MEANS FOR MEASURING THE THERMAL QUANTITIES

TWO-COLOR COMPENSATIVE THERMOMETRY WITH CORRECTED ADJUSTMENT USING NONLINEARITY EQUATION OF EMISSIVITY SPECTRAL DISTRIBUTION

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Abstract. The article is directed on metrological characteristics increase and extension of the optical thermometry field of use, including two-color compensative thermometry with a priori averaged adjustment.

The investigations have been performed for the tungsten. This metal studied in thermometry and metal optics has tabulated quantitative estimations of emissivity which are similar to the most widespread in metallurgy iron-carbon alloys. To increase the reliability and extend the field of use of obtained results, approximated and linearized spectral distributions of tungsten, as well as their mirror representations with decreasing and increasing, convex, linear, and concave distributions of emissivity have been researched.

The influence of qualitative and quantitative characteristics of the spectral distributions of emissivity on their nonlinearity coefficient has been studied. The equation of nonlinearity has been obtained. This equation connects the nonlinearity coefficient at the middle wave with the emissivity value at one of the boundary waves through the measured one-color radiation temperatures at 3 operating waves. With a priori knew quantitative estimates of the nonlinearity coefficient at the middle wave and measured one-color radiation temperatures, the obtained equation can be used for the calculation of emissivity values at the boundary waves. For example, in the linear spectral distributions of emissivity, the nonlinearity coefficient is equal to 0. The number of solutions for linear distributions of emissivity varies from 1 to 2, and for nonlinear – from 1 to 3.

The influence of measurement errors of one-color radiation temperatures at operating waves on the errors of emissivity determination by nonlinearity equation is established. The metrological advantages of two-color compensative thermometry using the emissivity values, corrected by the nonlinearity equation, are proved. It was found, that at the nonselective distribution of measurement errors of one-color radiation temperatures, measurement errors of the object temperature for two-color compensative, spectral ratio, and energy thermometry are insignificant for technical measurements. Under conditions of selective distribution of measurement errors of one-color radiation temperatures, these errors respectively are 0.04-0.25 %; 1.66-9.30 %; 0.18-0.34 %.

For nonlinear emissivity spectral distributions, real for tungsten and iron-carbon alloys, the methodical component due to the nonlinearity doesn't exceed 0.48 %, which is also acceptable for technical measurements. The method has been developed for practically acceptable conditions of primary pyrometric information obtaining.

Key words: Two-color compensative thermometry, Temperature, Spectral distribution of emissivity, Equation of nonlinearity.

1. Introduction

Optical thermometry has no alternative for continuous temperature measurements in high-temperature, including metallurgical, processes. Therefore, the first optical thermometers were created and implemented in metallurgy almost simultaneously with the development of the theory of thermal radiation.

At that time, the concept and term "radiation temperature" were not used, which led to the terms "conditional temperature" or "pseudo-temperature". They do not correspond to the physical content and definition of temperature in thermodynamics, molecular-kinetic theory, and statistical physics. For example, if we consider the fundamental direction of heat transfer by radiation, then a body with a lower "pseudo-temperature" under certain conditions can transfer energy to a body with a higher "pseudo-temperature" and heat it. This contradicts the definition of temperature in physics, which determines the direction of heat transfer.

Therefore, L. Zhukov has proposed to apply in optical thermometry a physically well-founded "radiation temperature" concept in due time [1]. The proposed concept is also followed by the authors in this article.

Classical energy radiation thermometry (ERT) and spectral ratio thermometry (SRT) in terms of instrumental errors are not worse than thermoelectric thermometry. These thermometry methods are widely applied for reference measurements, including the building of thermodynamic and practical temperature scales. Classical thermometry is used, without temperature corrections, in cases of the blackbody for ERT and black– and gray-bodies for SRT. Classical thermometry can be implemented for any colored bodies with stable optical characteristics with temperature corrections. With the increase of the operating length-waves number, the possibilities of optical thermometry expand. Thus, recently the investigations were reoriented on spectral and other "non-classical" technologies that improve the metrological characteristics of optical

thermometry under conditions of randomly changing optical characteristics of the radiating surface.

According to Wien's law, for 3 operating waves the system of pyrometric equations has the following form

$$\frac{1}{T} - \frac{1}{S_1} = \frac{\lambda_1}{C_2} \ln(\varepsilon_1);$$

$$\frac{1}{T} - \frac{1}{S_2} = \frac{\lambda_2}{C_2} \ln(\varepsilon_2); \qquad (1)$$

$$\frac{1}{T} - \frac{1}{S_3} = \frac{\lambda_3}{C_2} \ln(\varepsilon_3);$$

where S_1, S_2, S_3, K are the one-color radiation temperatures; λ_1, λ_2, λ_3, m are the operating length-waves of the pyrometric system; ε_1, ε_2, ε_3 is the object emissivity at the different operating length-waves; C_2=0,014388 K·m is the Plank's second constant; T is an object temperature.

The system (1) is not solved concerning T (has the infinite number of solutions), because 3 equations have 4 unknowns: ε_1, ε_2, ε_3, and T. The number of unknowns and equations can be balanced by the mutual replacement of the emissivity values using the dependent quantitative characteristics of the emissivity spectral distribution [1-4].

2. Disadvantages

Previously proposed such solutions demand high accuracy of the primary pyrometric information ($\delta S < 0.1 \delta T$), and also have the following disadvantages.

Linear symmetric-wave thermometry [1] is based on the $\varepsilon_2=(\varepsilon_1+\varepsilon_3)/2$ dependence for objects with linear $\varepsilon=f(\lambda)$, when operating waves are placed symmetrically on the spectrum. It excludes methodical errors of temperature measurements for objects with any linear emissivity spectral distributions and reduces them in other cases. However, the application of 3 fixed waves is not sufficient for the algorithmic determination of the system (1) correct solution.

In [3] three-color method of temperature measurement is considered. The method has the following disadvantages. A priori information about the spectral distribution of the object emissivity is required. According to this information, it is necessary to find λ_2, for which $\varepsilon_2=(\varepsilon_1+\varepsilon_3)/2$. The temperature is determined by the iteration method following the given formula. The proposed relationships complicate the analysis of the influence of measurement conditions on errors. Due to this, it is difficult to determine the application fields and metrological support of the proposed method.

As the development of [3], the method [4] is proposed, according to which the temperature is calculated by analytic formula without iteration method. However, to determine λ_2, where $\varepsilon_2 = \sqrt{\varepsilon_1 \cdot \varepsilon_3}$, except the case of $ln(\varepsilon)=f(\lambda)$ linear dependence, it is necessary to know a priori the quantitative characteristics of $\varepsilon=f(\lambda)$. If

we know these characteristics, it is possible to calculate T according to the pyrometric equations (1). There are also no requirements for the accuracy of λ_2 determination, as well as its connection with measurement error and the possibilities of modern optoelectronic technologies.

In [5] the directions and results of the development of spectral thermometric technologies and their metrological support are given. The authors argue that the "main errors" of primary pyrometric information obtaining in spectral thermometry should not exceed 0.3%. At the same time, it should be taken into consideration that the instrumental error of spectral thermometry depends on the distribution of errors of the primary pyrometric information over the operating spectral channels. For example, for any number of spectral channels, the relative error of spectral temperature measurements does not exceed the relative errors of radiation temperatures, in the case of their uniform distribution over the spectrum.

There are two known methods of double spectral ratio, based on the algorithm of primary pyrometric information processing for linear emissivity distributions. We have found that the used simplification, even for linear $\varepsilon=f(\lambda)$, leads to an additional component of the methodical error (-0.53 and -0.54 %). For objects with linear $\varepsilon=f(\lambda)$, this component is excluded only for gray distributions of ε.

The authors of [6] proposed a system of 3 equations. The system relates, through the emissivity, the brightnesses of the object on two operating waves with its temperature. An algorithm for the numerical solution of this system for T is given. But, in essence, the system of two equations and identity is proposed. It includes three unknowns. Therefore, the conclusion about the infinite number of solutions can be done.

To unambiguously determine the temperature of colored objects for the two-color thermometry, the relationship between emissivities on the operating waves is required. Such conditions take place for some production processes in metallurgy. For example, the ε_2 on ε_1 dependence for "galvanized steel specimens" was experimentally determined in [7]. The authors estimated the error of such measurements in "± 3 K on λ_1=1.6 μm and λ_2=3.0 μm in the temperature range 700-800 K".

There is a known three-color method. The main problem, that excludes its practical use, is the infinite number of solutions (for T) of the proposed pyrometric equations system.

Another 3-color method, parameters, and the concept of "relative emissivity $\varepsilon_f=f(\lambda, T_f)$" and "fictitious temperature T_f" have been also proposed. The method is purposed for the objects with linear emissivity distributions. The author states that the function

$$f_W(\lambda, T, T_f) = \exp(\frac{C_2}{\lambda}(\frac{1}{T_f} - \frac{1}{T}))$$

has no physical meaning.

Thermometry is a branch of physics and metrology. Therefore, to understand, evaluate, use and develop new solutions, it is necessary to physically justify the introduced parameters and terms. Also, to determine the correct solution ($T_f=T$) of the generalized equation, the author proposed to scan the registered radiation spectrum of the object by the operating range λ_1-λ_3 with all following consequences. He also does not provide an analysis of the errors of his method, but only compares its result (2504.5 K) with the result of the "reference method" (2504.7 K). As a "reference method", a previously developed method is used; its error is estimated at 0.88 K, with a confidence level of 0.95.

The authors of this article have proposed the method and algorithm of two-color compensative thermometry (TCCT) earlier. They are based on the established relationship between the measured and mirror two-color radiation temperatures of the object and its temperature. Performed in the conditions of selectively changing emissivity, the complex of investigations has confirmed much higher metrological characteristics of TCCT in comparison with SRT and ERT [8]. However, when the emissivity significantly non-selectively changes under conditions of a priori average adjustment, the errors of TCCT reach significant and even unacceptable values.

3. The aim of the work

The work is directed on enhancing the metrological characteristics of optical thermometry, including two-color compensative thermometry, in conditions of significant non-selective changes of emissivity.

4. Derivation of the nonlinearity equation of the emissivity spectral distribution

The method is based on the obtained nonlinearity equation of monotonous spectral emissivity distributions.

This equation determines the dependence of the nonlinearity coefficient Kn_2 [1] on the previously measured radiation temperatures of the object S_1, S_2, S_3, and the emissivity ε_1 or ε_3, respectively, at the operating λ_1, λ_2, λ_3, and boundary λ_1 or λ_3 waves. The physical sense of Kn and other quantitative characteristics of $\varepsilon=f(\lambda)$, i.e. the selectivity coefficient Ks and the average emissivity ε_{sr}, follows from Fig. 1.

$Kn_1=Kn_3=0$ at the boundary waves λ_1 and λ_3. For convex and concave spectral distributions of ε, $Kn_2>0$ and $Kn_2<0$, respectively, and for any linear distributions of ε, $Kn_2=0$.

The dependence of λ_2 on λ_1 and λ_3, which excludes the influence of T on Kn_2, is determined

$$\lambda_2 = f(\lambda_1, \lambda_3) = 2 / (\frac{1}{\lambda_1} + \frac{1}{\lambda_3}) \qquad (2)$$

Quite obviously, that the operating waves should be selected in the parts of the spectrum with the most stable optical characteristics, that do not violate the monotonicity of $\varepsilon=f(\lambda)$.

Kn_2 of the emissivity spectral distribution is determined by formula

$$Kn_2 = \varepsilon_2 - \varepsilon_{2l}, \qquad (3)$$

where ε_2 emissivity on λ_2, ε_{2l} linearized, in the spectral range λ_1-λ_3, emissivity on λ_2.

Formula for ε_2 can be obtained from equivalent emissivity definition $\varepsilon_{eq.} = \dfrac{\varepsilon_1 \cdot \varepsilon_3}{\varepsilon_2^2}$, i.e.

$$\varepsilon_2 = \sqrt{\frac{\varepsilon_1 \cdot \varepsilon_3}{\varepsilon_{eq.}}} \cdot \qquad (4)$$

ε_{2l} is expressed via ε_1 and ε_3 by the following expression

$$\varepsilon_{2l} = \varepsilon_1 + \frac{\varepsilon_3 - \varepsilon_1}{\lambda_3 - \lambda_1} \cdot (\lambda_2 - \lambda_1) \cdot \qquad (5)$$

Substituting (4) and (5) into (3), we obtain

$$Kn_2 = \sqrt{\frac{\varepsilon_1 \cdot \varepsilon_3}{\varepsilon_{eq.}}} - \varepsilon_1 - \frac{\varepsilon_3 - \varepsilon_1}{\lambda_3 - \lambda_1} \cdot (\lambda_2 - \lambda_1) \cdot \qquad (6)$$

$$Kn_2 = \varepsilon_2 - \varepsilon_{2l};$$
$$Kn_c = \varepsilon_c - \varepsilon_{c.l};$$
$$Ks = \frac{\varepsilon_1}{\varepsilon_3};$$
$$\varepsilon_{sr.} = \frac{\varepsilon_1 + \varepsilon_3}{2}$$
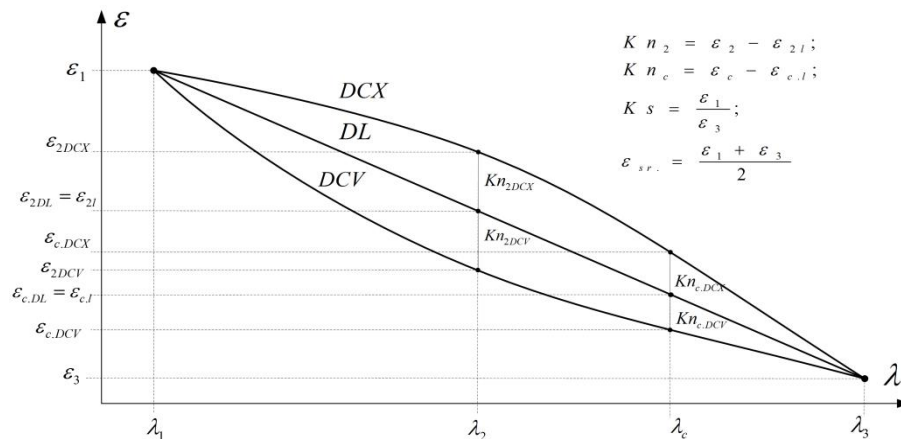
Figure 1. $\varepsilon=f(\lambda)$ for decreasing convex, linear and concave emissivity distributions in operating spectral range λ_1-λ_3.

To reduce equation (6) to one unknown ε_3, ε_1 can be expressed via ε_3 and color emissivity $\varepsilon_{col.} = \sqrt[\lambda_1 - \lambda_3]{\dfrac{\varepsilon_1^{\lambda_1}}{\varepsilon_3^{\lambda_3}}}$ [9]

$$\varepsilon_1 = \varepsilon_{col.}^{\frac{\lambda_1 - \lambda_3}{\lambda_1}} \cdot \varepsilon_3^{\frac{\lambda_3}{\lambda_1}} \quad (7)$$

Substituting formula (7) into (6), we can write the nonlinearity equation of the ε spectral distribution

$$K_{H_2} = f(\varepsilon_{eq.}, \varepsilon_{col.}, \varepsilon_3) = \sqrt{\dfrac{\varepsilon_{col.}^{\frac{\lambda_1 - \lambda_3}{\lambda_1}} \cdot \varepsilon_3^{1+\frac{\lambda_3}{\lambda_1}}}{\varepsilon_{eq.}}} -$$

$$- \varepsilon_{col.}^{\frac{\lambda_1 - \lambda_3}{\lambda_1}} \cdot \varepsilon_3^{\frac{\lambda_3}{\lambda_1}} - \dfrac{\varepsilon_3 - \varepsilon_{col.}^{\frac{\lambda_1 - \lambda_3}{\lambda_1}} \cdot \varepsilon_3^{\frac{\lambda_3}{\lambda_1}}}{\lambda_3 - \lambda_1} \cdot (\lambda_2 - \lambda_1). \quad (8)$$

To find the equation in the final form, it is necessary to express $\varepsilon_{eq.}$ and $\varepsilon_{col.}$ via S_1, S_2, S_3. At random value λ_2, Kn_2 can depend on T, because $\varepsilon_{eq} = f(E_1, E_2, E_3, T) = f(S_1, S_2, S_3, T)$. If we substitute (2) in (9), ε_{eq} dependence on T will be excluded. This can be demonstrated as follows

$$\varepsilon_{eq.} = \dfrac{\varepsilon_1 \cdot \varepsilon_3}{\varepsilon_2^2} = \dfrac{E_1 \cdot E_3}{E_2^2} / \dfrac{\lambda_1^{-5} \cdot e^{-\frac{C_2}{\lambda_1 \cdot T}} \cdot \lambda_3^{-5} \cdot e^{-\frac{C_2}{\lambda_3 \cdot T}}}{(\lambda_2^{-5} \cdot e^{-\frac{C_2}{\lambda_2 \cdot T}})^2}, \quad (9)$$

where E_1, E_2, E_3 object brightnesses on $\lambda_1, \lambda_2, \lambda_3$ in accordance with the Wien's law

$$E_1 = \varepsilon_1 \cdot C_1 \cdot \lambda_1^{-5} \cdot e^{-\frac{C_2}{\lambda_1 \cdot T}} = C_1 \cdot \lambda_1^{-5} \cdot e^{-\frac{C_2}{\lambda_1 \cdot S_1}}, \quad (10)$$

$$E_2 = \varepsilon_2 \cdot C_1 \cdot \lambda_2^{-5} \cdot e^{-\frac{C_2}{\lambda_2 \cdot T}} = C_1 \cdot \lambda_2^{-5} \cdot e^{-\frac{C_2}{\lambda_2 \cdot S_2}}, \quad (11)$$

$$E_3 = \varepsilon_3 \cdot C_1 \cdot \lambda_3^{-5} \cdot e^{-\frac{C_2}{\lambda_3 \cdot T}} = C_1 \cdot \lambda_3^{-5} \cdot e^{-\frac{C_2}{\lambda_3 \cdot S_3}}, \quad (12)$$

$C_1 = 3{,}7413 \cdot 10^{-16}$ W·m² Plank's first constant.

After logarithm of (9), we find the expression, which determines $\lambda_{eq.} = f(\lambda_1, \lambda_2, \lambda_3)$ and T influence on $\varepsilon_{eq.}$

$$\ln(\varepsilon_{eq.}) = \ln(\dfrac{E_1 \cdot E_3}{E_2^2}) + 5 \cdot \ln(\dfrac{\lambda_1 \cdot \lambda_3}{\lambda_2^2}) + \dfrac{C_2}{T \cdot \lambda_{eq.}}, \quad (13)$$

Under the condition of (2) $\lambda_{eq.} = 1 / (\dfrac{1}{\lambda_1} - \dfrac{2}{\lambda_2} + \dfrac{1}{\lambda_3}) \to \infty$, the temperature-dependent member in (13) $\dfrac{C_2}{T \cdot \lambda_{eq.}} \to 0$ and $\varepsilon_{eq.}$ does not depend on T.

After potentiating of (13) and substitution of (10-12) into (13), we obtain $\varepsilon_{eq.}$, expressed via S_1, S_2, S_3

$$\varepsilon_{eq.} = e^{\ln(\frac{E_1 \cdot E_3}{E_2^2}) + 5 \cdot \ln(\frac{\lambda_1 \cdot \lambda_3}{\lambda_2^2})} = e^{\frac{2 \cdot C_2}{\lambda_2 \cdot S_2} - \frac{C_2}{\lambda_1 \cdot S_1} - \frac{C_2}{\lambda_3 \cdot S_3}} \quad (14)$$

$\varepsilon_{col.}$ at the boundary waves, λ_1 and λ_3 is calculated via S_1 and S_3 [9]

$$\varepsilon_{col.} = e^{\frac{C_2 \cdot (\frac{1}{S_1} - \frac{1}{S_3})}{\lambda_3 - \lambda_1}} \quad (15)$$

After substitution of (14) and (15) in (8), we derive the nonlinearity equation of emissivity spectral distribution in final form

$$Kn_2 = f(S_1, S_2, S_3, \varepsilon_3) = e^{\frac{C_2}{2 \cdot \lambda_1 \cdot S_3} - \frac{C_2}{\lambda_2 \cdot S_2} + \frac{C_2}{2 \cdot \lambda_3 \cdot S_3}} \cdot \varepsilon_3^{\frac{1}{2} + \frac{\lambda_3}{2 \cdot \lambda_1}} -$$

$$- e^{\frac{C_2}{\lambda_1 \cdot S_3} - \frac{C_2}{\lambda_1 \cdot S_1}} \cdot \varepsilon_3^{\frac{\lambda_3}{\lambda_1}} - \dfrac{\varepsilon_3 - e^{\frac{C_2}{\lambda_1 \cdot S_3} - \frac{C_2}{\lambda_1 \cdot S_1}} \cdot \varepsilon_3^{\frac{\lambda_3}{\lambda_1}}}{\lambda_3 - \lambda_1} \cdot (\lambda_2 - \lambda_1). \quad (16)$$

Similarly to (8) and (16), variants of the nonlinearity equation, which connect Kn_2 with ε_1, have the following form

$$Kn_2 = f(\varepsilon_{eq.}, \varepsilon_{col.}, \varepsilon_1) = \sqrt{\dfrac{\varepsilon_1^{1+\frac{\lambda_1}{\lambda_3}}}{\varepsilon_{col.}^{\frac{\lambda_1 - \lambda_3}{\lambda_3}} \cdot \varepsilon_{eq.}}} - \varepsilon_1 -$$

$$- \varepsilon_1 - \dfrac{\dfrac{\varepsilon_1^{\frac{\lambda_1}{\lambda_3}}}{\varepsilon_{col.}^{\frac{\lambda_1 - \lambda_3}{\lambda_3}}} - \varepsilon_1}{\lambda_3 - \lambda_1} \cdot (\lambda_2 - \lambda_1) \quad (17)$$

$$Kn_2 = f(S_1, S_2, S_3, \varepsilon_1) = \dfrac{\varepsilon_1^{\frac{1}{2} + \frac{\lambda_1}{2 \cdot \lambda_3}}}{e^{\frac{C_2}{\lambda_2 \cdot S_2} - \frac{C_2}{2 \cdot \lambda_3 \cdot S_1} - \frac{C_2}{2 \cdot \lambda_1 S_1}}} -$$

$$- \varepsilon_1 - \dfrac{\dfrac{\varepsilon_1^{\frac{\lambda_1}{\lambda_3}} - \varepsilon_1 \cdot e^{\frac{C_2}{\lambda_3 \cdot S_3} - \frac{C_2}{\lambda_3 \cdot S_1}}}{e^{\frac{C_2}{\lambda_3 \cdot S_3} - \frac{C_2}{\lambda_3 \cdot S_1}} \cdot (\lambda_3 - \lambda_1)}}{} \cdot (\lambda_2 - \lambda_1). \quad (18)$$

At a priori known quantitative estimates of Kn_2 and measured S_1, S_2, S_3 following equations (16) and (18) one can calculate ε_3 and ε_1. For example, for objects with any linear $\varepsilon = f(\lambda)$, Kn_2 in (16) and (18) is equal to 0.

## 5. The number of solutions of the nonlinearity equation. Determination of the right solution

The nonlinearity equation in its various variants (8, 16, 17, 18) cannot be analytically solved concerning ε_3 and ε_1. The equation has the only numerical solution. For this purpose, it is necessary to set the values of ε_3 and ε_1 in the range of their a priori known values or in the absence of such information – in the range from 0 to 1. The classic question for the iteration method is the question about multiple solutions. I.e. the numerical values of Kn_2 can be the same for different qualitative and quantitative characteristics of $\varepsilon = f(\lambda)$. Hence we need to study the influence regularities of these characteristics on Kn_2 (Fig. 2).

Taking into account the quantitative characteristics of $\varepsilon = f(\lambda)$ (from Fig. 1), Table 1 demonstrates the classification of possible emissivity spectral distributions of the temperature control objects. The classification has been performed from the position of three-color thermometry.

The investigations have been performed for tungsten [10, 11], the most studied in optics of metals. It has tabulated quantitative estimates of ε, and emissivity spectral distribution similar to iron-carbon alloys, the most common in metallurgy. To increase the reliability and widen the implementation field of the obtained results, we have studied the approximate spectral distributions of tungsten, as well as their mirror representations with decreasing and increasing convex, linear, concave $\varepsilon = f(\lambda)$ [1]. The grey distribution of tungsten and thermodynamically equilibrium distribution have also been considered (Table 2).
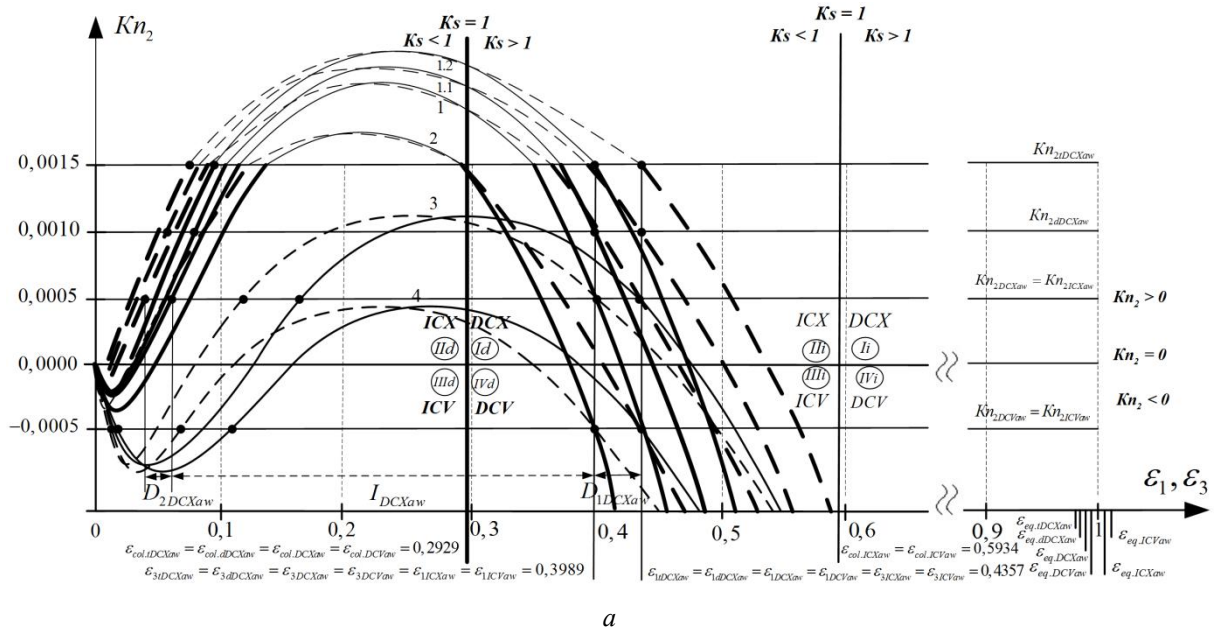
dependences $Kn_2=f(\varepsilon_3)$, **– – –** dependences $Kn_2=f(\varepsilon_1)$ at $\varepsilon_{eq.}$, $\varepsilon_{col.}$, calculated for:
1.2 – tDCXaw-distribution; 1.1 – dDCXaw-distribution; 1 – DCXaw-distribution; 2 – DCVaw-distribution; 3 – ICXaw-distribution; 4 – ICVaw-distribution (Table 2)
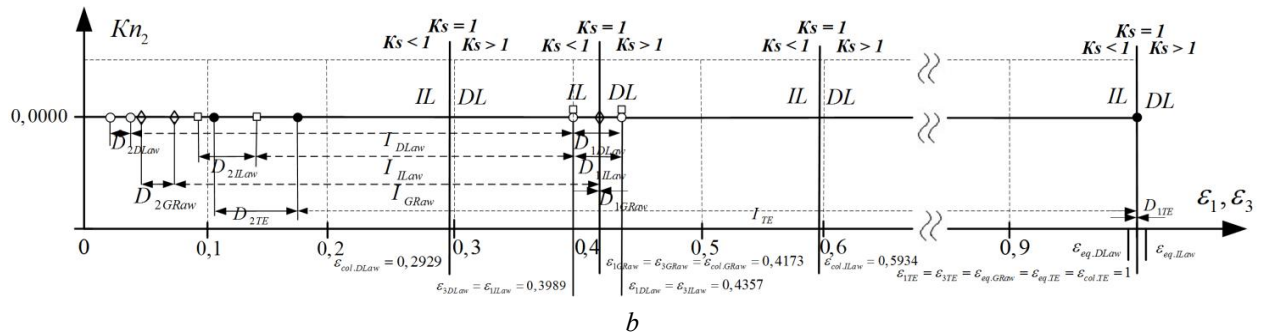
solutions of equations (8), (17) for:
○ – DLaw-distribution; □ – ILaw-distribution; ◇ – GRaw-distribution; ● – TE-distribution.

Figure 2. The influence regularities of qualitative and quantitative characteristics of spectral real and derivative emissivity distributions of tungsten on Kn_2 (λ_1=0,700 μm, λ_2=0,788 μm, λ_3=0,900 μm, T=1600 K).

Table 1

Types of emissivity spectral distributions, their notations, and quantitative characteristics.

Qualitative characteristics of ε spectral distributions			Quantitative characteristics of ε spectral distributions		
Object	Type of ε spectral distribution	Notation of ε spectral distribution	Kn_2	Ks	$\varepsilon_{sr.}$
Grey	Grey	GR	0	1	$0 < \varepsilon_{sr.} < 1$
	Thermodynamically equilibrium	TE	0	1	1
Colored	Decreasing convex	DCX	>0	>1	$0 < \varepsilon_{sr.} < 1$
	Decreasing linear	DL	0	>1	
	Decreasing concave	DCV	<0	>1	
	Increasing convex	ICX	>0	<1	
	Increasing linear	IL	0	<1	
	Increasing concave	ICV	<0	<1	

Table 2

Investigated emissivity spectral distributions and nonlinearity equation solutions at given Kn_2.

Qualitative and quantitative characteristics of ε spectral distributions							Solutions of equations (8, 17)		
Notation	ε_1	ε_2	ε_3	Kn_2	ε_{eq}	$\varepsilon_{col.}$	First $\varepsilon_{1.1} = \varepsilon_1$; $\varepsilon_{3.1} = \varepsilon_3$	Second $\varepsilon_{1.2}$; $\varepsilon_{3.2}$	Third $\varepsilon_{1.3}$; $\varepsilon_{3.3}$
tDCXaw	0.4357	0.4211	0.3989	0.0015	0.98012	0.29291	0.4357; 0.3989	0.0773; 0.0997	-
dDCXaw	0.4357	0.4206	0.3989	0.0010	0.98246	0.29291	0.4357; 0.3989	0.0554; 0.0803	-
DCXaw	0.4357	0.4201	0.3989	0.0005	0.98480	0.29291	0.4357; 0.3989	0.0380; 0.0599	-
DLaw	**0.4357**	**0.4196**	**0.3989**	**0.0000**	**0.98714**	**0.29291**	**0.4357; 0.3989**	**0.0205; 0.0370**	**-**
DCVaw	0.4357	0.4191	0.3989	-0.0005	0.98950	0.29291	0.4357; 0.3989	-	-
ICXaw	0.3989	0.4155	0.4357	0.0005	1.00672	0.59336	0.3989; 0.4357	0.1247; 0.1764	-
ILaw	**0.3989**	**0.4150**	**0.4357**	**0.0000**	**1.00915**	**0.59336**	**0.3989; 0.4357**	**0.0919; 0.1391**	**-**
ICVaw	0.3989	0.4145	0.4357	-0.0005	1.01159	0.59336	0.3989; 0.4357	0.0579; 0.0971	0.0021; 0.0074
GRaw	**0.4173**	**0.4173**	**0.4173**	**0.0000**	**1.00000**	**0.41730**	**0.4173; 0.4173**	**0.0435; 0.0718**	**-**
TE	**1.0000**	**1.0000**	**1.0000**	**0.0000**	**1.00000**	**1.00000**	**1.0000; 1.0000**	**0.1041; 0.1722**	**-**

According to the nonlinearity equation in its two variants (8, 17), Kn_2 depends on ε_3 and ε_1, as well as on ε_{eq} and ε_{col}. Therefore, the regularities have been studied for 4 main pairs of ε_{eq} and $\varepsilon_{col.}$, calculated by the specified emissivity distributions (DCXaw, DCVaw, ICXaw, ICVaw). Besides, for the first pair of ε_{eq} and $\varepsilon_{col.}$, which corresponds to the DCXaw-distribution, the influence of ε_2 on Kn_2 have been researched. For this purpose 2 additional pairs of ε_{eq} and $\varepsilon_{col.}$ for tDCXaw– and dDCXaw-distributions have been introduced. Intersections of the graphs of equations (8, 17) and the corresponding levels of Kn_2 (that Kn_2 values of given emissivity distributions, for which ε_{eq} and $\varepsilon_{col.}$ have been calculated) determine the number of solutions of equations (8, 17). One solution means a pair of ε_1, ε_3. If we have more than 1 solution, ranges D of ε_1, ε_3, and intervals I between pairs of ε_1, ε_3 can be determined. Moreover, for ICVaw– and DCVaw-distributions there are 3 and 1 solutions, respectively (Fig. 2a, Table 2).

Plane in the coordinate system $Kn_2-\varepsilon_1,\varepsilon_3$ by values $Kn_2=0$ and $\varepsilon_{col.}=0.2928$, as well as $Kn_2=0$ and $\varepsilon_{col.}=0.5934$ is divided into 4 quadrants. In these quadrants $Kn_2=f(\varepsilon_3)$, $Kn_2=f(\varepsilon_1)$ dependences, obtained at ε_{eq} and $\varepsilon_{col.}$ for decreasing and increasing emissivity distributions, respectively, are situated. To build continuous dependences in the range of ε_1, ε_3 from 0 to 1, it is necessary to utilize the appropriate fixed pairs of ε_{eq} and $\varepsilon_{col.}$. To comply with the requirement of constancy of the ε_{eq} and $\varepsilon_{col.}$ pair, in each quadrant the

corresponding types of emissivity distributions are denoted. For example, pair of ε_{eq} and $\varepsilon_{col..}$, calculated for DCXaw-distribution, can be computed not only on DCXaw-distribution but also on other types of distributions which have various qualitative and quantitative characteristics: DCX (I quadrant), GR (at $\varepsilon_1=\varepsilon_3$), ICX (II quadrant), IL (at $Kn_2=0$ between II and III quadrants), ICV (III quadrant), DCV (IV quadrant) and DL (at $Kn_2=0$ between IV and I quadrants). This can be concluded from the formulae for ε_{eq} and $\varepsilon_{col.}$.

To unload Fig. 2a, the main one-dimensional case at $Kn_2=0$, which is the subject of this article, is illustrated by Fig. 2b. Solutions of the nonlinearity equation in its two variants (8, 17) have been investigated for DL-, IL-, GR– and TE-distributions. For each of these distributions, the corresponding value of $\varepsilon_{col.}$ divides the field of solutions (values of ε_1, ε_3) on the left and right parts. Moreover, the ε_1 and ε_3 in the left and right parts give the same value of $\varepsilon_{col.}$ respectively for IL– and DL-distributions.

For example, there are 2 solutions for DLaw– and ILaw-distributions: $D_{1DLaw}=0.4357-0.3989$, $D_{2DLaw}=0.0205-0.0370$ and $D_{1ILaw}=0.3989-0.4357$, $D_{2ILaw}=0.0919-0.1391$. Intervals between these solutions respectively are $I_{DLaw}=0.3619$ and $I_{ILaw}=0.2598$. For GRaw-distribution: $D_{1GRaw}=0.4173-0.4173$, $D_{2GRaw}=0.0435-0.0718$, $I_{GRaw}=0.3455$. For TE-distribution: $D_{1TE}=1.0000-1.0000$, $D_{2TE}=0.1041-0.1722$, $I_{TE}=0.8278$. The ranges and intervals depend more significantly on the ε_1, ε_2, ε_3 values of the investigated linear

distributions and are less related to the qualitative characteristics of the distributions.

The ranges and intervals in combination with additional a priori information about the quantitative characteristics (Ks and $\varepsilon_{sr.}$) of the linear emissivity distributions allow the correct solution to be chosen. For example, for the DLaw-distribution, such identification can be reliably done because the correct pair of ε_1, ε_3 is situated in the field of the corresponding to tungsten tenths parts of ε, and the incorrect one is in field of hundredths parts of ε. Also, the correct solution is placed in the part of the DL-distributions. In general, for DL-distributions, when Ks increases (>1), D_1 expands, D_2 narrows, and I widens. For IL-distributions, when Ks decreases (<1), D_1 and D_2 expand, and I becomes smaller.

Depending on the qualitative and quantitative characteristics of the emissivity distributions, including linear ones, both solutions may be located in the field of really possible values of ε_1 and ε_3. In addition, both of these solutions can be situated in the same part of the solution field (at the left of $\varepsilon_{col.}$, i.e. in the part of IL-distributions). For example, for the IL-distribution with ε_1=0.4670, ε_3=0.5675 "correct" D_1=0.4670-0.5675, "incorrect" D_2=0.2811-0.3823, I=0.0847. The component of the error due to the choice of the incorrect pair of ε_1, ε_3 reaches 4.11% at T=1600 K. Partial or complete overlap of D_1 and D_2 is possible, in case of IL-distributions, depending on Ks and ε_{sr}. This feature can be explained due to $\varepsilon_{eq.}$ and $\varepsilon_{col.}$ for IL-distributions always exceed the values of $\varepsilon_{eq.}$ and $\varepsilon_{col.}$ for GR– and DL– distributions. $\varepsilon_{eq.}$ and ε_{col} correspondingly, through the equation of nonlinearity at Kn_2=0, determine the positions of D_1 and D_2. There is a single solution (D_1=D_2, $\varepsilon_{1.1}=\varepsilon_{1.2}; \varepsilon_{3.1}=\varepsilon_{3.2}$) at the full overlap. If intervals are sufficient and a priori information about Ks and $\varepsilon_{sr.}$ is available, one can determine the correct solution by analytical approach.

To unambiguously choose the correct solution, for example, with the incomplete overlap of D_1 and D_2, insufficient I between them and insufficient a priori information about Ks and $\varepsilon_{sr.}$, an algorithmic method of an additional wave (a comparison wave λ_c) has been proposed. λ_C (Fig.1) is placed between λ_2 and λ_3 or λ_1 and λ_2 in compliance with the above requirements for λ_1, λ_2, λ_3. For variant «between λ_2 and λ_3», $\varepsilon_{col.~(\lambda C-\lambda 3)}$ is calculated through previously measured one-color radiation temperatures S_c and S_3

$$\varepsilon_{col.(\lambda c-\lambda 3)} = e^{\dfrac{C_2\cdot\left(\frac{1}{S_c}-\frac{1}{S_3}\right)}{\lambda_3-\lambda_c}} . \qquad (19)$$

From the obtained two solutions we choose, for example, the first. The pair ($\varepsilon_{1.1}; \varepsilon_{3.1}$) corresponds to this first solution. For chosen pair ($\varepsilon_{1.1}; \varepsilon_{3.1}$) and linear emissivity distribution, we can calculate $\varepsilon_{c.L1}$

$$\varepsilon_{c.L1} = \varepsilon_{1.1} + \frac{\varepsilon_{3.1}-\varepsilon_{1.1}}{\lambda_3-\lambda_1}\cdot(\lambda_c-\lambda_1) + K н_{c.l}, \qquad (20)$$

where $K н_{c.l} = 0$ is the nonlinearity coefficient of $\varepsilon=f(\lambda)$ on λ_C.

Using the value of $\varepsilon_{c.L1}$, obtained from (20), we can determine $\varepsilon_{col.p(\lambda C-\lambda 3)}$

$$\varepsilon_{col.p(\lambda c-\lambda 3)} = \lambda_c-\lambda_3\sqrt{\frac{\varepsilon_{c.L1}^{\lambda_c}}{\varepsilon_{3.1}^{\lambda_3}}} . \qquad (21)$$

The difference between expressions (19) and (21) can be find

$$\Delta\varepsilon_{col.} = \varepsilon_{col.p(\lambda c-\lambda 3)} - \varepsilon_{col.(\lambda c-\lambda 3)} . \qquad (22)$$

If $\Delta\varepsilon_{col.} = 0$, the chosen solution ($\varepsilon_{1.1}; \varepsilon_{3.1}$) is right. If $\Delta\varepsilon_{col.} \neq 0$, the other solution (pair ($\varepsilon_{1.2}; \varepsilon_{3.2}$)) should be right.

According to the right solution with the specified values of ε_1, ε_3 and the measured S_1, S_2, S_3, we determine the T. The pyrometric equations of TCCT, SRT, and ERT can be applied for such calculations [8].

Previously performed investigations of the TCCT with a priori average adjustment have demonstrated the metrological advantages of this method. Under conditions of selectively variable emissivity, the TCCT errors are in 4.4-40.2 times lower than the classical thermometry errors. The source of instrumental errors of the compared thermometry methods is the errors of measurements of S_1, S_2, S_3. At the current level of optical thermometry, it is quite possible to measure S_1, S_2, S_3 with errors δS_1, δS_2, δS_3, which do not exceed 0.2 %. The investigations have also been performed for measurement errors of 0.5 and 1.0% and different distributions of their signs and modules.

Analysis of investigations results for DLaw-distribution shows the following. In case of increasing accuracy of S_1, S_2, S_3 measurements and nonselective distribution of δS_1, δS_2, δS_3 advantages and disadvantages of the studied methods are leveled, as their errors are in the range of insignificant for technical measurements. For example, for $\delta S_1=\delta S_2=\delta S_3$=0.2 %, $\delta S_1=\delta S_2=\delta S_3$=0.5 % and $\delta S_1=\delta S_2=\delta S_3$=1.0 % the errors are respectively within 0.005-0.019 %, 0.014-0.048 % and 0.028-0.095 %. However, when selective distributions of δS take place, the measurement errors for TCCT, SRT, and ERT respectively are 0.04-0.25 %, 1.66-9.30 % i 0.18-1.34 %. If the emissivity distribution changes from DLaw to DCXaw, the methodical component due to the nonlinearity of $\varepsilon=f(\lambda)$ does not exceed 0.48%, which is also acceptable for technical measurements.

6. Conclusions

As a result of investigations, directed on improving the metrological characteristics and widening optical thermometry field of application, including two-color compensative thermometry, under conditions of significant nonselective changes of emissivity, the nonlinearity equation of emissivity spectral distribution

is obtained. With a proper known nonlinearity coefficient on the middle wave, the equation allows the values of emissivity on boundary waves to be determined. The methods of identification of the equation correct solution have been developed.

The equation of nonlinearity has been used to correct emissivity values. It has been proved, that under conditions of nonselective measurement errors of one-color radiation temperatures, advantages and disadvantages of two-color compensative, spectral ratio, and energy thermometry are leveled, as their errors are in the range of insignificant for technical measurements.

For selective distributions, the measurement errors of two-color compensative thermometry are lower 5-42 times than the errors of classical thermometry. For nonlinear distributions of emissivity, real for tungsten and iron-carbon alloys, the methodical component due to nonlinearity does not exceed 0.48%. The method is implemented based on practically achievable accuracy of primary pyrometric information obtaining.

7. Gratitude

8. Conflict of interest

The authors declare that there is no financial or other possible conflict related to the current paper.

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