Vol. 6, No. 1, 2016

SOLVING INVERSE PROBLEMS OF DYNAMICS OF NON-LINEAR OBJECTS WITH THE USE OF VOLTERRA SERIES

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Abstract. The article deals with the method of resolving inverse problems of dynamics of nonlinear dynamical objects described by the Volterra series. As an example the case of the Volterra series with two members has been considered. The proposed approach is based on the quadrature method. As a result the methods of resolving of Volterra polynominal integral equation of the first kind and second degree based on the left rectangle method and trapezoidal method were developed. Based on the offered approach, the software for restoration of signals of nonlinear dynamical objects was developed in the Matlab environment. The effectiveness of the means has been investigated in the course of the series of computing experiments including the possibility of their application while noise is superimposed on the input signal. Computational errors significantly depend on the type of the input signal, in particular for smooth signals the errors vary from 1 % to 5 % and with 10 % of superimposednoise – to 15 %.

Thus, the results of computing experiments have shown that the proposed method can be effectively used in the restoration of input signals of nonlinear dynamical systems described by the integro-powerVolterra series with two members.

Key words: the inverse problem, Volterra series, quadrature method, Volterra integral equation of the first kind.

1. Introduction

In solving many physical and technological problems, the necessity of solving inverse problems occurs. All general inverse problems regardless of physical processes or technical systems can be divided into three classes: inverse problems arising during the diagnosis and identification of physical processes; inverse problems arising during the design of technical objects; inverse problems arising during the control of processes and objects [1, 2, 3].

A universal approach to the development of mathematical models of non-linear dynamical systems of "black box" type is the representation of the response of the system to external influence in the form of the integro-power Volterra series [4, 5, 6, 7]:

$$y(t)=\int_0^t K_1(t,s)x(s)ds+ \int_0^t\int_0^t K_2(t,s_1,s_2)x(s_1)x(s_2)ds_1ds_2+..., \quad (1)$$

where $x(t)$, $y(t)$ are correspondingly input and output signals of an object, t is transient time, $K_i(t_1,\mathbf{K},t_i)$ are Volterra kernels.

Solving inverse problems leads to the Volterra polynomial integral equations of the first kind (1). Currently, no effective methods and means for solving such a class of equations exist. That is why the improvement of existing methods and the development of new ones for solving the Volterra integral equations of the first kind is so essential.

2. The purpose of the research

The purpose of this paper is to develop a method for solving inverse problems of dynamics of non-linear dynamical objects by solving the Volterra polynomial integral equations of the first kind.

3. Task definition

The problem of controlling a nonlinear system which consists in the determination of input influence $x(t)$ corresponding with desired response $y(t)$ is considered [3].

It is proposed to solve the posed task by the replacement of integrals in (1) with a quadrature formula. This operation provides a number of advantages, in particular, the simplicity of implementation and high stability of computational algorithms owing to quadrature step regulating properties [8].

This method is considered to be applied to solving the Volterra polynomial integral equation of the first kind and second degree:

$$y(t)=\int_0^t K_1(t,s)x(s)ds+ \int_0^t\int_0^t K_2(t,s_1,s_2)x(s_1)x(s_2)ds_1ds_2 \quad (2)$$

4. Left rectangle method

By applying the quadrature method to (2), the integral $\int_0^t K_1(t,s)x(s)ds$ being approximated by the trapezoidal method, and double integral $\int_0^t\int_0^t K_2(t,s_1,s_2)x(s_1)x(s_2)ds_1ds_2$ by the left rectangle method, the following recurrence equation is obtained for $x(t_i)$:

$$x(t_i)=\left(\frac{1}{2}hK_1(t_i,t_i)\right)^{-1}\times$$
$$\times\left(y(t_i)-\frac{hK_1(t_i,t_0)x(t_0)}{2}-\sum_{j=1}^{i-1}hK_1(t_i,t_j)x(t_j)-\right. \quad (3)$$
$$\left.-\sum_{j=0}^{i-1}\sum_{g=0}^{i-1}h^2K_2(t_i,t_j,t_g)x(t_j)x(t_g)\right),$$

where $i=\overline{1..n}$, $h=t_i-t_{i-1}$. The value in point t_0 is obtained after differentiation (2) in respect with t. Thus,

$$y_t'(t)=\int_0^t\frac{\partial K_1(t,s)}{\partial t}x(s)ds+K_1(t,s)x(t)+$$
$$+\int_0^t\int_0^t\frac{\partial K_2(t,s_1,s_2)}{\partial t}x(s_1)x(s_2)ds_1ds_2+ \quad (4)$$
$$+\int_0^t K_2(t,s_1,t)x(s_1)ds_1+\int_0^t K_2(t,t,s_2)x(s_2)ds_2,$$

where at $t=0$ i $K_1(0,0)\neq 0$

$$x(t_0)=\frac{y_t'(t_0)}{K_1(t_0,t_0)}. \quad (5)$$

5. Trapezoidal method

By applying the trapezoidal method to (1), we obtain:

$$y(t_i)=\frac{1}{2}hK_1(t_i,t_i)x(t_i)+\sum_{j=1}^{i-1}hK_1(t_i,t_j)x(t_j)+$$
$$+\frac{1}{2}hK_1(t_i,t_0)x(t_0)+\frac{1}{4}h^2K_2(t_i,t_0,t_0)x(t_0)x(t_0)+$$
$$+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(t_i,t_{0,},t_j)+K_2(t_i,t_{j,},t_0)\right)x(t_0)x(t_j)+$$
$$+h^2\sum_{j=1}^{i-1}\sum_{g=1}^{i-1}K_2(t_i,t_{j,},t_g)x(t_j)x(t_g)+ \quad (6)$$
$$+\frac{1}{4}h^2\left(K_2(t_i,t_i,t_0)+K_2(t_i,t_0,t_i)\right)x(t_0)x(t_i)+$$
$$+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(t_i,t_{i,},t_j)+K_2(t_i,t_{j,},t_i)\right)x(t_j)x(t_i)+$$
$$+\frac{1}{4}h^2K_2(t_i,t_{i,},t_i)x(t_i)x(t_i),$$

where $i=\overline{1..n}$, $h=t_i-t_{i-1}$. Let us rewrite (6) by grouping summands for the unknown quantity $x(t_i)$:

$$y(t_i)=\frac{1}{4}h^2K_2(t_i,t_{i,},t_i)x(t_i)x(t_i)+$$
$$+\left(\frac{1}{4}h^2\left(K_2(t_i,t_i,t_0)+K_2(t_i,t_0,t_i)\right)x(t_0)+\right.$$
$$+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(t_i,t_{i,},t_j)+K_2(t_i,t_{j,},t_i)\right)x(t_j)+$$
$$\left.+\frac{1}{2}hK_1(t_i,t_i)\right)x(t_i)+\sum_{j=1}^{i-1}hK_1(t_i,t_j)x(t_j)+ \quad (7)$$
$$+\frac{1}{2}hK_1(t_i,t_0)x(t_0)+\frac{1}{4}h^2K_2(t_i,t_0,t_0)x(t_0)x(t_0)+$$
$$+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(t_i,t_{0,},t_j)+K_2(t_i,t_{j,},t_0)\right)x(t_0)x(t_j)+$$
$$+h^2\sum_{j=1}^{i-1}\sum_{g=1}^{i-1}K_2(t_i,t_{j,},t_g)x(t_j)x(t_g),$$

Let`s introduce a notation:

$$A_i=\frac{1}{4}h^2K_2(t_i,t_{i,},t_i), \quad (8)$$

$$B=\frac{1}{4}h^2\left(K_2(t_i,t_i,t_0)+K_2(t_i,t_0,t_i)\right)x(t_0)+$$
$$+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(t_i,t_{i,},t_j)+K_2(t_i,t_{j,},t_i)\right)x(t_j)+\frac{1}{2}hK_1(t_i,t_i), \quad (9)$$

$$C_i=\sum_{j=1}^{i-1}hK_1(t_i,t_j)x(t_j)+$$
$$+\frac{1}{2}hK_1(t_i,t_0)x(t_0)+\frac{1}{4}h^2K_2(t_i,t_0,t_0)x(t_0)x(t_0)+$$
$$+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(t_i,t_{0,},t_j)+K_2(t_i,t_{j,},t_0)\right)x(t_0)x(t_j)+ \quad (10)$$
$$+h^2\sum_{j=1}^{i-1}\sum_{g=1}^{i-1}K_2(t_i,t_{j,},t_g)x(t_j)x(t_g).$$

Then, taking into consideration notions (8)–(10), (7) can be represented as follows:

$$A_ix_i^2+B_ix_i+C_i=0. \quad (11)$$

The value $x(t_1)$ is calculated by formula (5). n quadratic equations (11) are calculated sequentially with the use of the iterative method, where the root of the previous equation is taken as an initial estimate.

6. Software implementation of the technique

Based on the proposed algorithms in the environment of Matlab, software for restoration of signals was developed and represented in the form of two functions invvolterraseries2l and invvolterraseries2t.

x=invvolterraseries2l(kern,y,t,h) is the function for solving the Volterra polynomial integral equation of the first type and second degree based on the left rectangle method.

x=invvolterraseries2t(kern,y,t,h) is the function for solving the Volterra polynomial integral equation of the first type and second degree based on the trapezoidal method, where *kern* are kernels in the Volterra series which are set as the array of functions; y is a vector of function value $y(t)$; t is a vector of values of time variables *t*; h is a modelling step.

7. Computational experiment

Let's consider a non-linear dynamical object described by the following mathematical model:

$$\begin{cases} \dfrac{du}{dt} = -u + x; \\ y = u + u^2 \end{cases}. \qquad (12)$$

Model (12) is represented in the form of block diagram (Fig. 1).

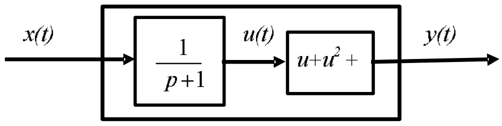


Fig. 1. Block diagram of the model (12).

By means of model equivalent transformations (12) [9] the model in the form of integro-power Volterra series is obtained:

$$y(t) = \int_0^t K_1(s)x(t-s)\,ds + \int_0^t\int_0^t K_2(s_1,s_2)\,x(t-s_1)\,x(t-s_2)\,ds_1 ds_2, \qquad (13)$$

where first-order kernel is:

$$K_1(s) = e^{-s},$$

second-order kernel is:

$$K_2(s_1,s_2) = e^{-(s_1+s_2)}.$$

Applying the compression theorem to (13), we will obtain the Volterra polynomial integral equation of the first kind and second degree with different kernels:

$$y(t) = \int_0^t K_1(t-s)x(s)\,ds + \int_0^t\int_0^t K_2(t-s_1,t-s_2)\,x(s_1)\,x(s_2)\,ds_1 ds_2. \qquad (14)$$

With the use of the designed software, a number of computing experiments on signals restoration were carried out.

Experiment No.1. In Fig. 2 the input signal fed during the experiment and output signal are presented. The application of the proposed method allowed obtaining the restored signal shown in Fig. 3, *a*, and restoration error shown in Fig. 3, *b*. The test signal was also considered with a 10 % of superimposed noise, and the result of its restoration is shown in Fig. 4.

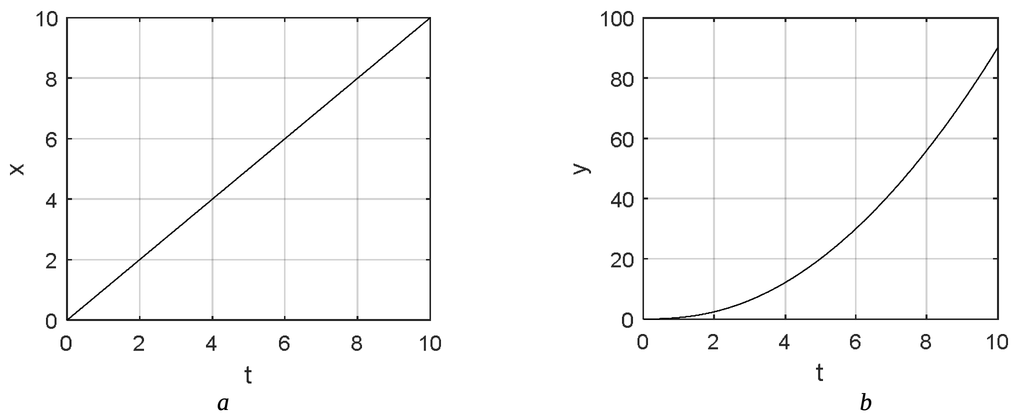


Fig. 2. Experiment No. 1: a) input signal No. 1; b) output signal No. 1.

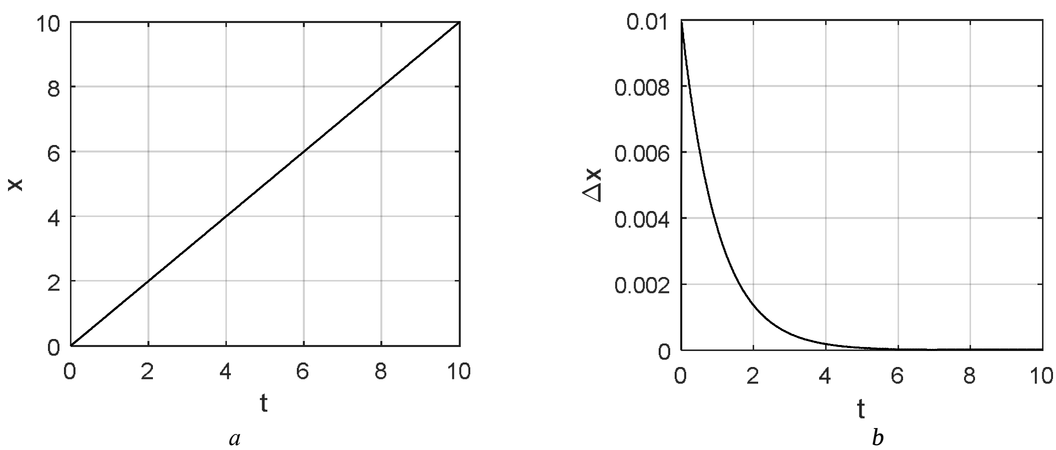
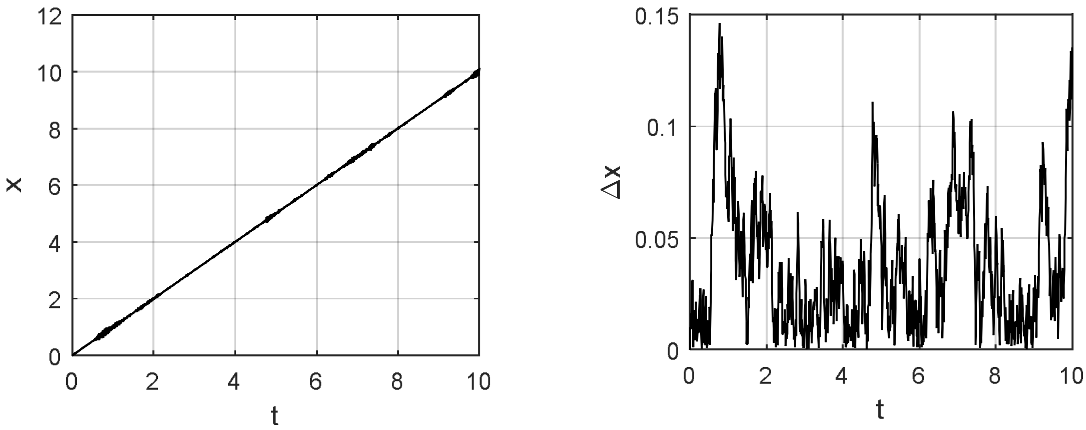


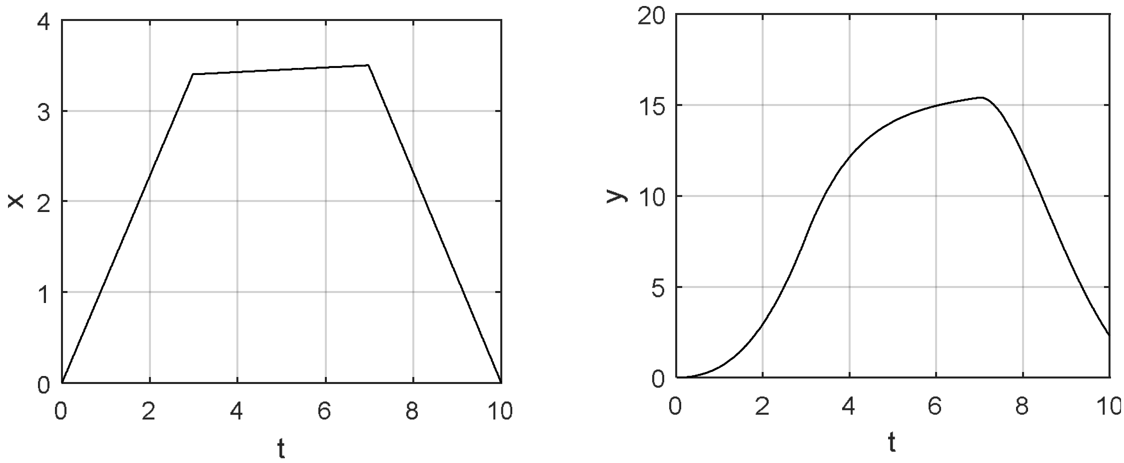
Fig. 3. Results of experiment No. 1: a – restored signal No. 1; b – restoration error No. 1.



a *b*

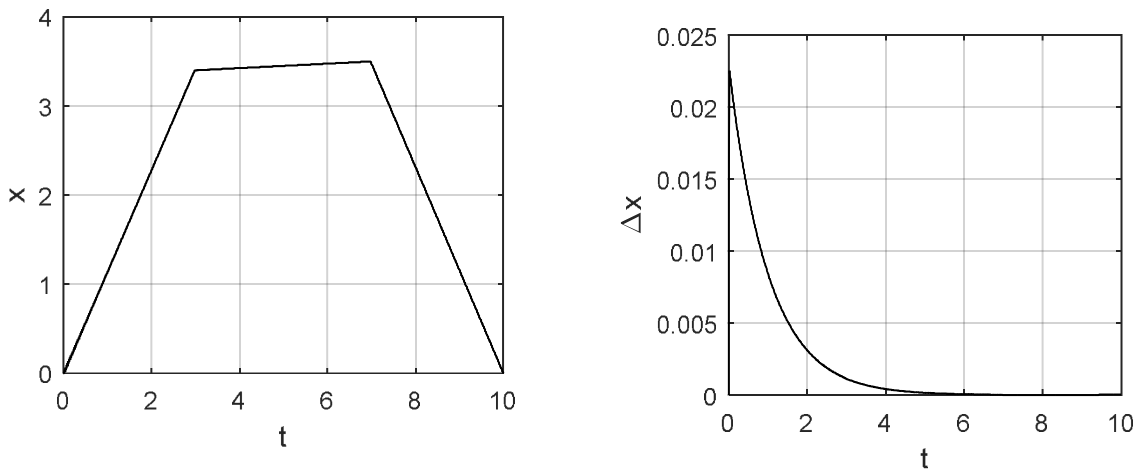
Fig. 4. Results of experiment No. 1 with superimposed noise:
a – restored signal No. 1; b – restoration error No. 1.

Experiment No 2. In fig. 5 the input and output signals are presented. Restored signal is shown in Fig. 6, *a*, restoration error is shown in Fig. 6, *b*. The result of signal restoration in case of 10 % noise superimposing is shown in Fig. 7.



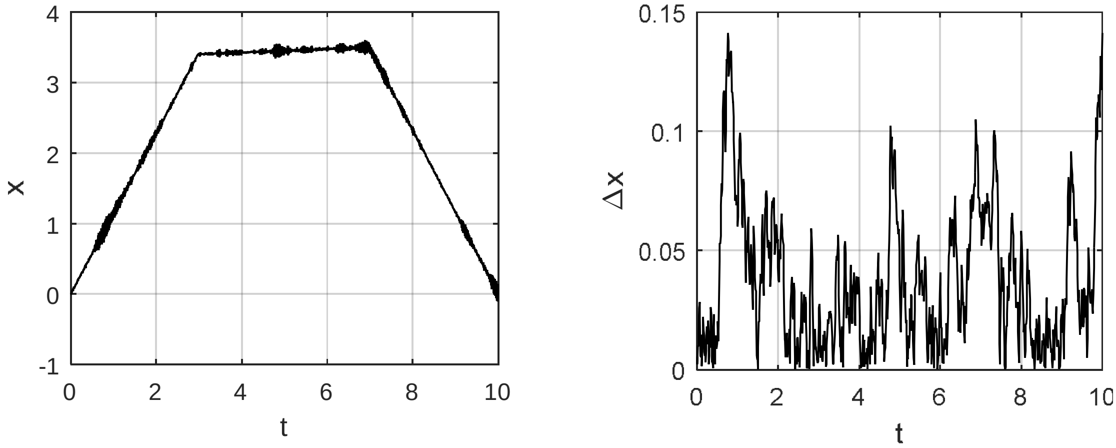
a *b*

Fig. 5. Experiment No. 2:
a – input signal No. 2; b – output signal No. 2.



a *b*

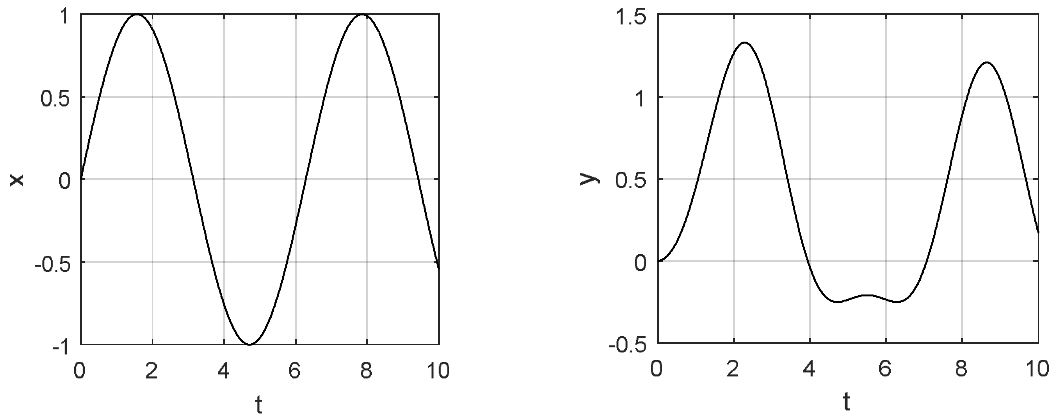
Fig. 6. Results of experiment No. 2:
a – restored signal No.2; b – restoration error No. 2.



a *b*

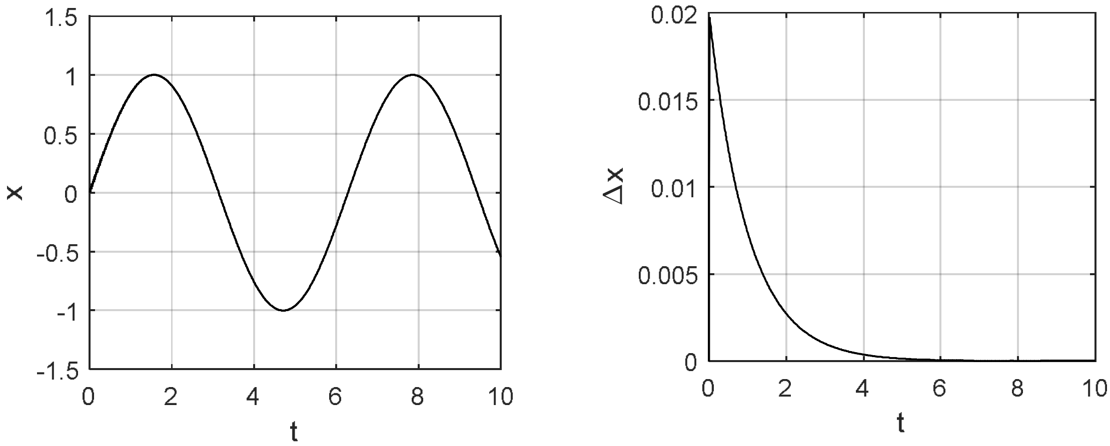
Fig. 7. Results of experiment No. 2 with superimposed noise:
a – restored signal No. 2; b – restoration error No. 2.

Experiment No. 3. Within the experiment, a sinusoidal signal was fed to the system (Fig. 8). Using developed software, the restored signal shown in Fig. 9, *a* and restoration error shown in Fig. 9, *b* were obtained. The result of restoration in case of 10 % of superimposed noise is shown in Fig. 10.



a *b*

Fig. 8. Experiment No. 3:
a – input signal No. 3; b – output signal No. 3.



a *b*

Fig. 9. Results of experiment No. 3:
a – restored signal No. 3; b – restoration error No. 3.

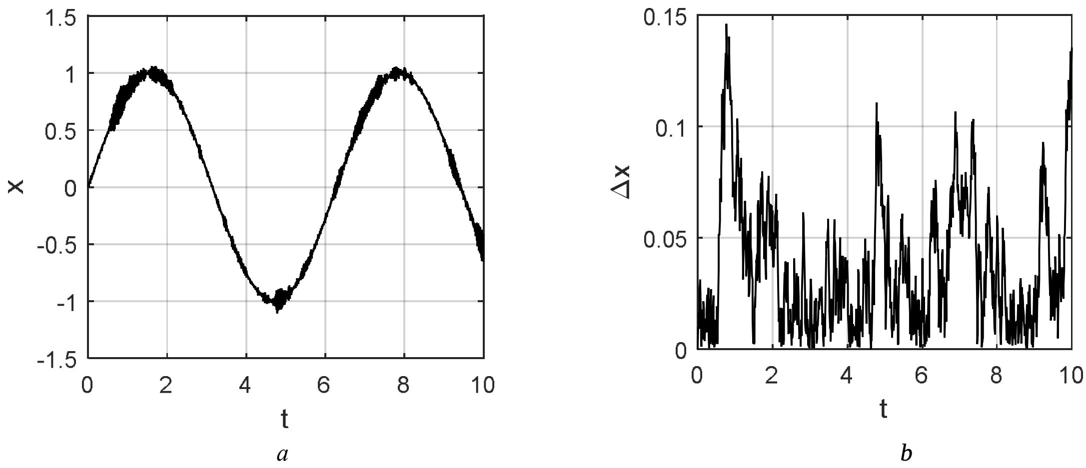


Fig. 10. Results of experiment No. 3 with superimposed noise:
a – restored signal No. 3; b – restoration error No. 3.

Experiment No. 4. Within the experiment, a unit step signal was fed to the system (Fig. 11). With the use of the developed software, the restored signal shown in Fig. 12, *a* and restoration error shown in Fig. 12, *b* were obtained. The result of restoration and the error in case of 10 % noise superimposing are shown in fig.13.

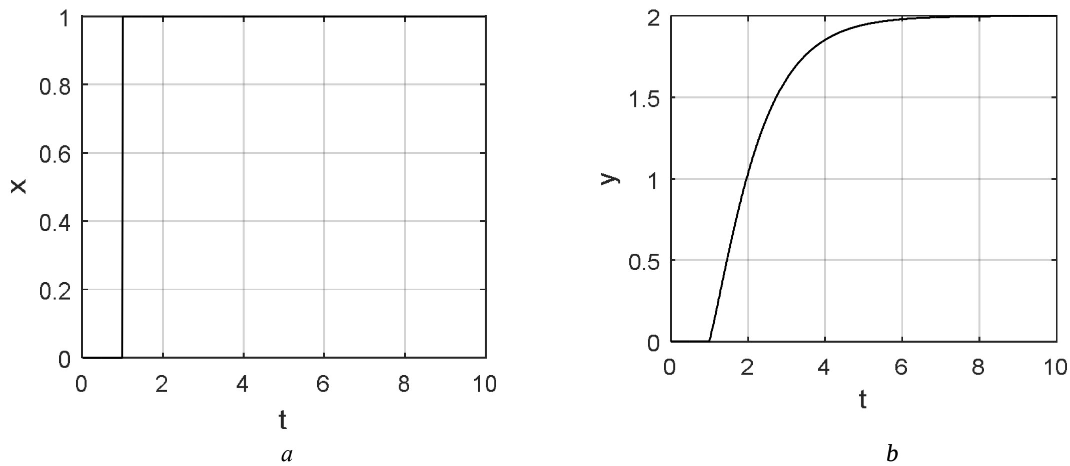


Fig. 11. Experiment No. 4:
a – input signal No. 4; b – output signal No. 4.

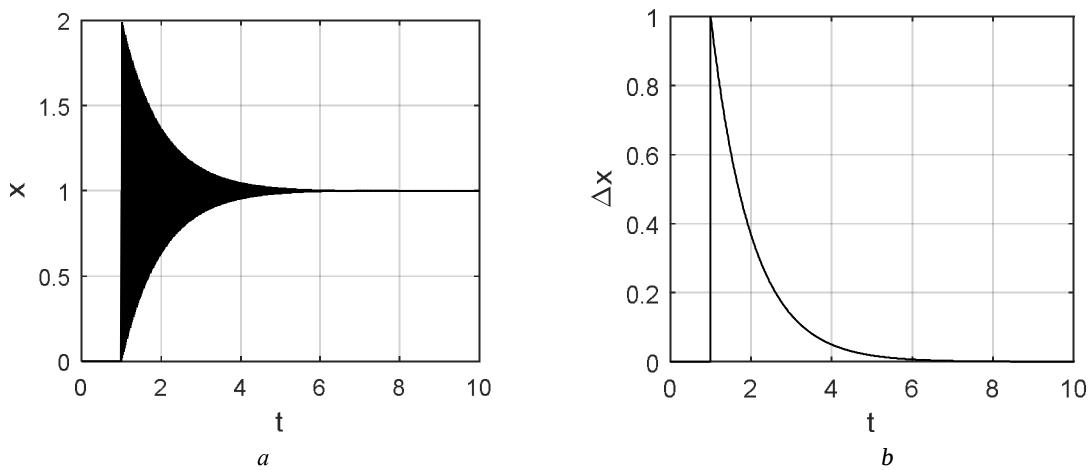
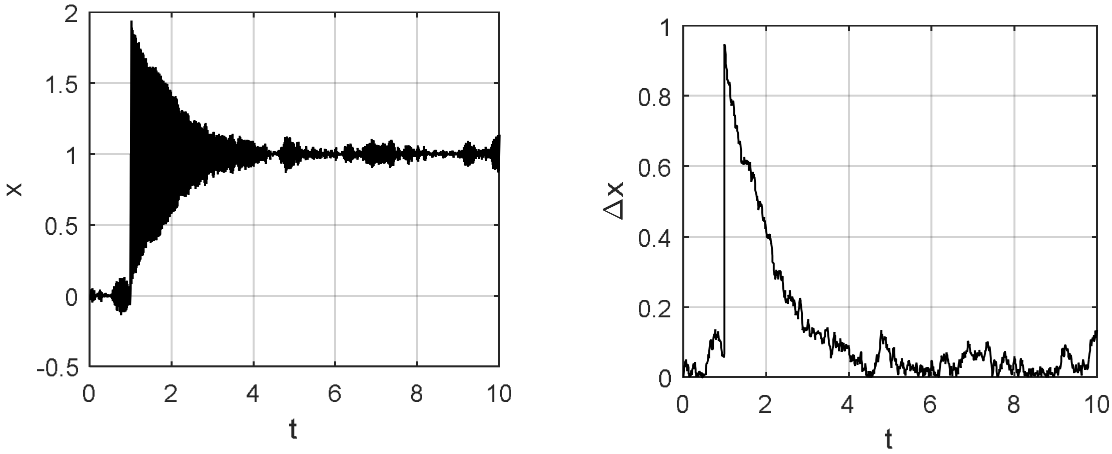


Fig. 12. Results of experiment No. 4:
a –restores signal No. 4; b – restoration error No. 4.



a *b*

Fig. 13. Results of experiment No. 4 with superimposed noise: a – restored signal No. 4; b – restoration error No. 4.

Conclusion

The results of the research have shown that the proposed method can be effectively used for the restoration of input signals of nonlinear dynamical systems described by the integro-power Volterra series with two members. Further studies will be applied to develop methods and algorithms for restoring the input signals of nonlinear dynamical systems described by the integro-power Volterra series with any number of members.

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РОЗВ'ЯЗУВАННЯ ОБЕРНЕНИХ ЗАДАЧ ДИНАМІКИ НЕЛІНІЙНИХ ОБ'ЄКТІВ НА ОСНОВІ РЯДІВ ВОЛЬТЕРРИ

Віталій Іванюк, Вадим Понеділок, Джо Стертен

Розглянуто метод розв'язування обернених задач динаміки нелінійних динамічних об'єктів, які описуються рядами Вольтерри. Як приклад розглянуто випадок ряду Вольтерри з двома членами. Запропонований підхід ґрунтується на квадратурному методі. У результаті розроблено методи розв'язування поліноміальних інтегральних рівнянь Вольтерри I роду 2-го степеня на основі методу лівих прямокутників та методу трапецій. На основі запропонованого підходу розроблено програмні засобив середовищі Matlab для відновлення сигналів нелінійних динамічних об'єктів. Ефективність засобів досліджено на ряді обчислювальних експериментів, зокрема досліджувалась можливість їх застосування в разі накладання шуму на

вхідний сигнал. Похибки обчислень значною мірою залежать від типу вхідного сигналу, зокрема для гладких сигналів похибки коливаються від 1 % до 5 %, а із накладанням 10 % шуму – до 15 %.

Отже, результати обчислювальних експериментів показали, що запропонований метод можна ефективно використовувати для відновлення вхідних сигналів нелінійних динамічних систем, які описуються інтегро-степеневим рядом Вольтерри із двома членами.

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