - detail or generalization, taking into account the recognition of external and internal events and actions. The model makes it possible to determine the conditions for behavioral synchronization of multilevel DIS processes, to reduce the computational complexity of synchronization analysis in comparison with single-level synchronization of DIS behavior.

III. CHOOSING A HIERARCHICAL MODEL AND ITS MAPPINGS

In multilevel behavioral synchronization of DIS processes, a hierarchical extended Petri net (HEPN) is adopted as a formal *IS* model [48]:

$IS = (S(f), \cup_{i\in I}S(f)_i^p, \cup_{i\in J}S(f)_i^t, Sg_{iS})$,

in its interlevel mappings as the senior systemic Petri net *S(f)* and the lower Petri subnets $\cup_{i\in I}S(f)_i^p$, $\cup_{j\in J}S(f)_j^t$, detailing positions, transitions and tokens, with the necessary synchronization-translation of alphabets, events, actions, interval-time properties of macro positions / macro transitions and subnets between them, based on the signature of operations of hierarchical mappings $Sg_{iS}=\{\chi^{\to p}, \chi^{p\to}, \nu^{\to t}, \nu^{t\to}\}$ [48].

The hierarchy satisfies the mappings $iS\subseteq IS$ $iS=(\cup_{i\in I}is_ip)\cup(\cup_{i\in J}is_it)=(\cup_{i\in I}(p_{\chi i},S(f)_ip,\{\chi_i\to p,\chi_ip\to\}))\cup$ $\cup(\cup_{i\in J}(t_{\nu i},S(f)_i^t,\{\nu_i^{\to t},\nu_i^{t\to}\}))$ for macro positions P_χ and macro transitions T_ν between adjacent levels of the model.

The two-level decomposition yields a detailed lower Petri net $S(f)+=((S(f)\backslash((\cup_{i\in I}p_i)\cup(\cup_{i\in J}t_i)))\cup$ $\cup((\cup_{i\in I}S(f)_i^p)\cup(\cup_{i\in J}S(f)_i^t))$, obtained from the highest net *S(f)* after the replacement mappings $Sg_{iS}=\{\chi^{\to p},\chi^{p\to},\nu^{\to t},\nu^{t\to}\}$ for detailing Petri subnets from $\cup_{i\in I}S(f)_i^p$, $\cup_{i\in J}S(f)_i^t$ instead of substituted positions $P_\chi=\cup_{i\in I}p_{\chi i}$ and transitions $T_\nu=\cup_{j\in J}t_\nu$ of the highest *S(f)*.

IV. INTERLEVEL SYNCHRONIZATION OF THE HIERARCHICAL MODEL

In the hierarchical asynchronous-event extended Petri net *IS*, internal synchronization is performed by relations of quasi-order ψ and compatibility ξ for events and actions, time intervals, probability coefficients. The relations ψ and ξ are extended for *S(f)* and $\cup_{i\in I}S(f)_i^p$, $\cup_{i\in J}S(f)_i^t$, as level internal relations of the quasi-order $\psi^+=\psi\cup(\cup_{i\in I}\psi_i^p)\cup(\cup_{i\in J}\psi_i^t)$ and compatibility $\xi^+=\xi\cup(\cup_{i\in I}\xi_i^p)\cup(\cup_{i\in J}\xi_i^t)$ events and actions, time intervals and probability coefficients for the lower Petri net $S(f)^+=S(f)\cup(\cup_{i\in I}S(f)_i^p)\cup(\cup_{j\in J}S(f)_j^t$. Compatibility of ξ^+ also includes indifference.

In the *IS* hierarchy, the Petri net *S(f)* and the Petri subnets $\cup_{i\in I}S(f)_i^p$, $\cup_{i\in J}S(f)_i$, are connected by the operations of interlevel mappings, that also perform interlevel synchronization. Moreover, the upper relations of the quasi-order ψ and compatibility ξ must also be inherited in the lower relations of the quasi-order $(\cup_{i\in I}\psi_i^p)\cup(\cup_{i\in J}\psi_j^t)$ and compatibility $(\cup_{i\in I}\xi_i^p)\cup(\cup_{i\in J}\xi_i^t)$ for the *iS* hierarchy.

Operations of the mappings $\{\chi^{\to p},\chi^{p\to},\nu^{\to t},\nu^{t\to}\}$ for $S(f)^+=S(f)\cup(\cup_{i\in I}S(f)_i^p)\cup(\cup_{i\in J}S(f)_i^t)$ in addition to the level relations of the quasi-order ψ^+ and compatibility ξ^+ (and with them the asynchronous-event time with time intervals as well) for the entities from *iS* assume their own conditions for the relations of hierarchical inter-level quasi-order $i\psi=(\cup_{i\in I}i\psi_i^p)\cup(\cup_{i\in J}i\psi_i^t)$ and compatibility $i\xi=(\cup_{i\in I}i\xi_i^p)\cup(\cup_{i\in J}i\xi_i^t)$ for positions P, $\cup_{i\in I}P_i^p$, and transitions T, $\cup_{i\in J}T_i^t$, events and actions represented by input X, $\cup_{i\in I}X_i^p$, $\cup_{i\in J}X_i^t$, and output Y, $\cup_{i\in I}Y_i^p$, $\cup_{i\in J}Y_i^t$ symbols, time intervals In, $\cup_{i\in I}In_i^p$, $\cup_{i\in J}In_i^t$, probability coefficients Pb, $\cup_{i\in I}Pb_i^p$, $\cup_{j\in J}Pb_i^t$ in the behavior of the *IS* hierarchy.

Replacement in the highest Petri net *S(f)* of macro positions P_χ and macro transitions T_ν when generating a hierarchical detailing Petri net of the form $S(f)^-=S(f)\backslash((\cup_{i\in I}p_{\chi i})\cup(\cup_{i\in J}t_{\nu i}))\cup(\cup_{i\in I}S(f)_i^p)\cup(\cup_{i\in J}S(f)_i^t)$ presupposes the preservation / inheritance of quasi-order relations ψ and compatibility ξ of the highest *S(f)* under the mappings $\{\chi^{\to p},\chi^{p\to},\nu^{\to t},\nu^{t\to}\}$ in relations of the quasi-order $\cup_{i\in I}\psi_i^p$, $\cup_{j\in J}\psi_j^t$ and compatibility $\cup_{i\in I}\xi_i^p$, $\cup_{j\in J}\xi_j^t$, detailing Petri subnets from $(\cup_{i\in I}S(f)_i^c)\cup(\cup_{j\in J}S(f)_j^t)$ and vice versa, asynchronous-event time intervals for net $S(f)$ and any subnet from $(\cup_{i\in I}S(f)_i^c)\cup(\cup_{j\in J}S(f)_j^t)$ are quasi-ordered in levels with respect to $\psi^+=\psi\cup(\cup_{i\in I}\psi_i^p)\cup(\cup_{j\in J}\psi_j^t)$ and between levels with respect to по $i\psi=(\cup_{i\in I}i\psi_i^p)\cup(\cup_{j\in J}i\psi_j^t)$, are compatible in levels by $\xi^+=\xi\cup(\cup_{i\in I}\xi_i^p)\cup(\cup_{j\in J}\xi_j^t)$ and between levels by $i\xi=(\cup_{i\in I}i\xi_i^p)\cup(\cup_{j\in J}i\xi_j^t)$.

The statement is true:

Statement. In hierarchical transitions $iS=(\cup_{i\in I}is_i^p)\cup(\cup_{j\in J}is_j^t)=(\cup_{i\in I}(p_{\chi i},S(f)_i^p,\{\chi_i^{\rightarrow p},\chi_i^{p\rightarrow}\}))\cup$ $\cup(\cup_{j\in J}(t_{\nu j},S(f)_j^t,\{\nu_j^{\rightarrow t},\nu_j^{t\rightarrow}\}))$ from $IS=(S(f),(\cup_{i\in I}S(f)_i^p),(\cup_{j\in J}S(f)_j^t,Sg_{iS}))$, when replacing macro positions P_χ and macro transitions T_ν of the older network $S(f)$ with detailing subnets from $(\cup_{i\in I}S(f)_i^p)\cup(\cup_{j\in J}S(f)_j^t)$, which is performed when the behavior is decomposed in *IS* in accordance with the mappings $\{\chi^{\rightarrow p},\chi^{p\rightarrow},\nu^{\rightarrow t},\nu^{t\rightarrow}\}$, the highest relations of the quasi-order ψ and compatibility ξ for events, actions, time intervals, probability coefficients from the network $S(f)$ are preserved / inherited in the lowest relations of the quasi-order $(\cup_{i\in I}\psi_i^p)$, $(\cup_{j\in J}\psi_j^t)$ and compatibility $(\cup_{i\in I}\xi_i^p)$, $(\cup_{j\in J}\xi_j^t)$ for events, actions, time intervals, probability coefficients from subnets $(\cup_{i\in I}S(f)_i^p)\cup(\cup_{j\in J}S(f)_j^t)$ and vice versa.

For each hierarchical transition $is\in iS$ from $iS=(\cup_{i\in I}is_i^p)\cup(\cup_{j\in J}is_j^t)$ of the *IS* hierarchy from $S(f)^+=(S(f)\cup(\cup_{i\in I}S(f)_i^p)\cup(\cup_{j\in J}S(f)_j^t)$ according to the relations ψ and ξ of the higher net $S(f)$ and the relations $(\cup_{i\in I}\psi_i^p)\cup(\cup_{j\in J}\psi_j^t)$ and $(\cup_{i\in I}\xi_i^p)\cup(\cup_{j\in J}\xi_j^t)$ of lower subnets $(\cup_{i\in I}S(f)_i^c)\cup(\cup_{j\in J}S(f)_j^t)$ the following conditions are accepted:

instantaneous occurrence-start of its event, actions, taking into account their time intervals and probability coefficients for the higher net $S(f)$ S (f) at a certain moment of asynchronous-event time corresponding to relations ψ, ξ and to mappings $\{\chi^{\rightarrow p},\chi^{p\rightarrow},\nu^{\rightarrow t},\nu^{t\rightarrow}\}$ to them of events, actions taking into account their time intervals and probability coefficients for the corresponding detailing subnets from $(\cup_{i\in I}S(f)_i^c)\cup(\cup_{j\in J}S(f)_j^t$;

the uniqueness of the occurrence-start of the event, the action, taking into account their time intervals and probability coefficients for the higher net $S(f)$ at some corresponding to the relations ψ and ξ, the moment of asynchronous-event time is inherited in the uniqueness of the occurrence-starts of the corresponding (by the mappings $\{\{\chi^{\rightarrow p},\chi^{p\rightarrow},\nu^{\rightarrow t},\nu^{t\rightarrow}\}$) events, actions taking into account their time intervals and probability coefficients for the corresponding detailing Petri subnets from $(\cup_{i\in I}S(f)_i^c)\cup(\cup_{j\in J}S(f)_j^t)$;

the occurrence-start of events, actions taking into account their time intervals and probabilistic coefficients for the higher net $S(f)$ at a certain moment of asynchronous-event time corresponding to the relations ψ and ξ with the possibility of saving it until a certain corresponding to the ratios ψ and ξ of the moment $\varpi+\zeta$, here ζ = 0,1,2,… are the numbers of subsequent events, actions taking into account their time intervals and probability coefficients associated with the initial occurrence-start of the events, actions at $\zeta = 0$, is inherited in the occurrences-starts of the corresponding (by the mappings $\{\{\chi^{\rightarrow p},\chi^{p\rightarrow},\nu^{\rightarrow t},\nu^{t\rightarrow}\}$) events, actions taking into account their time intervals and probability coefficients with common initial times $(\cup_{i\in I}\varpi_i)\cup(\cup_{j\in J}\varpi_j)$ and the possibility of their preservation to some common (total) for all sets of limiting moments $(\cup_{i\in I}\varpi_i+\zeta_i)\cup(\cup_{j\in J}\varpi_j+\zeta_j)$ for the corresponding detailing subnet from $(\cup_{i\in I}S(f)_i^c)\cup(\cup_{j\in J}S(f)_j^t)$;

the occurrence of non-persistent events, actions taking into account their time intervals and probability coefficients (for one moment of asynchronous-event time) and stored events, actions taking into account their time intervals and probability coefficients (for more than one moment of asynchronous-event time).

Synchronization of level relations $\psi^+=\psi\cup(\cup_{i\in I}\psi_i)\cup(\cup_{j\in J}\psi_j)$ and $\xi^+=\xi\cup(\cup_{i\in I}\xi_i)\cup(\cup_{j\in J}\xi_j)$, in hierarchical transitions of the *IS* hierarchy for $S(f)^+$ determines the options for synchronization of the higher and lower levels for the sets: a) input events $X'^+=X'\cup(\cup_{i\in I}X_i^{c}{}')\cup(\cup_{j\in J}X_j^{t}{}')$; b) output actions $Y'^+=Y'\cup(\cup_{i\in I}Y_i^{c}{}')\cup(\cup_{j\in J}Y_j^{t}{}')$; c) positions $P^+=P\cup(\cup_{i\in I}P_i^c)\cup(\cup_{j\in J}P_j^t)$ and their subsets $B(P^+)=B(P)\cup(\cup_{i\in I}B(P_i^c))\cup(\cup_{j\in J}B(P_j^t))=$ $Sp^+=Sp\cup(\cup_{i\in I}Sp_i^p)\cup(\cup_{j\in J}Sp_j^t)$; d) transitions $T^+=T\cup(\cup_{i\in I}T_i^c)\cup(\cup_{j\in J}T_j^t)$; e) time intervals $In^+=In\cup(\cup_{i\in I}In_i^c)\cup(\cup_{j\in J}In_j^t)$ for events and actions; f) probability coefficients $Pb^+=Pb\cup(\cup_{i\in I}Pb_i^c)\cup(\cup_{j\in J}Pb_j^t)$ for events X'^+ and actions Y'^+; g) multiplicities $Qu^+=Qu\cup(\cup_{i\in I}Qu_i^c)\cup(\cup_{j\in J}Qu_j^t)$ for events and actions; h) distributions of tokens $Ql^+=Ql\cup(\cup_{i\in I}Ql_i^c)\cup(\cup_{j\in J}Ql_j^t)$ for positions P^+ and their subsets, as well as transitions T^+.

Consequently, it is necessary to analyze and classify these sets in their specific incidence with Petri nets from $S(f)^+$, as well as to coordinate time intervals and probability coefficients to obtain a combined lower detailed subnets $S(f)^-=S(f)\backslash((\cup_{i\in I}p_{\chi i})\cup(\cup_{j\in J}t_{\nu j}))\cup(\cup_{i\in I}S(f)_i^p)\cup(\cup_{j\in J}S(f)_j^t)$.

As in the graphs of attainable states $G(S(f))$ [41-43], for all Petri nets from $S(f)^+=S(f)\cup(\cup_{i\in I}S(f)_i^p)\cup(\cup_{j\in J}S(f)_j^t)$, the sets of subsets of positions $Sp^+=Sp\cup(\cup_{i\in I}Sp_i^p)\cup(\cup_{j\in J}Sp_j^t)$, input and output for the corresponding transitions $T^+=T\cup(\cup_{i\in I}T_i^p)\cup(\cup_{j\in J}T_j^t)$. The subsets of positions Sp^+ are collections of containers, at the boundaries of which aggregates of events appear, and transitions T^+ are containers, at the boundaries of which actions for Petri nets from $S(f)^+$ are manifested.

In an arbitrary interlevel transition of the *IS* hierarchy for the higher Petri net $S(f)$ and detailing Petri subnets from $(\cup_{i\in I}S(f)_i^p)\cup(\cup_{j\in J}S(f)_j^t)$ for nets, detailing macro positions from $P_\chi=\cup_{i\in I}p_{\chi i}{}^p$ or transitions from $T_\nu=\cup_{j\in J}t_{\nu j}{}^t$ from $S(f)$ for subsets of positions and transitions, several options are possible.

So, for detailing a certain macroposition $p_{\chi i}{}^p$, the following cases are possible: a) $Sp\cap Sp_i^p=\varnothing$ and $T\cap T_i^p=\varnothing$; b) $Sp\cap Sp_i^p=\varnothing$ and $T\cap T_i^p\neq\varnothing$; c) $Sp\cap Sp_i^p\neq\varnothing$ and $T\cap T_i^p=\varnothing$; d) $Sp\cap Sp_i^p\neq\varnothing$ and $T\cap T_i^p\neq\varnothing$.

Case 1. $Sp\cap Sp_i^p=\varnothing$ и $T\cap T_i^p=\varnothing$. In this case, the net $S(f)$, the subnets $S(f)_i^p$, and the resulting lower detailed net $S(f)^-$, which is equivalent to *IS*, do not reduce the determinism of the incidence relations of positions and transitions (within its state preceding the hierarchical transition). As a consequence, the prior analysis of the higher network is not destroyed. Further, at the lower level for the lower detailed net $S(f)^-$ the analysis is reduced to the organization of the recognition experiment, without requiring any special actions, except for the synchronization of time intervals, probability coefficients and distributions of tokens in hierarchical inter-level relations of the quasi-order $i\psi=(\cup_{i\in I} i\psi_i^p)\cup(\cup_{i\in J} i\psi_i^t$ and compatibility $i\xi=(\cup_{i\in I} i\xi_i^p)\cup(\cup_{i\in J} i\xi_i^t)$. The relationship between the output alphabets Y and Y_i^p does not affect the previously performed check analysis.

Case 2. $Sp\cap Sp_i^p=\varnothing$ and $T\cap T_i^p\neq\varnothing$.. In this case, the net $S(f)$ and the subnets $S(f)_i^p$ do not create a new non-determinism in the specially extended Petri net, defining alternative input subsets of positions (and events) to trigger some transition (many-to-one incidence). The resulting detailing, equivalent with respect to IS, is the lower detailed net $S(f)^-$ - retains the determinism of the incidence relations of positions and transitions. Further, at the lower level, for the lower detailed net $S(f)^-$ the analysis is reduced to the organization of the recognition experiment, without requiring any special actions, except for the synchronization of time intervals, probability coefficients and distributions of tokens in relations of hierarchical inter-level quasi-order $i\psi=(\cup_{i\in I} i\psi_i^p)\cup(\cup_{i\in J} i\psi_i^t)$ and compatibility $i\xi=(\cup_{i\in I} i\xi_i^p)\cup(\cup_{i\in J} i\xi_i^t)$. The relationship between the output alphabets Y and Y_i^p does not affect the check analysis performed earlier.

Case 3. $Sp\cap Sp_i^p\neq\varnothing$ and $T\cap T_i^p=\varnothing$. In this case, a certain subset of positions (and events) for any $sp\in Sp\cap Sp_i^p$ can put one of several transitions from T and T_i^p, that are identical for sp to each other (incidence "one-to-many") to the ready-to-execute state. As a result, the net $S(f)$ and the subnets $S(f)_i^p$, as well as the resulting detailed lower net $S(f)^-$, which is equivalent to IS, reduce the determinism of the incidence relations of positions and transitions. The previous analysis is not destroyed, but for the lower detailed network $S(f)^-$, in addition to synchronization of time intervals, probability coefficients and distributions of tokens in the relations of hierarchical interlevel quasi-order $i\psi=(\cup_{i\in I} i\psi_i^p)\cup(\cup_{i\in J} i\psi_i^t)$ and compatibility $i\xi=(\cup_{i\in I} i\xi_i^p)\cup(\cup_{i\in J} i\xi_i^t)$, additional analysis of probabilistic statistics or top-down design actions, that provide it may be required, involving the output alphabets of actions Y and и Y_i^p and level relations of quasi-order ψ^+ and compatibility ξ^+, interlevel relations of quasi-order $i\psi=(\cup_{i\in I} i\psi_i^p)\cup(\cup_{i\in J} i\psi_i^t)$ and compatibility $i\xi=(\cup_{i\in I} i\xi_i^p)\cup(\cup_{i\in J} i\xi_i^t)$.

Case 4. $Sp\cap Sp_i^p\neq\varnothing$ and $T\cap T_i^p\neq\varnothing$. In this case, the conclusions of options 2 and 3 are combined. Some arbitrary subsets of positions (and events) for any $sp\in Sp\cap Sp_i^p$ can be put in a state of readiness for execution of several transitions from T and T_i^p, which are identical for sp, and alternate each other, including those from $T\cap T_i^p$ (many-to-many incidence). As a result, the net $S(f)$ and the subnets $S(f)_i^p$, as well as the resulting detailed lower detailed net $S(f)^-$, which is equivalent to IS, reduce the determinism of the incidence relations of positions and transitions due to the possible appearance of the multiplicity of pairs “subset of positions-transition” simultaneously present in the net $S(f)$ and subnets $S(f)_i^p$. The previous analysis can be broken, for the lower detailed network $S(f)^-$, in addition to the synchronization of time intervals, probability coefficients and distributions of tokens in the relations of hierarchical inter-level quasi-order $i\psi=(\cup_{i\in I} i\psi_i^p)\cup(\cup_{i\in J} i\psi_i^t)$ and compatibility $i\xi=(\cup_{i\in I} i\xi_i^p)\cup(\cup_{i\in J} i\xi_i^t)$, it may be necessary to narrow the analysis based on identifiers and recognition primitives, rather than just additional analysis of probabilistic statistics or top-down design actions that provide it, involving the output alphabets of actions Y and Y_i^p and level relations of the quasi-order ψ^+ and compatibility ξ^+, inter-level relations of the quasi-order $i\psi=(\cup_{i\in I} i\psi_i^p)\cup(\cup_{j\in J} i\psi_j^t)$ and compatibility $i\xi=(\cup_{i\in I} i\xi_i^p)\cup(\cup_{i\in J} i\xi_i^t)$.

In end-to-end design, you can have all four possibilities.

The first option can be considered as retaining properties (checkable) for end-to-end top-down design.

Partly retaining properties (partly checkable) with a minimum number of additional checks and no check redesign – the second and third possibilities separately.

Partially not preserving properties (not checkable) is the fourth combinable possibility, where the completeness of the previous analysis of the recognition of the higher net can be reduced, in which a case of redesign is required.

V. SYNCHRONIZATION MODEL DIMENSION

To estimate the dimension of the synchronization model, a representation of the Petri net graph $S(f)$, using doubly linked list structures, was chosen. Let $|P|=n_p$, $|T|=n_t$, $|M|=n_m$, $n=n_p+n_t+2n_m$. Here $2n_m$ are two fields with the index of the energy-loaded type label and their number), $|Ev|=n_e$, $|X|=n_x$, $|Ac|=n_a$, $|Y|=n_y$, where $X\subseteq Ev$ и $Y\subseteq Ac$. For the upper bound on the number of conditional fields, it turns out:

$$cS(f)=nt(2np+1t+1a+2m+2Addr)+np(2nt+1p+1e+2m+2Addr)=$$
$$=4npnt+(2m+2Addr)(nt+np)+1tnt+1ant+1pnp+1enp\cong$$
$$\cong 4npnt+6(nt+np).$$

The graph of reachable states $A_{S(f)}$ with the representation of a multiple for positions and parallel for transitions of the operation of the Petri net $S(f)$ includes 2^{nt} adjacent multi-transitions and 2^{np} adjacent multi-positions with their sets:

$$2n_p(2^{nt})-1=n_p2^{nt+1}-1,$$
$$2n_t(2^{np})-1=n_t2^{np+1}-1.$$

For all multi-transitions and multi-positions, the number of conditional cells and the maximum length, when searching is:

$$cell_{AS(f)}=2n_tn_p(n_t)+2n_pn_t(n_p)=2n_pn_t(n_p+n_t),$$
$$cell_{AS(f)multi}=n_p2^{nt+1}-1+n_t2^{np+1}-1=n_p2^{nt+1}+n_t2^{np+1}-2.$$

In the limit, the number of conditional cells and the maximum length, when searching are:

$$c_{AS(f)}=\ d_{AS(f)}=\ 2*6n_tn_p(n_t)+2*6n_pn_t(n_p)=12n_pn_t(n_p+n_t),$$
$$c_{AS(f)multi}=\ d_{AS(f)multi}=\ 6n_t(n_p2^{nt+1}-1)+6n_p(n_t2^{np+1}-1)=\ =3n_t(n_p2^{nt+2}-2)+3n_p(n_t2^{np+2}-2).$$

The estimates show the upper bounds for the application of the abstract synchronization model; their reduction is performed during the decomposition in the hierarchical Petri net IS.

VI. CONCLUSION

The paper presents the development of a formal model for the synchronization of DIS processes, based on the analysis of the asynchronous-event behavior of hierarchical enhanced Petri nets, the difference of which is the hierarchies of quasi-order and compatibility relations, interlevel mapping operations to coordinate the properties and functions of DIS components during their end-to-end translation - detailing or generalization in hierarchical Petri net.

The synchronization model makes it possible to determine the conditions in the behavioral synchronization of single-level and multi-level events and actions in the DIS processes, represented by the behavior of a hierarchical Petri net.

Due to the decomposition, the model makes it possible to reduce the computational complexity of synchronization analysis in comparison with single-level synchronization and can be used to construct methods of end-to-end behavioral synchronization of DIS processes.

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