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SOUND TRANSMISSION PROPERTIES OF COMPOSITE LAYERED STRUCTURES IN THE LOWER FREQUENCY RANGE

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Abstract. This study aims to predict the sound transmission properties of composite layered plates structures in the lower frequency range. In present paper a novel procedure to derive the sound isolation parameters for layered plates is under discussion. This paper presents a new stress analysis method for the accurate determination of the detailed stress distributions in laminated plates subjected to cylindrical bending. Some approximate methods for the stress state predictions for laminated plates are presented here. The theoretical model described here incorporates deformations of each sheet of the lamina, which account for the effects of transverse shear deformation, transverse normal strain-stress and nonlinear variation of displacements with respect to the thickness coordinate. The main advantage of the present method is that it does not rely on strong assumptions about the model of the plate. Comparison with the Timoshenko beam theory is systematically made for analytical and approximation variants. The geometrical and mechanical parameters dependent frequency response functions and damping are presented for an arbitrary layered beam. The effective stiffness constants of equivalent to lamina Timoshenko beam (TB) and their damping properties have been determined by using a procedure based on multi-level numerical schemes and eigenfrequencies comparison. Numerical evaluations obtained for the vibration of the equivalent TB have been used to determine the sound transmission properties of laminated composite beams with the system of dynamic vibration absorbers (DVA's). The optimization of beams – DVA's system sound absorption properties is performed in the low frequency range. The results have shown that the presence of a DVA causes a decrease in the sound transmission in the low-frequency range. The extension of the present approach to various layered plates with various DVA's systems will be performed in order to obtain optimal sound insulation.

Keywords: composite materials, laminated plates, elastic moduli, damping, eigenfrequencies, sound insulating parameters, plate model, Timoshenko beam.

Introduction

Structures composed of laminated materials are among the most important systems used in modern engineering and, especially, in the automobile and aerospace industries. Aircraft such as the Airbus A340 and Boeing 787 Dreamliner make extensive use of laminated composite materials. Such lightweight and highly reinforced structures are also being used increasingly in civil, mechanical and transportation engineering applications. The rapid increase in the industrial use of these structures has necessitated the development of new analytical and numerical tools that are suitable for the analysis and study of the mechanical behavior of such structures. The determination of stiffness parameters for complex materials such as fiber-reinforced composites is much more complicated than for isotropic materials, since composites are anisotropic and non-homogeneous. Many different approaches have now been produced for the identification of the physical parameters which directly characterize structural behavior.

Considering their light weight and high strength they become more used also in civil engineering, road transport and mechanical engineering. To model the composite laminated plates, it is important to have an effective general theory to precisely assess the effects of transverse shear stress on work of the plate. To achieve higher vibration and noise reduction settings it is appropriate to apply the plate which attached to the DVA.

Knowledge of the sound transmission properties of structures composed of laminated materials including aircraft, vehicle and ship cabin walls and building walls is also important in order that occupants can be protected from external noise sources. Many such structures are comprised of beam and plate like elements. The vibration of beam and plate systems can be reduced by the use of passive damping, once the system parameters have been identified [1–6]. In some cases of forced vibration, the passive damping that can be provided is insufficient and the use of active damping has become attractive. The rapid development of micro-processors and control algorithms has made the use of active control feasible in some practical situations [7]. In most cases, however, passive control is preferred to reduce vibration and sound transmission through structures.

The purpose of this paper is analyze noise insulation performance of an arbitrary layered structure, among them sandwich plates. Nowadays, sandwich structure can be made with different core materials and shapes for particular applications in aircrafts, ships and buildings, such as foam cores [1], corrugated cores [2], textile core [3], aluminum honeycomb cores [4], truss cores [5] and lattice cores [6]. A great deal of study (7–15) had proved that sandwich panel can be used to minimize the noise transmission of external*sources. Kurtze and Watters [7] firstly undertook a pioneering acoustic analysis of sandwich panel and obtained transmission loss (TL) assuming the core incompressible. Since then, there had been continuous effort (by [8–14] directed at improving analytical approaches of sound transmission characteristic through sandwich panel.

Noise and vibration are of concern with many mechanical systems including: industrial machines, home appliances, surface vehicle transportation systems, aerospace systems, and building structures. Many such mechanical system components are comprised of beam and plate like elements and the vibration of such systems is the focus of several detailed recent studies. The vibration of beam and plate structural systems can be reduced by the use of passive damping, once the system parameters, such as dynamic stiffness of the plate or beam, have been identified. This is the subject of the research described in the present paper. In some cases of forced vibration, the passive damping that can be provided is insufficient and the use of active damping has become attractive. Active damping is mostly only used with high first cost items such as automobiles and aircraft, since it is still too expensive to use with low cost items such a household appliances.

Since the late 1950s, many papers have been published on the vibration of sandwich structures [15–18]. All of the models discussed so far are based on the following assumptions: a) the viscoelastic layer undergoes only shear deformation and hence the extensional energy of the core is neglected; b) the face sheets are elastic and isotropic and their contribution to the shear energy is neglected, and c) in the face-sheets, plane sections remain plane and normal to the deformed centre lines of the face-sheets. However, as the frequency increases, the results calculated from these models disagree strongly with measurements.

For modeling laminated composite plates, it is important to have an effective general theory for accurately evaluating the effects of transverse shear stresses on the plate performance. It has long been recognized that higher- order laminated plate theories may provide an effective solution tool for accurately predicting the deformation behavior of composite laminates subjected to bending loads [19–28]. It is well known that higher-order theories, which account for transverse shear and transverse normal stresses, generally provide a reasonable compromise between accuracy and simplicity although they are usually associated with higher-order boundary conditions that are difficult to interpret in practical engineering applications. Simple theories for laminated plates are most often incapable of determining the three-dimensional (3-D) stress field in the laminates. Thus, the analysis of composite laminates may require the use of a laminate-independent theory or a 3-D elasticity theory. Exact 3-D solutions [29–31] have shown the fundamental role played by the continuity conditions for the displacements and the transverse stress components at the interfaces between two adjacent layers for making an accurate analysis of multilayered composite thick plates. Further, these elasticity solutions demonstrate that the transverse normal stress plays a predominant role in these analyses. However, accurate solutions based on 3-D elasticity theory are often intractable. The limitation in the analysis which based on the displacement formulation has motivated some recent researches in which it has been used the theorem of mixed variation for the dynamic analysis of multilayered plates [32, 33].

A review of refined theories of laminated composite plates has been presented at [34–36]. Damping analysis of composite materials and structures has been presented at [37–48]. A review of vibration damping in sandwich composite structures has been presented in [49].

But this classical refined theory has some limitations. For comparatively thick plates it is not sufficiently precise. Other limitations of this theory are found with investigations not only with freely supported plates, but also rigidly or elastically clamped ones. This is caused by the assumption in this theory of zero free surface stresses. For rigidly clamped plates, the exact 3-D solution near the clamped edges is not submerged. The unification of formulation of schemes of calculations, which order of equalizations is unreserved, are offered in (the arbitrary number of approximations which are examined on the thickness of plate) [50–53]. This study aims to predict the elastic and damping properties of composite laminated plates. The present method for the modeling of laminated composite plates does not rely on strong assumptions about the model of the plate. In this paper, numerical evaluations obtained for vibrations in isotropic, orthotropic and composite laminated plates have been used to determine the displacement field for the efficient analysis of vibrations in laminated composite plates. The numerical method developed follows a semi-analytical approach with an analytical field applied in the longitudinal direction and a layer-wise displacement field employed in the transverse direction. A semi-analytical method has been developed to obtain the natural frequencies of vibration of simply-supported and clamped laminated composite plates. Further, these models have been formulated by considering a local Cartesian coordinate system at the mid-surface of each individual layer. Six degrees of freedom: three displacement components, u , v and w (along the x , y and z axis directions, respectively) and three transverse stress components are expressed at the bottom as well as the top surface of each individual layer. The time dependent axial and transverse displacements along the x , y and z axis directions at any point can be expressed using power series expansions. The continuity of the displacements has been explicitly satisfied at the lamina interfaces in these models. The continuity of lamina plate interfaces and zero state of transverse strains was satisfied in these models by the appropriation degree of approximation.

Higher order asymptotic approach

Various high-order displacement models have been developed in the literature by considering combinations of displacement fields for in-plane and transverse displacements inside a mathematical sub-layer. In order to obtain more accurate results for the local responses, another class of laminate theories, commonly named as the layer-wise theories, approximate the kinematics of individual layers rather than a total laminate using the 2-D theories. These models have been used to investigate the phenomena of wave propagation as well as vibrations in laminated composite plates. Numerical evaluations obtained for wave propagation and vibrations in isotropic, orthotropic and composite laminated plates have been used to determine the efficient displacement field for economic analysis of wave propagation and vibration. The numerical method developed in this paper follows a semi-analytical approach with an analytical field applied in the longitudinal direction and a layer-wise power series displacement field employed in the transverse direction. The goal of the present paper is to develop a simple numerical technique, which can produce very accurate results compared with the available analytical solution. The goal is also to provide one with the ability to decide upon the level of refinement in higher order theory that is needed for accurate and efficient analysis.

Let us consider now such kinematic assumptions ($U=U_e+U_d$) for a symmetrical three-layered plate of thickness $2H_p$ (only cylindrical bending is considered):

$$U_e - \begin{cases} u = \sum_{i,k} u_{ik}^e z^{2i-1} \phi_k(x), \\ w = \sum_{i,k} w_{ik}^e z^{2i-2} \gamma_k(x), \end{cases} \quad \begin{matrix} 0 < z < H, \\ 0 < x < L, \end{matrix} \quad U_d - \begin{cases} u = \sum_{i,k} u_{ik}^d (z-H)^i \phi_k(x), \\ w = \sum_{i,k} w_{ik}^d (z-H)^i \gamma_k(x) \end{cases} \quad \begin{matrix} H < z < H_p, \\ 0 < x < L. \end{matrix} \quad (1)$$

Here $\phi_k(x)$, $\gamma_k(x)$ – are *a priori* known coordinate functions (for every beam clamp conditions),

$u_{ik}^e, w_{ik}^e, u_{ik}^d, w_{ik}^d$ – unknown set of parameters.

By substituting Eqs. (1) into the Hamilton variation equation

$$\int_{t_1}^{t_2} \int_V (\sigma_{xx} \delta \epsilon_{xx} + \sigma_{zz} \delta \epsilon_{zz} + \tau_{xz} \delta \epsilon_{xz} - \rho \frac{\partial u}{\partial t} \delta \frac{\partial u}{\partial t} - \rho \frac{\partial w}{\partial t} \delta \frac{\partial w}{\partial t}) dV dt = \int_{t_1}^{t_2} \int_S P \delta U, \quad (2)$$

and also assuming single frequency vibration ($u_{ik}^e = \bar{u}_{ik}^e e^{i\omega t}$, $w_{ik}^e = \bar{w}_{ik}^e e^{i\omega t}$, $u_{ik}^d = \bar{u}_{ik}^d e^{i\omega t}$, $w_{ik}^d = \bar{w}_{ik}^d e^{i\omega t}$) we obtain the set of linear algebraic equations for the amplitudes

$$[A] \bar{U} = \begin{bmatrix} A_1 & A_d \\ A_d^T & A_2 \end{bmatrix} \begin{bmatrix} \bar{U}_e \\ \bar{U}_d \end{bmatrix} = f. \quad (3)$$

For a greater number of lamina this equation has the following form for each additional layer

$$U_d^n = \begin{cases} u = \sum_{i,k} u_{ik} \left(z - H^{(n)} \right)^i \phi_k(x), & H_n < z < H_P, \\ w = \sum_{i,k} w_{ik}^n \left(z - H^{(n)} \right)^i \gamma_k(x), & 0 < x < L, \\ & n = 1, \dots, N, \end{cases} \quad (4)$$

Here H_n are the low bounds of the n -th layer, respectively. The matrix $[A]$ is found by double integration through the thickness and along the length of the beam. Note that, $N = 1$ and $N = 2$ represent the cases of symmetrical three- and five-layered plates, respectively. The corresponding frequency equation for the material with the viscous damping should be written as

$$-\omega^2 [M] \bar{U} + i\omega [C] \bar{U} + [K] \bar{U} = [A] \bar{U} = \bar{f}. \quad (5)$$

This is the traditional frequency domain method which is normally used in linear elastic system investigations. By having taken into account the first terms in (1), we obtain the kinematic assumptions for a TB. Details of this approach may be found in [50–53].

Damping properties in the frequency domain

Analogous theories and order-dependent results may be obtained also for the prediction of damping [36–49]. This result may be achieved by direct computation by use of the stiffness matrix if the damping matrix is congruent to the stiffness matrix.

$$\eta = \frac{\eta_1 [q_1]^T [K_1] [q_1] + \eta_2 [q_2]^T [K_2] [q_2] + \dots + \eta_N [q_N]^T [K_N] [q_N]}{[q]^T [K] [q]}. \quad (6)$$

Here: $[K]$ is the stiffness matrix, $|q|$ is a vector of displacement components, $[K_i]$ is a stiffness matrix component corresponding to the (i^{th}) layer ($[K] = \sum_i [K_i]$). The components of the damping matrix C are, us

usual, taken to be proportional to the components of the rigidity matrix: $C_i = \eta_i [K_i]$. In the previous section of this paper, the influence of the properties of the sandwich panels in the frequency domain may be seen. For this reason, fewer investigations were made of the damping properties in the frequency domain.

Transition to TB. Numerical examples. Sandwich panel

A three-layer beam is considered. Parameters are: length (L) = 0.6 m, the thickness of the filler (H) = 0.0254 m, the thickness of outer layer $H_F = 0.0005$ m, modules of filler $C_{xx} = C_{zz} = 180$ MPa, $G = 35$ MPa, $C_{xz} = 40$ MPa, density (ρ) = 240 kg/m³, modules of rigid outer layer – $C_{xx} = 43$ GPa; $C_{xz} = 6$ GPa; $G = 0.6$ GPa, $\rho = 2000$ kg/m³. To transmit a three-layer beams to a homogeneous TB the same thickness and weight per unit length the following criteria is used:

$$C = \min \sum_{E_T, G_T} \left| f_S^i - f_T^i(E_T, G_T) \right| \quad \text{in the range of frequencies} \quad f_k - \Delta_k/2 < f < f_k + \Delta_k/2. \quad (7)$$

Here E_T, G_T are the Young's and shear modulus of an equivalent beam.

The numerical calculation of the vibration eigenfrequencies was performed on the basis of Eqs. (1)–(5), taking into account the elastic and inertial properties of the beam. Fig. 1 shows $E - G$ maps – the level lines of the error function Eq. (6) as a function of the moduli of homogeneous beams. For all layered beams, as shown at the Fig. 1, the equivalent TB is simply determined.

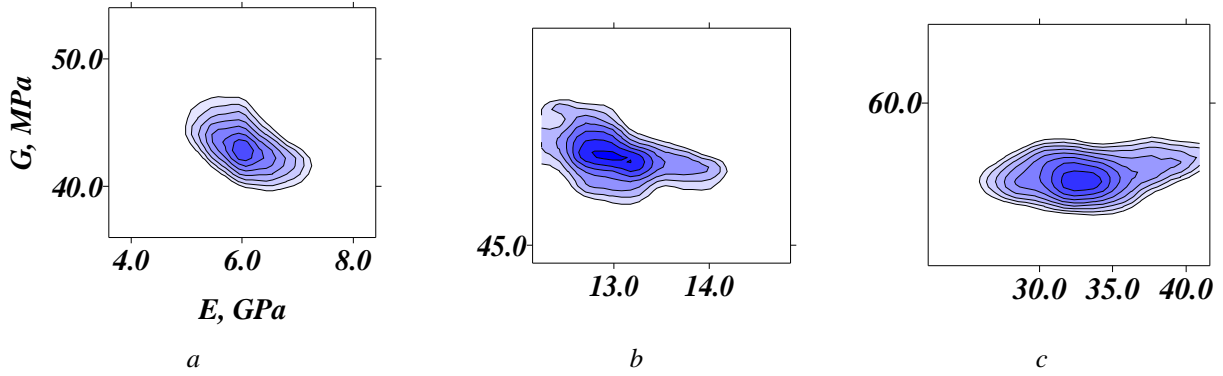


Fig. 1. $E_T - G_T$ maps – the contours of the error function, Eq. (23) as a function of the moduli of homogeneous beams for various face sheet thicknesses: *a* – one face sheet, $H_F = 0.0005\text{ m}$; *b* – two face sheets, $H_F = 0.001\text{ m}$; *c* – four face sheets, $H_F = 0.002\text{ m}$

In Fig. 2, the sandwich beam FRF, damping and the equivalent TB FRF are presented. The core damping is 1.2 %.

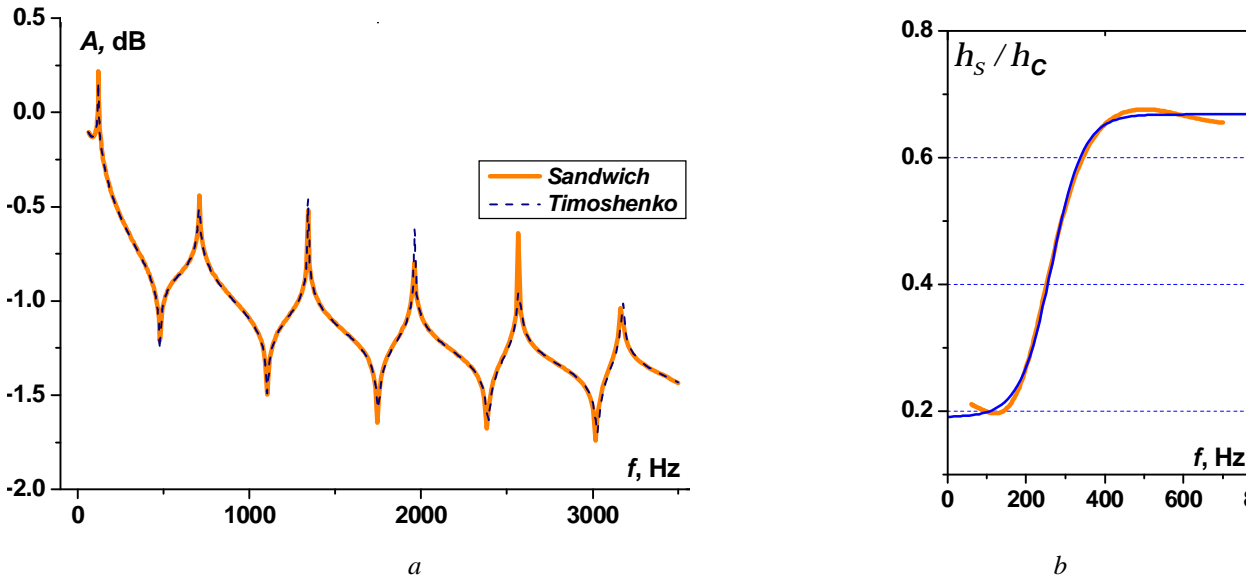


Fig. 2. Sandwich beam and equivalent TB FRF (*a*); relative damping and Boltzman approximation of the damping (one face sheet, $H_F = 0.0005\text{ m}$) (*b*)

Here for the relative damping η_Σ / η_C – the relationship between the total sandwich damping and the core damping with the Boltzmann approximation (thin blue line) is presented.

$$y = \frac{A_1 - A_2}{1 + \exp\left(\frac{x - x_0}{dx}\right)}. \quad (8)$$

In Fig. 3, the sandwich beam FRF with the double face sheets $H_F = 0.001\text{ m}$, damping and the equivalent TB FRF and in Fig. 4 the four face sheets $H_F = 0.002\text{ m}$ are presented.

As we can see, in a wide frequency range observed almost complete coincidence of the (FRF). Only at higher frequencies observed deviations. The parameters of equivalent TBs used to determine the absorption of sound in the sandwiches plate.

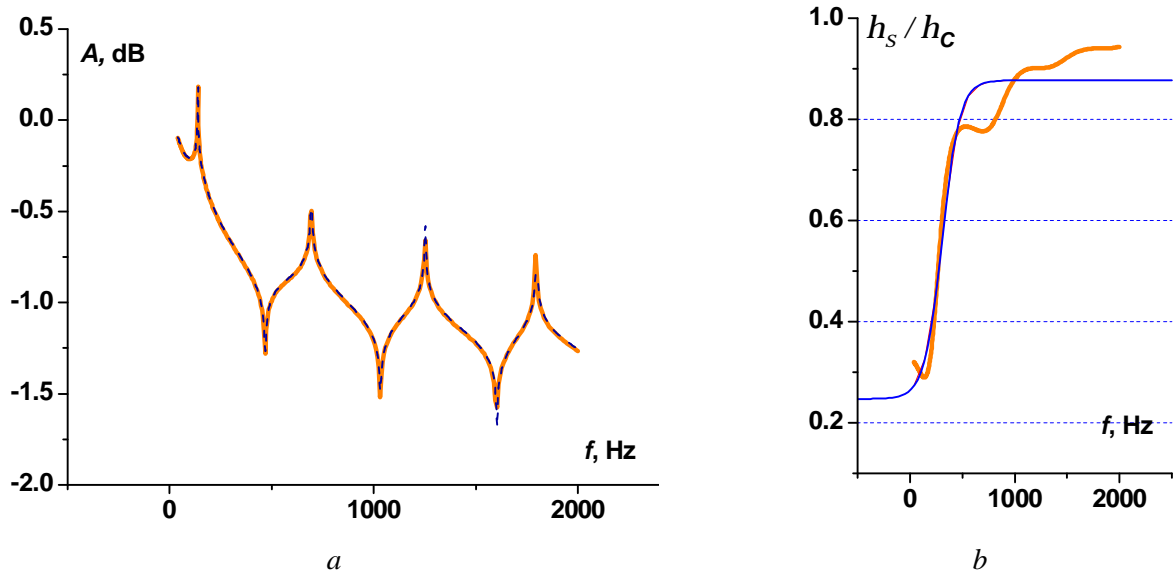


Fig. 3. Sandwich beam and equivalent TB FRF (a);
Relative damping and Boltzmann approximation of the damping (double face sheets $H_F = 0.001m$) (b)

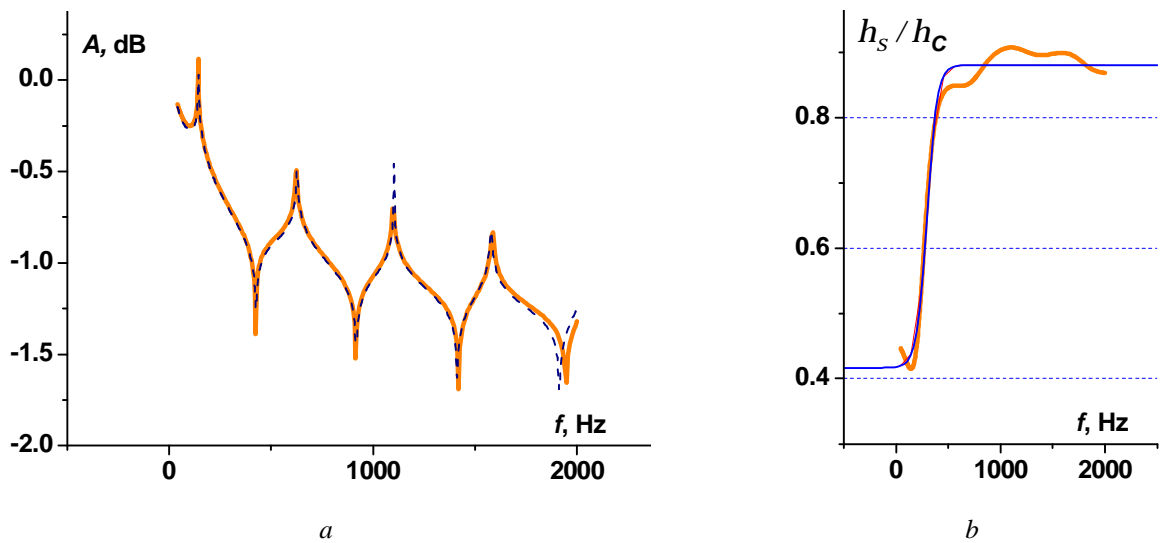


Fig. 4. Sandwich beam and equivalent TB FRF (a);
Relative damping and Boltzmann approximation of the damping (four face sheets $H_F = 0.002m$) (b)

The main ratio of TB sound transmission properties

When a panel is excited acoustically, the frequency at which the speed of the forced bending wave in the panel is equal to the speed of the free bending wave in the panel is called the coincidence frequency. It is expected that the sound power transmission coefficient is very high at the coincidence frequency of the panel.

Let us consider a panel with an incident sound field (Fig. 5).

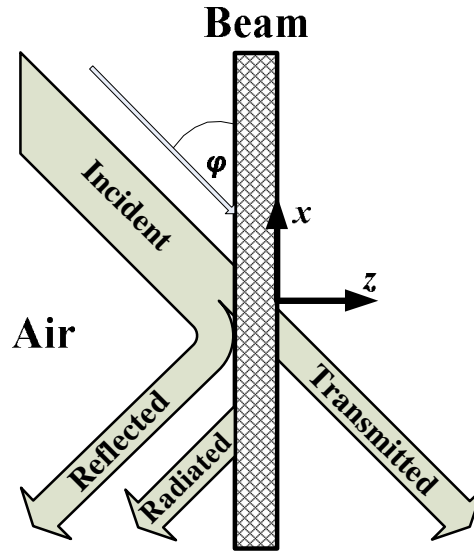


Fig. 5. Beam in plane sound field (plane waves)

An external excitation in the form of a plane sound wave at the angular frequency ω is assumed to be incident on the first face sheet layer. The sound power transmission coefficient is defined as the ratio of the intensity of the transmitted sound to the intensity of the incident sound. If I_i is the intensity of the incident sound wave and I_t is the intensity of the transmitted sound wave, the sound power transmission coefficient τ is defined by $\tau = I_t / I_i$.

Let us consider study state harmonic vibrations of TB. Panel is regarded as Timoshenko plate and its parameters were based on comparison of frequencies of sandwich panel and TBs [54] which were identical thickness and weight. Kinematic hypothesis for TBs are:

$$U(x, z, t) = z \gamma(x, t), \quad W(x, z, t) = w(x, t). \quad (9)$$

The equations of dynamic balance of TB's bending are:

$$EI \frac{\partial^2 \gamma}{\partial x^2} - SG \left(\frac{\partial w}{\partial x} + \gamma \right) - \rho I \frac{\partial^2 \gamma}{\partial t^2} = 0, \quad SG \left(\frac{\partial \gamma}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right) - \rho S \frac{\partial^2 w}{\partial t^2} = -q. \quad (10)$$

Where the normal load will be ([55]):

$$q = 2(p_i - p_t). \quad (11)$$

For steady vibrations

$$p_i = A_i e^{i\omega t} e^{ikx \sin \phi - ikz \cos \phi}, \quad p_t = A_t e^{i\omega t} e^{ikx \sin \phi - ikz \cos \phi}, \\ \gamma = \gamma_0 e^{i\omega t} e^{ikx \sin \phi}, \quad w = w_0 e^{i\omega t} e^{ikx \sin \phi}, \quad q = q_0 e^{i\omega t} e^{ikx \sin \phi}. \quad (12)$$

Considering the boundary conditions for pressure [55] $\frac{\partial p_t}{\partial z} = \rho_a \omega^2 w$, we obtain a system of algebraic equations

$$\left(-EI k^2 - SG - \rho I \omega^2 \right) \gamma_0 - SG i k w_0 = 0, \quad SG i k \gamma_0 + \left(-SG k^2 + \rho S \omega^2 \right) w_0 = q_0, \\ q_0 = 2(A_i - A_t) - A_t i k \cos \phi = \rho_a \omega^2 w_0. \quad (13)$$

where $k = \omega / C_v$, C_v – speed of sound. Resolving the system of equations, we get

$$A_t = F(\omega) A_i. \quad (14)$$

Coefficient of power transfer through an obstacle given as $\tau = \frac{|A_t|}{|A_i|}$ and the absorption coefficient in decibels have look $T_L = 10 \log |\tau^{-1}|$. So as a result we have:

$$T_L = 10 \log \left| \frac{1}{F(\omega)^2} \right|. \quad (15)$$

Fig. 6 shows the coefficient values T_L for sandwiches with different number of layers of coating.

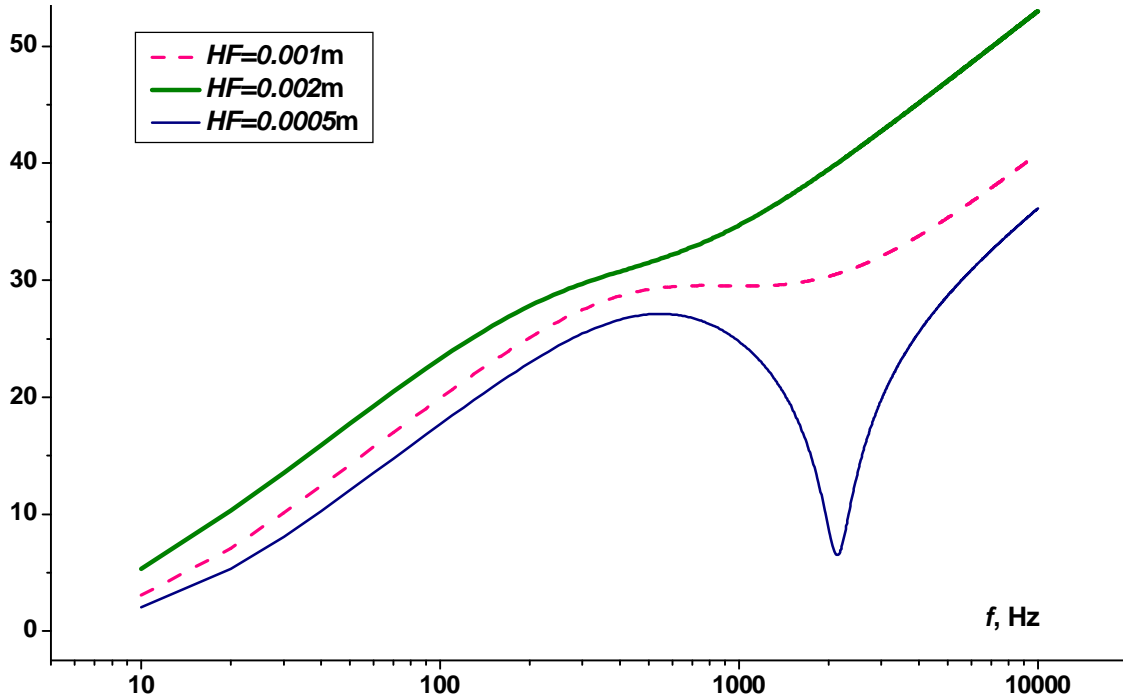


Fig. 6. Beam in plane sound field (plane waves)

Sandwich with soft outer layers

Let us consider now transition of a three-layered symmetrical beam with the soft thick face layer and thin rigid core: length $L = 0.6$ m and thickness $H = 0.02$ m, face layers thickness $h = 0.008$ m) (the foam face layers elastic moduli are assumed to be as follows: $C_{xx} = C_{zz} = 150$ MPa, $G = 33$ MPa, and $C_{xz} = 40$ MPa, density $\rho = 2000$ kg/m³) and rigid face layers (fibre- composite material: $C_{xx} = 50$ GPa; $C_{xz} = 6$ GPa; $G = 1.5$ GPa, $\rho = 2000$ kg/m³). In this case the equivalent beam can be found only for separate frequency range. In Fig. 7, a, b, c this equivalent module E_t , G_t are found for the different frequency regions (maps (7)).

In Fig. 8 the equivalent Young module E decreases and shear module G increases with the increasing frequency. In the Fig. 8 the appropriate FRF's are presented.

This Timoshenko beam FRF's also approximate the sandwich FRF, but only with various moduli in the frequency range.

Numerical scheme verification

Let us consider now slightly another beam. Modules of outer layers $C_{xx} = C_{zz} = 180$ MPa, $G = 35$ MPa, $C_{xz} = 40$ MPa, density $\rho = 240$ kg/m³; the hard inner layer modules – $C_{xx} = 43$ GPa; $C_{xz} = 6$ GPa; $G = 0.6$ GPa, $\rho = 2000$ kg/m³). In the Fig. 9 the (FRF) of varying degrees of the beams for approximating in the thickness of the beam is shown.

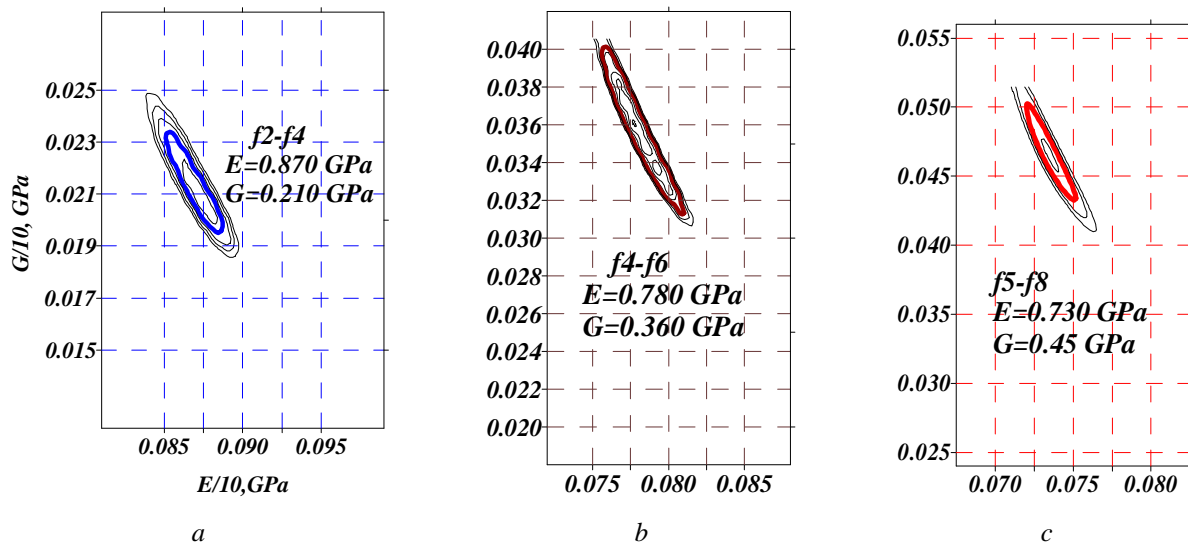


Fig. 7. The result of sandwich translation to the Timoshenko beam, – equivalent beam moduli E_t, G_t :

a – for $f_2 < f < f_4$; b – for $f_4 < f < f_6$; c – for $f_5 < f < f_8$

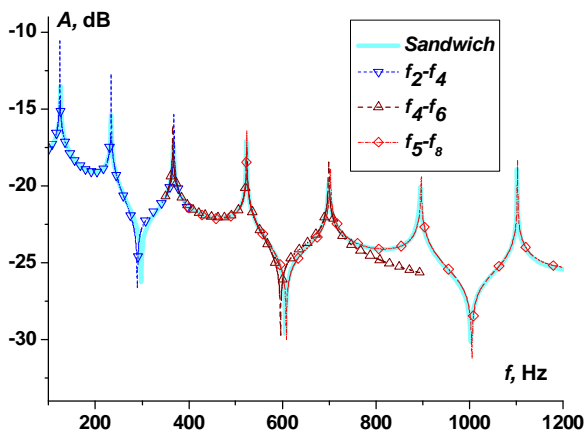


Fig. 8. Beam in plane sound field (plane waves)

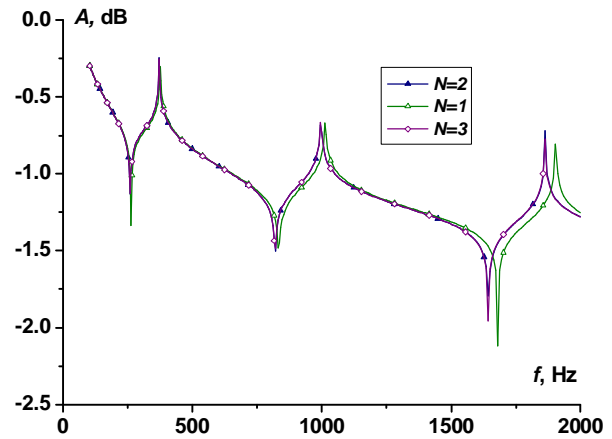


Fig. 9. FRF of the beam with soft outer layers for varying degrees approximation in the beams thickness

Now, we consider the values of the relative damping determined based on approximation theory of displacement in the thickness by the one expressions (1) for the whole package (Fig. 10).

For approximations of members of decomposition in thickness of the beam higher than 2 the results are identical.

In the Fig. 11 we see the FRF of the beams with different thicknesses of the internal hard layer. Fig. 12 illustrates the value of the relative damping.

Fig. 13 illustrate the results of transmission of the TB for the different soft outer layers thicknesses. the inner layer with thickness 0.001 m.

In Fig. 14, results of transmission of sandwich with soft outer layers to TB in different frequency ranges (a) and the map of equivalent modules E, G (b) are presented

Fig. 15 illustrates the damping in the beam and her approximation.

Fig. 16 illustrates an approximation of FRF of the three-layer beam to FRF of TB for different thickness of the inner layer.

In Fig. 17 we see values of the coefficient T_L for beam with varying thickness of the inner hard layer.

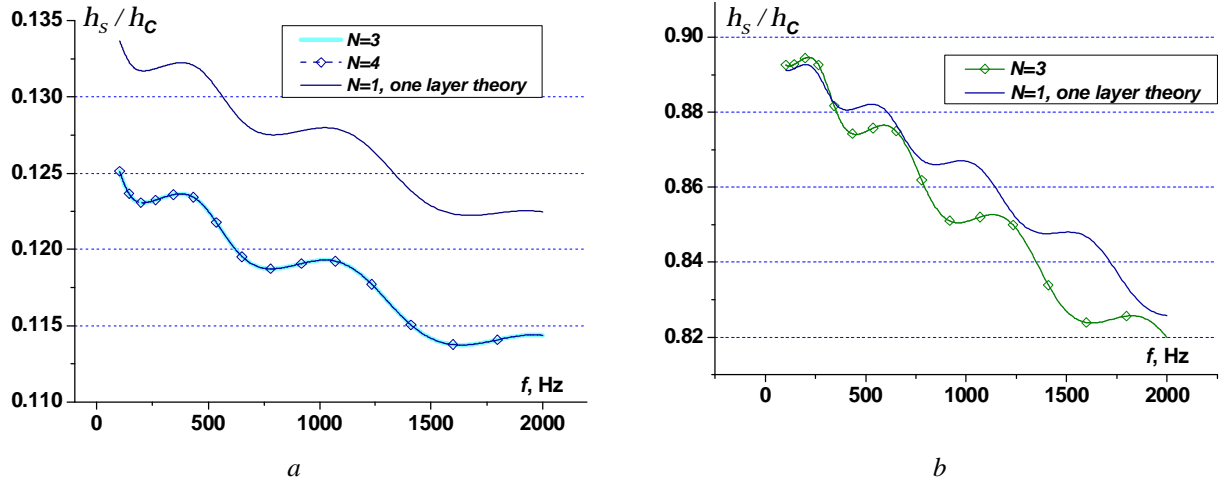


Fig. 10. The values of the relative damping determined based on theory of approximation f of displacement in the thickness by one polynomials: $a - 0.008$ m; $b - 0.002$ m

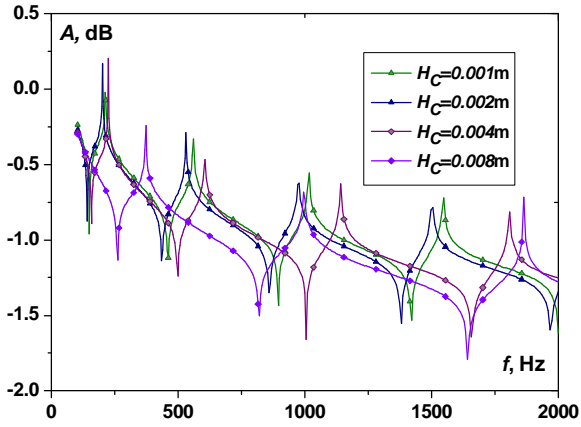


Fig. 11. FRF of the beam with different thicknesses H_C of the internal hard layer

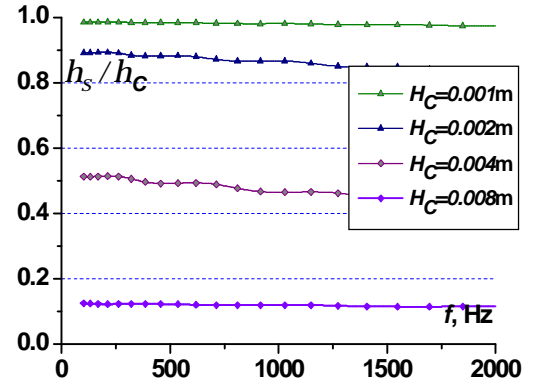


Fig. 12. The value of relative damping of the beam with different thicknesses H_C of the internal hard layer

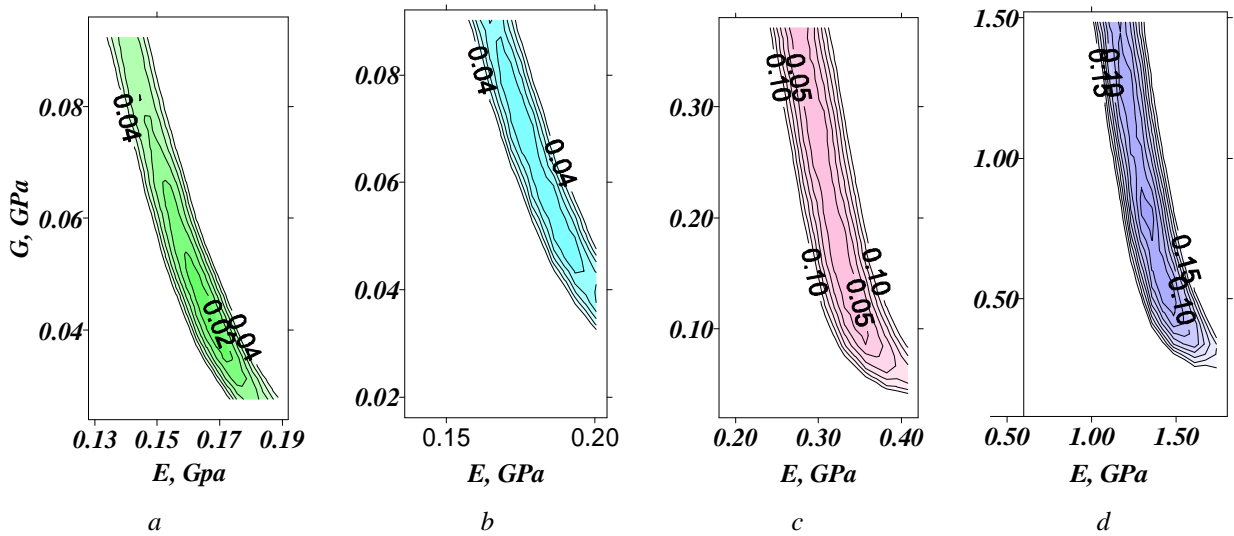


Fig. 13. The results of transmission of the beam with soft outer layers to the TB, equivalent modules E , G of the beam; $a -$ core thickness 0.001 m; $b -$ 0.002 m core thickness; $c -$ core thickness 0.004 m; $d -$ core thickness 0.008 m

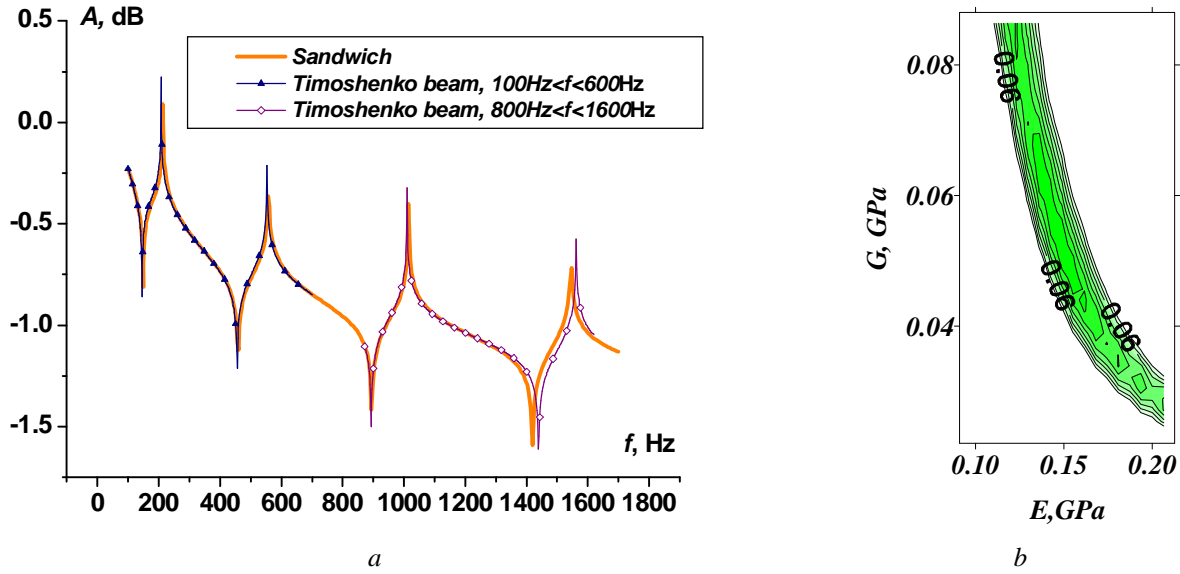


Fig. 14. Results of transmission of sandwich with soft outer layers to TB in different frequency ranges, with one layer covering (a); equivalent modules E , G of the beam (b)

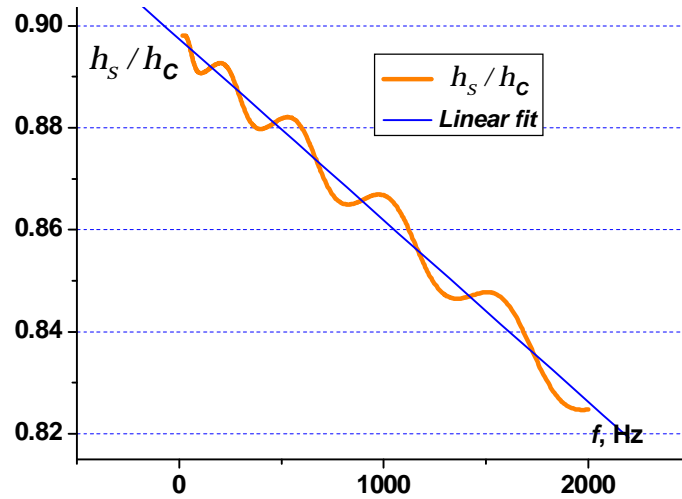


Fig. 15. The relative damping in the beam and its approximation

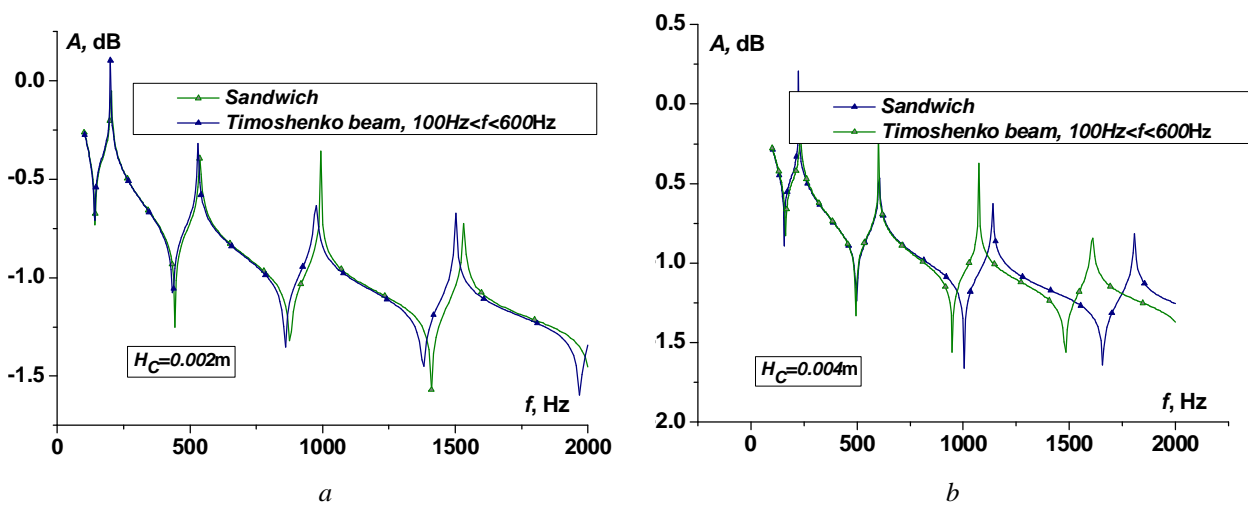


Fig. 16. Approximation of FRF of three-layer beam to FRF of the TB for the thickness of the inner layer: a – $H_C = 0.002$ m; b – $H_C = 0.004$ m

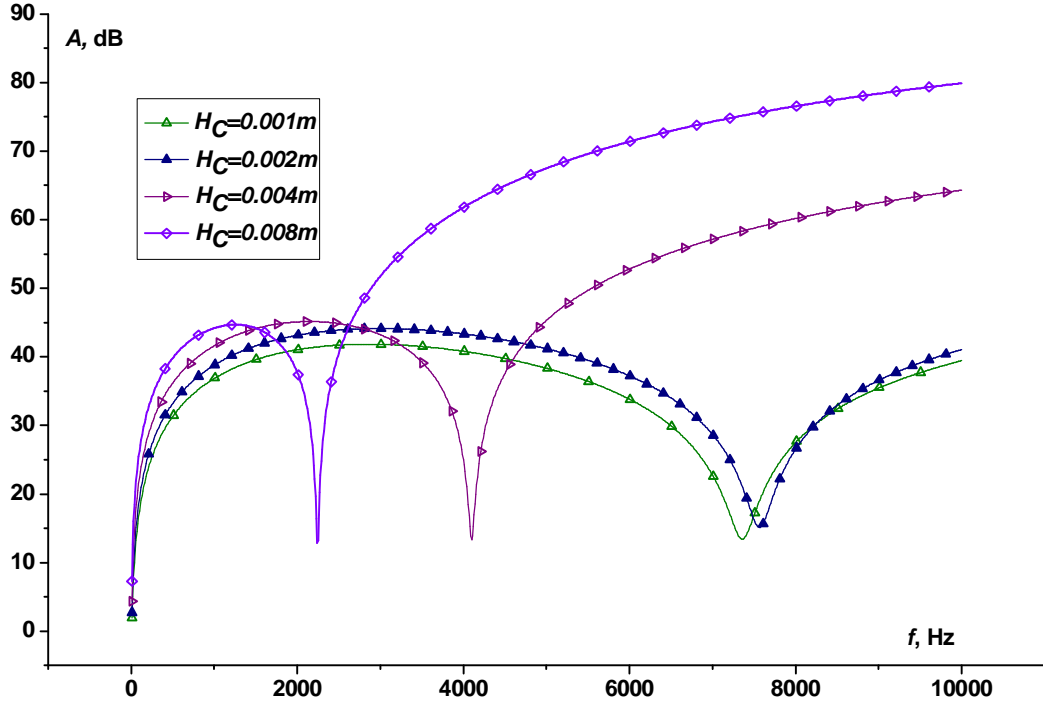


Fig. 17. The value of the coefficient T_L for the beams with varying thickness of the inner hard layer

Five-layer beam

Let us consider a five-layer beam with a hard core, facial layers and soft damping interim layer. Its parameters are: length $L = 0.6$ m, the thickness of the filler, the thickness of the core $H_C = 0.001$ m; interim layer thickness $H_I = 0.012$ m, thickness of outer modules $H_F = 0.001$ m interim layer moduli $C_{xx} = C_{zz} = 180$ MPa, $G = 35$ MPa, $C_{xz} = 40$ MPa, density $\rho = 240$ kg / m³; modules of outer and inner rigid layer – $C_{xx} = 4.3$ GPa; $C_{xz} = 0.6$ GPa; $G = 0.06$ GPa, $\rho = 2000$ kg / m³). In Fig. 18 there are FRF of the beams with different thickness of internal hard layer.

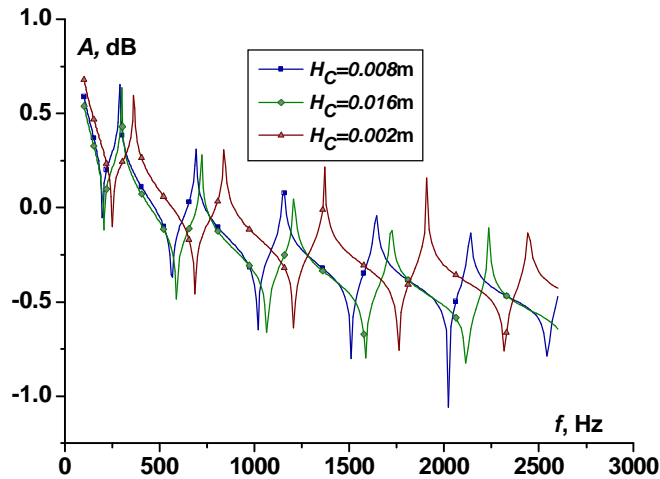


Fig. 18. FRF of the beams with different thickness of internal hard layer

This external hard layers with the thickness $H_F = 0.001$ m are the same. Fig. 19 shows the relative damping for five-layer beam with the different thickness of internal hard layer and, for comparison, three-layer soft outer layers.

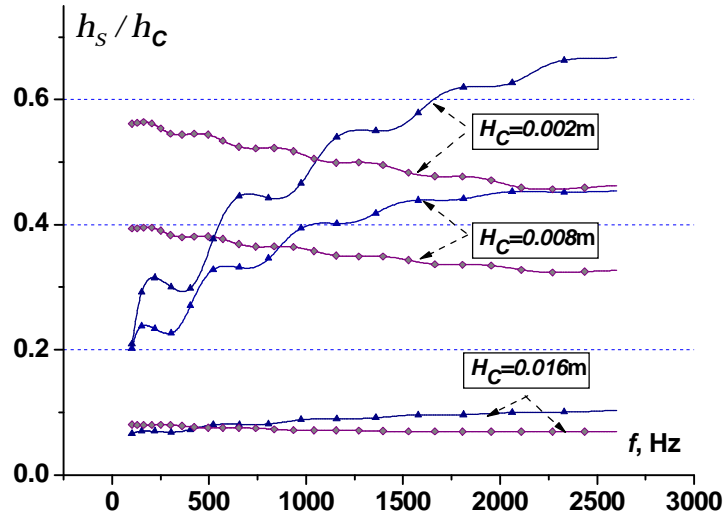


Fig. 19. The relative damping of sandwich beams with different thickness of the internal hard layer (marked with triangles) and three-layer soft outer layers (marked by diamonds)

In Fig. 20, *a*, for the smaller thickness H_{DEMP} of the interlaminar sheets damping in the symmetrical five-layered beam ($2H = 0.02$ m) with the various damping interlaminar sheets thickness H_{DEMP} and damping (small marks) of three layer beam with the soft face sheets equal thickness are presented.

Fig. 20, *b*, shows the results of the five-layer beams ($H = 0.0254$ m, $H_C = 0.001$ m, 0.001 m = HF) translation to TB, E, G modules of equivalent TB.

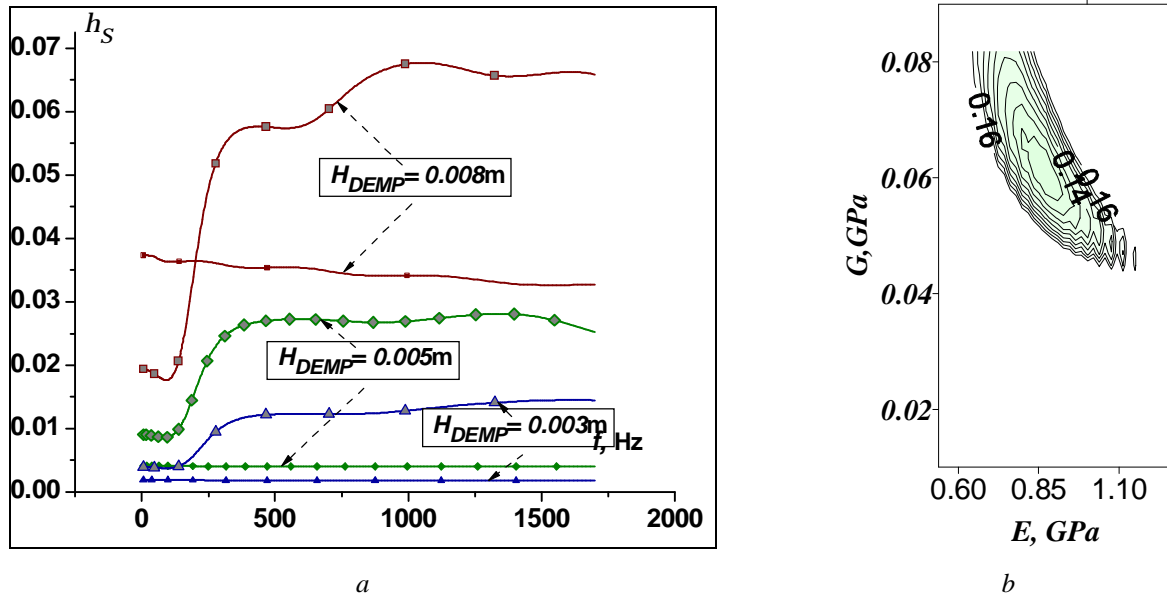


Fig. 20. Damping in the symmetrical five-layered beam ($2H=0.02$ m) with the various damping interlaminar sheets thickness H_{DEMP} and damping (small marks) of three layer beam with the soft face sheets equal thickness (*a*); The results of five-layer beams ($H = 0.0254$ m, $H_C = 0.001$ m, 0.001 m = HF) translation to TB (*b*)

Below in Fig. 21–25 the results of translation to the TB and relative damping. for the different five-layer laminated beams are shown.

In the Fig. 26 is presented coefficient T_L for five-layer beams with the different thickness of the inner layer.

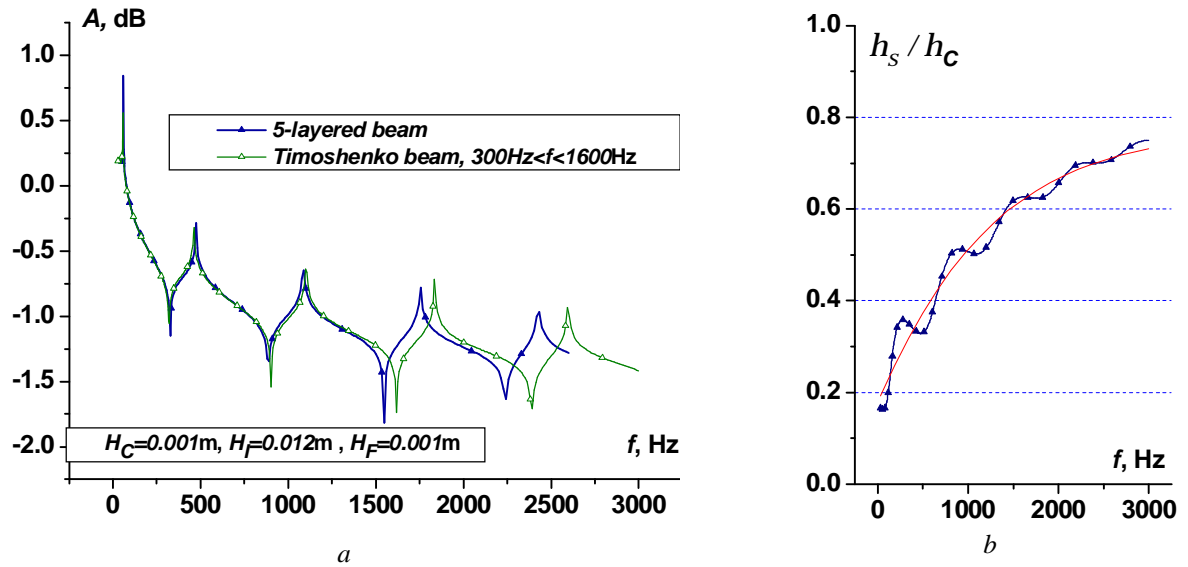


Fig. 21. The FRF of five-layer beam, an equivalent TB and relative damping in the beam

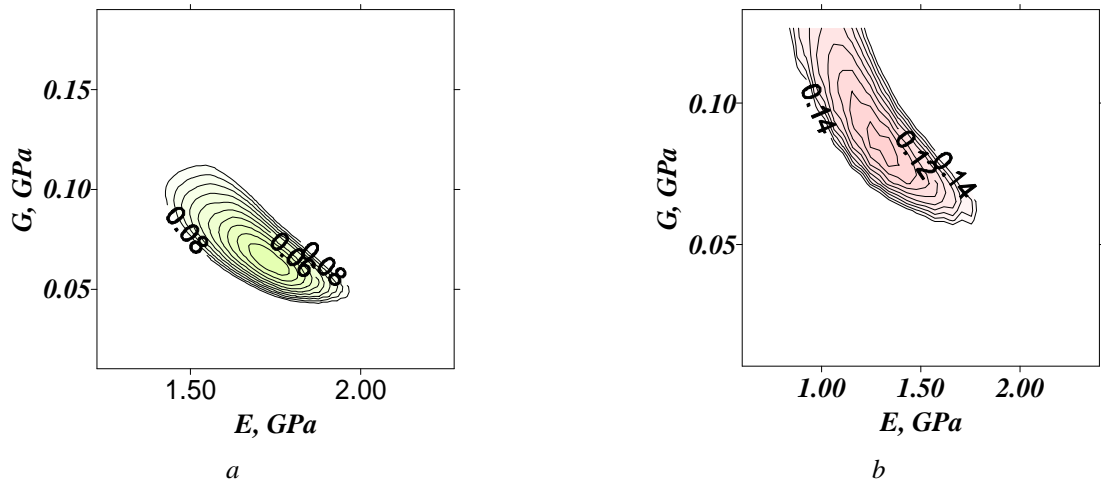


Fig. 22. Results of five-layer beam translation to TB, equivalent modules E , G of the beam:
 a – frequency range 0-500 Hts; b – frequency range y 500–1400 Hz

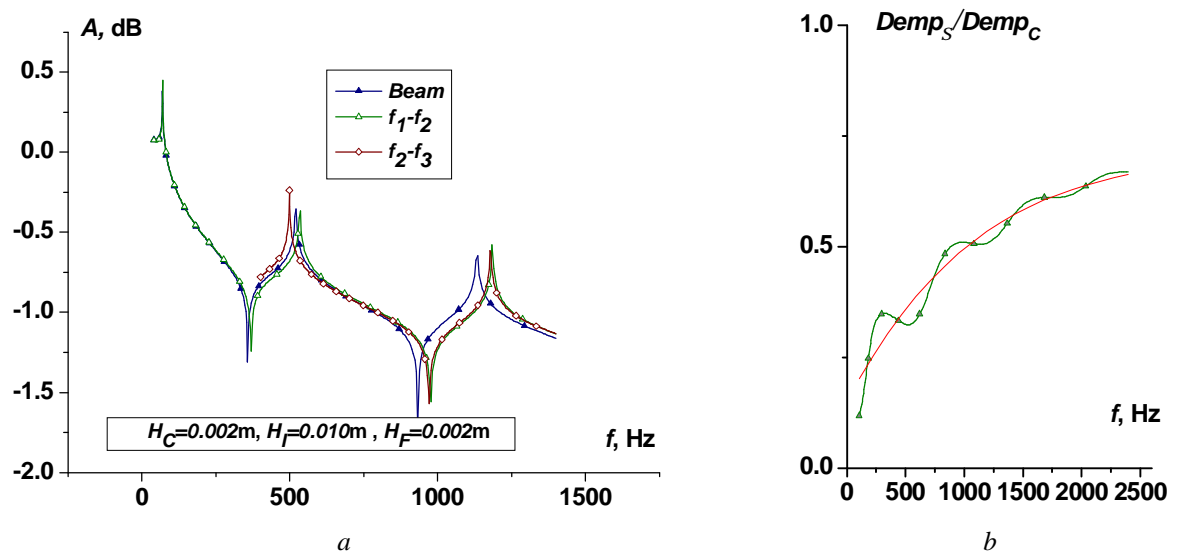


Fig. 23. FRF of the five-layer beam, FRF of the equivalent TB (for different frequency ranges) and relative damping in the beam

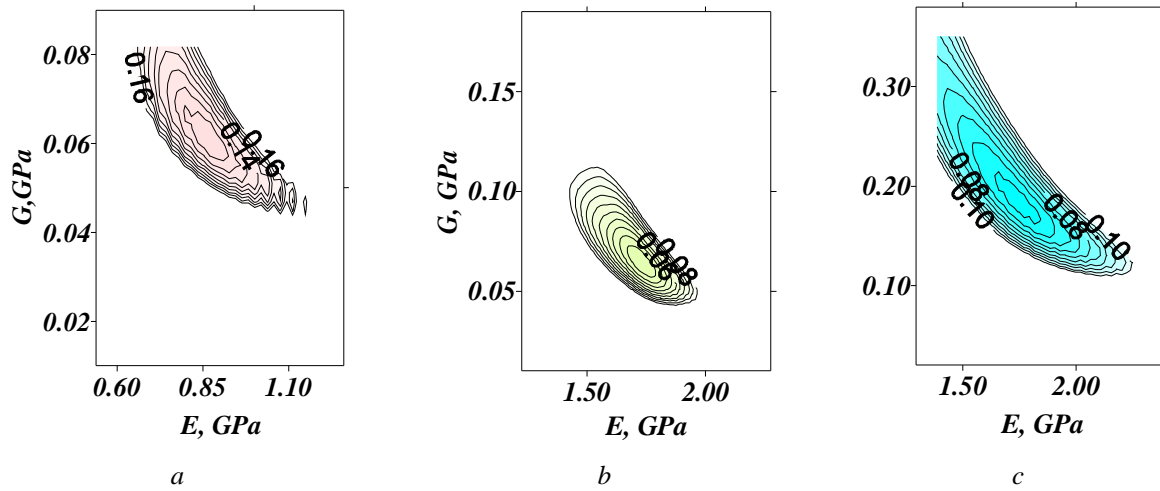


Fig. 24. Results of five-layer beam translation to TB, equivalent modules E , G of the beam:
 $a - H_C = 0.002 \text{ m}$; $b - H_C = 0.008 \text{ m}$; $c - H_C = 0.016 \text{ m}$

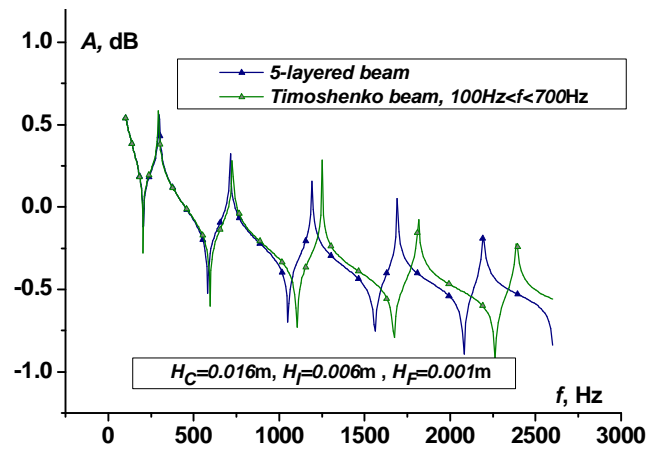


Fig. 25. Results of the translation of five-layer beam with $H_C = 0.016 \text{ m}$ to TB

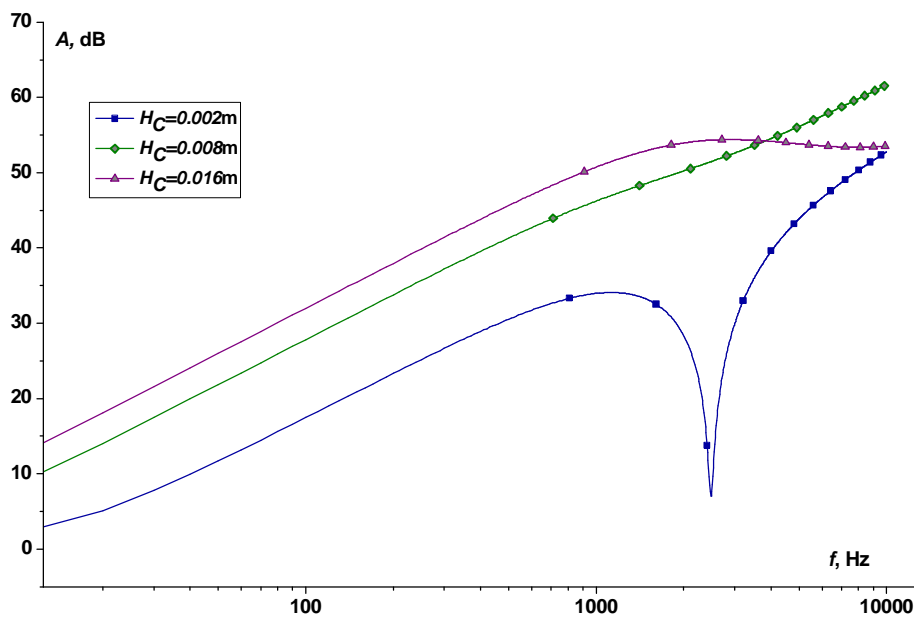


Fig. 26. Coefficient T_L for five-layer beams with the different thickness of the inner layer

Fig. 27 shows the coefficient of losses of sound for different beams: beams with a hard core, core material of beams and beams of the coating material.

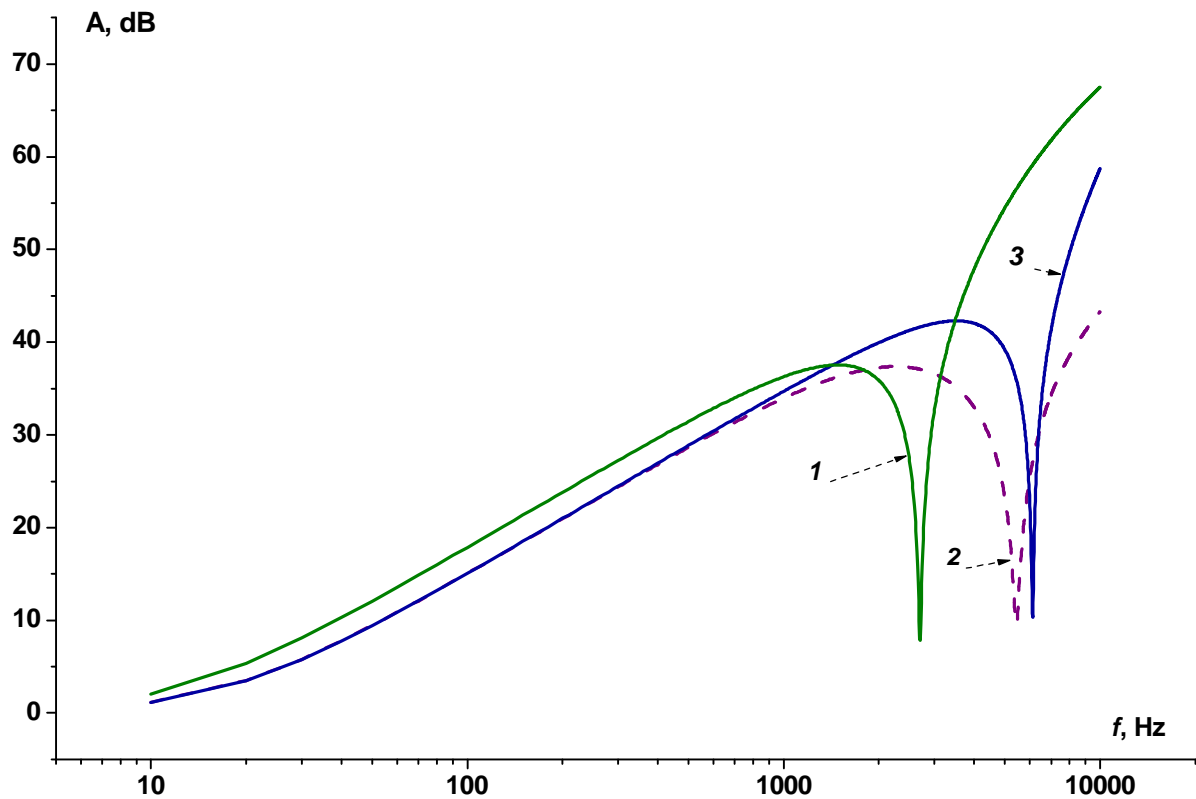


Fig. 27. The coefficient of sound losses for different beams: beams with a hard core (1), core material of beams (2) and beams of coating material (4)

Line weight of these beam was taken as a constant. That is, when the specific weight of coating is 10 times greater than weight of the core, the thickness of the coating material beam was taken 10 times lower.

Acoustical properties of the beam with DVA

Damped DVAs are used to provide energy dissipation, thereby motivating the term ‘absorber’. These realistic absorbers furthermore reduce their sensitivities to parameter variations from optimal values and reduce the primary system motion at its resonance frequencies, while increasing their effective bandwidth, as compared to undamped examples. Consider the viscously damped DVA with elastic and viscous damping elements, used between the masses. Detailed descriptions of the fundamentals of such DVA’s are given in [54].

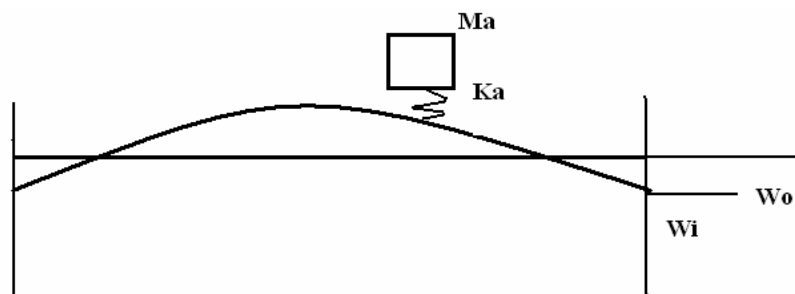


Fig. 28. Beam with the DVA

Let us now consider a layered beam with a locally attached DVA (Fig. 28): M_A – DVA mass; K_A – DVA clamping rigidity). Taking into account only the first type of vibration we obtain a similar set of equations as in Eq. (13). Only one additional equation for the DVA is needed:

$$-M_A \omega^2 w_A + (K_A + i\omega C_A)(w_A - w) = 0. \quad (3)$$

The TL is presented for various DVA parameters in Fig. 29.

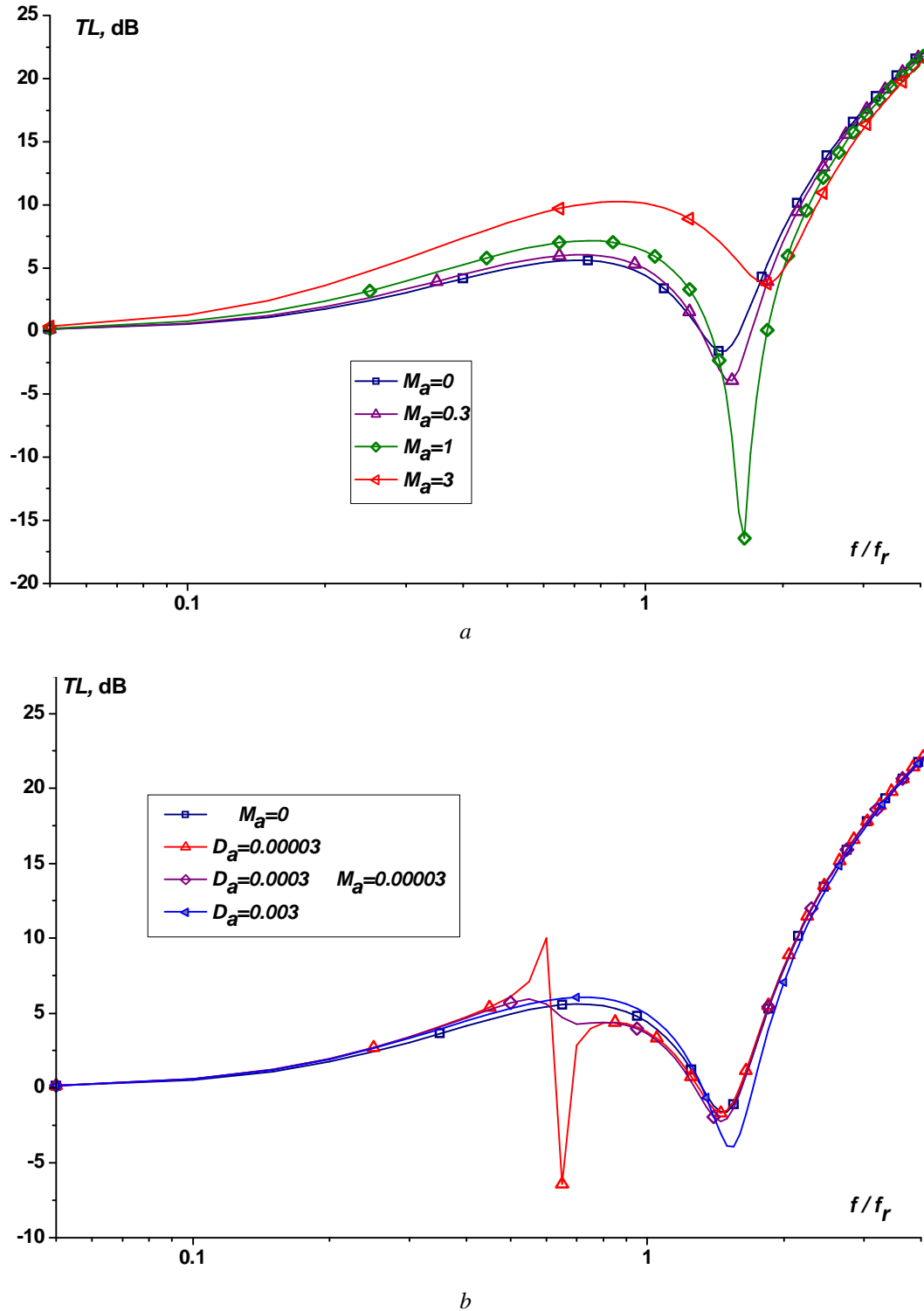


Fig. 29. Mass influence of the DVA (a); damping influence of DVA (b)

The parameter f_r is the critical frequency of the panel without considering the transverse shear deformation [55].

$$f_r = C_v^2 \sqrt{\frac{r}{EI}} / 2p . \quad (16)$$

The influence of the DVA mass on the TL is presented in Fig. 29, *a*. The influence of the damping of the DVA is presented in Fig. 29, *b*. The influence of DVA's number on TL is presented (Fig. 30).

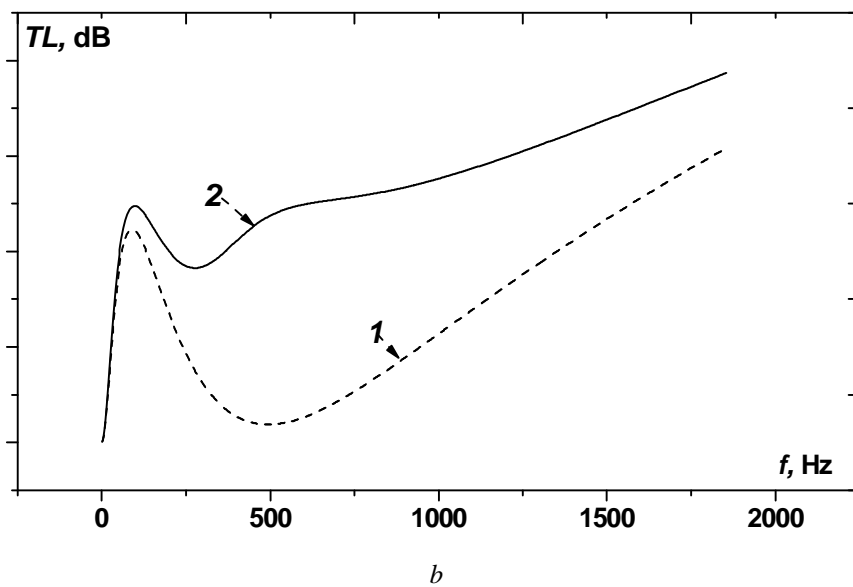
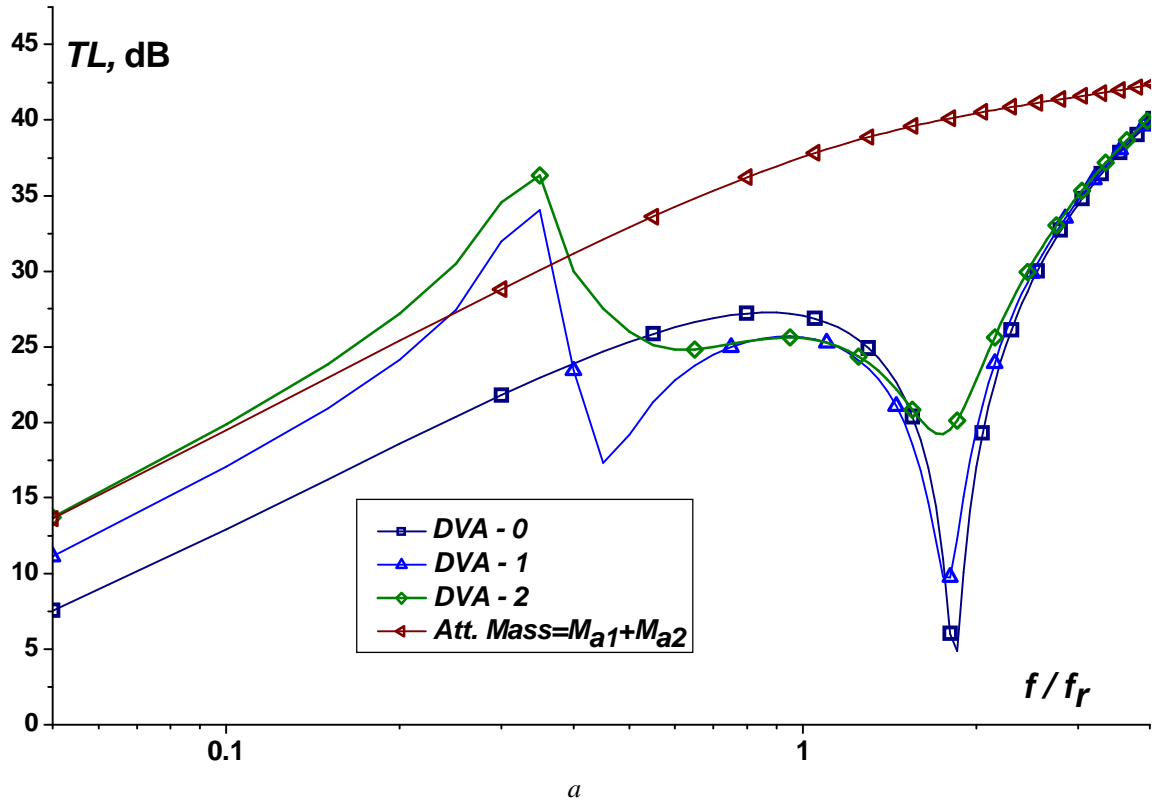


Fig. 30. Influence of DVA's number on TL (a);
DVA constructed for the most dangerous case (b)

Four cases are presented in Fig. 30, *a*: 1 – beam without the DVA's; 2 – one DVA; 3 – two DVA's; 4 – DVA's masses rigidly connected to beam. The last case present “mass law” for the heavy beam. The better sound transmission panel properties may be seen for the DVA's system. In Fig. 30b DVA constructed for the most dangerous case, the frequency of sound and own beam frequency equality is shown.

Conclusions

Theoretical models for the dynamics and damping of laminated structures have been developed. With the small number of parameters studied so far, this approach predicts the dynamic behavior of the beams investigated. Using this model for layered beams, higher order modeling was carried out, not only for the damping caused by the shear strain in the core, but also for the damping caused by normal and bending deformation. This is important for the middle and high frequency analysis of the damping properties of sandwich structures. The main advantage of the present method is that it does not rely on strong assumptions about the model of the plate. The key feature is that series of models can be applied for different vibration conditions of the plate by use of a suitable analytical or approximate method. Using numerical experiments the results for adaptive numerical schemes of lower degrees of approximation in the normal direction have been established.

The wide range of laminated composite plates had been analyzed. For sandwich with the damping soft core damping increases with frequency and Boltzmann function can serve as good approximation of damping. Timoshenko beam with the defined parameters of equivalence based on eigenfrequencies difference can serve for approximation of dynamic properties of the sandwich plates in a wide frequency range. For the beam with soft damping outer layers such approximation with Timoshenko beam is possible only in certain frequency bands.

For the damping in a three-layer beam the fact was confirmed, that the constrained damping layer increases the damping compared to the damping layer coated on the surface of the hard core material with low damping. However, this is not observed for all layer thicknesses and frequencies. For a thick damping layer an effect of the constrained layer is rather negligible and even negative in low frequency range. However, the effect for thin damping layers is much higher and is shown almost in the entire frequency range.

The questions updated modeling of the layered structures dynamics is discussed. Refined models used to determine the coefficients of sound losses sandwich panels. To do this, it was used the method of the equivalent Timoshenko beam. The loss coefficient for beams with the different thickness, and with the different materials, and for sandwiches with the different thickness of the coating is represented. When for the homogeneous beams and beams with a hard core the losses coefficients are comparable, for sandwich with increased thickness of the cover the losses significantly increase.

The present paper is a first attempt at proposing a novel procedure to derive the damping and sound insulation parameters for sandwich plates with the presence of a DVA. The main advantage of the present method is that it does not rely on strong assumptions about the model of the plate. The parameter dependent FRF and damping are presented for a three-layer beam. The results of this paper have shown that the presence of a DVA causes a decrease in the sound transmission in the low-frequency range. In the future, the extension of the present approach to various layered plates with various micro- and macro-inclusions will be performed in order to investigate various experimental conditions. To do this, it was used the method of the equivalent Timoshenko beam. The loss coefficients for beams with different thickness and with different materials and for sandwiches with different thickness of the coating are represented. When for the homogeneous beams and beams with a hard core the losses coefficients are comparable, for sandwich with increased thickness of the cover the losses significantly increase.

Out of consideration of this paper the questions of maximizing of damping and sound insulation in the package have lost. However, such structures as a dynamic vibration absorber with the elastic resiliency element, the optimal damping must have a certain value. It is important to reduce the tensions in some working frequency range. These questions we are going to consider in the future.

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