

MATHEMATICAL MODEL OF PERMANENT MAGNET DIRECT CURRENT MOTOR

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Abstract. Nowadays there is a tendency towards the replacement of small direct current motors with electromagnetic excitation by permanent magnet direct current (PMDC) motors. Application of field methods for simulation of PMDC motor complicates its mathematical model and requires considerable amount of time for the processing of the results obtained. The purpose of the paper is development of a mathematical model of PMDC motor on the basis of theory of electric and magnetic circuits. The mathematical model of PMDC motor has been developed on the basis of the ramified equivalent circuit of the motor magnetic core featured by high level of magnetic core detailing. In the paper, the procedure for the equivalent circuit development and the method used for determination of its elements are described. The equations of magnetic, electric, and mechanical equilibrium of PMDC motor have been obtained. The mathematical model enables us to calculate transients and steady state characteristics of a PMDC motor with sufficient accuracy.

Key words: permanent magnet direct current motor, magnetic core, equivalent circuit.

1. Introduction

Owing to the absence of excitation losses, PMDC motors have a higher output-input ratio and less mass per unit of power versus DC motors with electromagnetic excitation. Today there is a tendency towards replacing small electrically-excited DC motors with PMDC motors.

PMDC motor is widely used in such fields of industry as

- a) electrical appliances and portable electric tools;
- b) automobile industry (drive electric motors of automobile ventilators, automobile starters);
- c) computer disc drives.

There is a wide variety of the constructions of PMDC motors. The widespread one is a small motor with a parallelepiped-shaped permanent magnet that is magnetized radially (Fig. 1). This magnetic system is obtained by substituting of motor's excitation winding by the permanent magnets 1 attached to the ferromagnetic yoke of the stator 2 through which the magnetic flux flows. The armature winding 4 is located at the armature 3.

The procedure of optimum geometry selection for a brushless direct current (BLDC) motor on the basis of finite-element method was considered in [1]. Such motor consists of a slotless stator and a magnetized cylindrical rotor positioned in a uniform air gap. The methods of forming the magnetic systems created by varying magnetization vectors of adjacent magnets were shown in [2]. In [3], algorithms for computational analysis of BLDC motor with the magnetic systems described in [2] were developed on the basis of finite-element method. In [4], a simplified linear physical model of PMDC motor was considered. The obtained analytical equations enable us to calculate the overall dimensions of PMDC motor. In [5], there were given technical characteristics which determine the magnetic flux magnitude in PMDC motor with permanent magnets of different size after attenuation of overload currents effect. Their computation was performed using two- and three-dimensional models developed by combination of field and circuit methods. As one can see, methods based on the calculation of magnetic field [1–3, 5] are the most exact and quite labour-intensive ones. The application of the simplified models [4] enables obtaining equations which can be used for approximate engineering calculations.

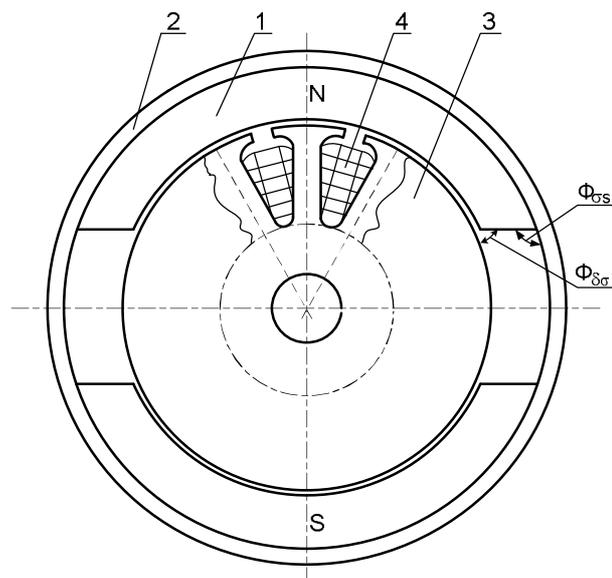


Fig. 1. PMDC motor with radial magnets.

While designing new PMDC motors it is often expedient to use armature plates, permanent magnets and stator yokes of existent PMDC motors or armature plates of DC motors with electromagnetic excitation. In this case, the process of design comes to the series of verification calculations. Using the field methods for those calculations results in the complication of mathematical models and considerable amount of time spent on the processing of the results obtained. Let us mention that PMDC motors are micromotors whose output characteristics are substantially influenced by various technological deviations (deviation of air gap size, properties of magnet materials, fringe effects and others). It can be logically assumed, that the influence of the totality of the noted technological deviations on the results of calculations makes the surplus complication of PMDC motor mathematical models needless.

Hereupon it is necessary to create an engineering mathematical model of PMDC motor with such features:

a) sufficient appropriateness, that is, taking into account basic processes in PMDC motor on the condition of neglecting secondary phenomena (e.g. commutation effects);

b) usability for practical design of PMDC motor.

The mathematical model of a shaded-pole induction (SPI) motor developed in [6] is based on representing the magnetic core by an equivalent circuit. This model enables us to calculate transients and steady-states of the SPI motor quite adequately.

In our opinion, the analysis of processes and calculation of technical characteristic of the PMDC motor can be conducted on the basis of theory of electric and magnetic circuits [6] with accuracy sufficient for engineering practice.

The purpose of the paper is the development of a mathematical model of PMDC motor on the basis of theory of electric and magnetic circuits.

2. Mathematical model of PMDC motor

The mathematical model of PMDC motor has been developed on the basis of the ramified equivalent circuit of the motor magnetic core featured by high level of magnetic core detailing.

Let us make the following assumptions for the mathematical model of PMDC motor to be developed:

a) the main magnetic field of the motor is plane-parallel;

b) the crosscoupling between the leakage flux of the armature winding and the main magnetic flux is neglectable;

c) the armature winding is represented by an infinitely thin layer of its current, evenly distributed on the surface of the slotless armature;

d) calculation of magnetization characteristics for different areas of the magnetic circuit of PMDC motor is carried out on the basis of the basic magnetization curve;

e) the tooth zone of the armature is represented by an equivalent homogeneous layer whose magnetization characteristic in the radial direction is the same as the characteristic of the real tooth zone;

f) the permanent magnet is represented by a lumped magnetomotive force (MMF) which is given by the curve $F_M = F_M[\Phi_M]$;

g) brushes are placed at the geometrical neutral axis; the direct-axis armature reaction is absent;

h) armature reaction to commutation is absent.

For the analysis of the magnetic circuit of the PMDC motor an equivalent circuit with lumped parameters (Fig. 2) is used. For convenience, the magnetic core of the PMDC motor is divided into separate areas where the field is considered to be homogeneous. The areas of magnetic core made of electric steel as well as the armature tooth zone are represented by nonlinear magnetic resistances (MR) given by the characteristics of $F[\Phi]$ as the dependences of magnetic voltage drop on magnetic flux. The areas of the air gap are represented by linear MR.

Within the limits of the pole pitch the armature with the air gap is divided by radial planes into s uniform areas. m areas out of those s areas correspond to the part of the armature located under the magnet, and n of them correspond to the part of the armature located in the space between magnets.

The area of the armature under the magnet, the air-gap and the tooth zone of the armature are represented in the equivalent circuit by a branch, where linear MR R_d , MMF of the armature T and nonlinear MR R_Z are connected in series. Nonlinear MR R_Z is described by the curve $F_Z[\Phi_d]$, where F_Z is magnetic voltage drop in the tooth zone and Φ_d is magnetic flux in the air gap adjacent to this area. In the equivalent circuit, the areas of the armature between poles which are adjacent to the areas of the armature under the magnet are represented by a branch with linear MR R_{ds} , MMF of the armature T and nonlinear MR R_Z connected in series. The linear MR R_{ds} corresponds to the magnetic flux Φ_{ds} (Fig. 1) that flows from the lateral side of the magnet to the armature. Other areas of the armature between poles are represented in the equivalent circuit by a branch consisting of MMF of the armature T and MR R_Z connected in series. Between those radial branches of the armature there are nonlinear MR R_a given by the curves $F_a[\Phi_a]$, where F_a and Φ_a are magnetic voltage drop and magnetic flux of the armature yoke in this area. In the circuit shown in Fig. 2 we have $m = 3$; $n = 3$. The

permanent magnet is represented by the lumped MMF described by the curve of demagnetization $F_M = F_M[\Phi_M]$. Areas of the stator yoke contacting with the magnet surface are represented by nonlinear MR $R_{s1}, R_{s2}, R_{s4}, R_{s5}$, what are described by the

curve $F_s[\Phi_s]$, where F_s and Φ_s are magnetic voltage drop and magnetic flux of the stator yoke in this area. Areas of the stator yoke located beyond the zone contacting with the magnet surface are represented by nonlinear MR R_{s3}, R_{s6} , described by the curve

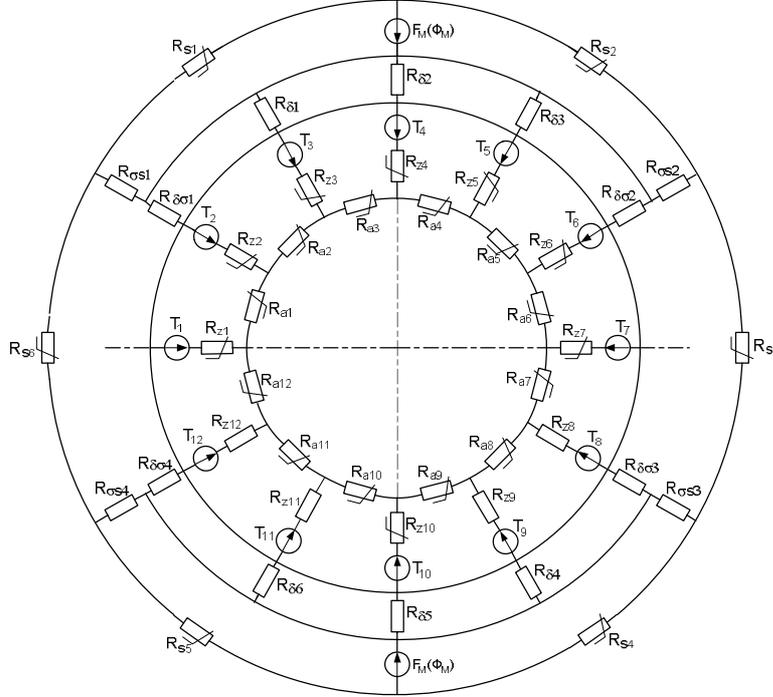


Fig. 2. Equivalent circuit of magnetic core of PMDC motor with radial magnets.

$F_{s1}[\Phi_{s1}]$, where F_{s1} and Φ_{s1} are magnetic voltage drop and magnetic flux of the stator yoke in this area. Linear magnetic leakage resistances $R_{Ss1}, R_{Ss2}, R_{Ss3}, R_{Ss4}$ correspond to the magnetic leakage flux Φ_{Ss} (Fig. 1) between the magnet and the internal surface of the stator yoke.

MMF T of the i -th area of the armature (Fig. 2) depends on the currents of the armature, its winding specification and the location of the area in relation to the direct axis d

$$T_i = T_M (2g_i/t); \quad T_M = (N/4p) (i_a/2a) \quad (1a,b)$$

where g_i stands for the angle that determines the position of the center line of the i -th area of the armature in relation to the axis d ; t is a pole pitch; i_a is current in the armature circuit; N and $2a$ are a number of armature conductors and a number of parallel branches of the armature winding; p is a number of pole pairs.

The equation of the electric state of the PMDC motor is presented as follows

$$u = \frac{d\Psi_a}{dt} + i_a r_a \quad (2)$$

where u is voltage of the motor; Ψ_a is total armature flux linkage; r_a is resistance of the armature winding.

Total armature flux linkage Ψ_a is presented as a sum of leakage flux linkage Ψ_s and main flux linkage Ψ_d

$$\Psi_a = \Psi_s + \Psi_d; \quad \Psi_s = L_s i_a; \quad (3a,b)$$

$$\Psi_d = \frac{N l}{2a t} \int_{-t/2}^{t/2} B_g dg \quad (3c)$$

where L_s is leakage inductance of the armature; l is the length of the magnetic core; B_g is flux density in the i -th area of the armature.

The system of equations of the magnetic equilibrium describing the equivalent circuit of the magnetic core of the PMDC motor (Fig. 2) can be presented as follows

$$\mathbf{W} \bar{\mathbf{i}} - \mathbf{G} \bar{\mathbf{F}}(\Phi) = 0, \quad (4)$$

where \mathbf{W} is a matrix of a number of coils; \mathbf{G} is a matrix of connections in the magnetic circuit; $\bar{\mathbf{F}}, \bar{\mathbf{\Phi}}$ are vectors of magnetic voltage drops and magnetic fluxes.

Having solved the system (5) using one of numeral methods, we obtain the values of flux density and flux itself on all areas of the magnetic core of the PMDC

motor. On the basis of these values we obtain the total armature flux linkage Ψ_a .

The equation of moments of the PMDC motor is presented as follows

$$M - M_l - J \frac{dw}{dt} = 0; \quad M = C_M i_a \Phi_d, \quad (5)$$

where M is torque; M_l is loading moment; J is inertia moment at the motor shaft; w is motor speed; C_M is torque constant; Φ_d is main flux linkage of the armature.

3. Conclusions

The mathematical model of PMDC motor has been developed on the basis of representation of the motor magnetic core by the ramified equivalent circuit. Input values of the model are instantaneous values of the armature current; applying the model, we obtain on their basis flux magnitudes in all areas of the magnetic core as well as linkage magnitudes of the armature winding. Moreover, the obtained data allow us to compute transients and steady-state characteristics of PMDC motor with accuracy sufficient for engineering practice.

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МАТЕМАТИЧНА МОДЕЛЬ КОЛЕКТОРНОГО ДВИГУНА ПОСТІЙНОГО СТРУМУ ЗІ ЗБУДЖЕННЯМ ВІД ПОСТІЙНИХ МАГНІТІВ

Василь Маляр, Ігор Гавдо

Сьогодні існує тенденція заміни мікродвигунів постійного струму з електромагнітним збудженням на двигуни постійного струму зі збудженням від постійних магнітів (ДПС ПМ). Використання для розрахунків ДПС ПМ польових методів призводить до надмірного ускладнення математичних моделей та значних затрат часу на обробку отриманих результатів. Метою статті є розроблення математичної моделі ДПС ПМ на основі теорії електричних та магнітних кіл. Математична модель ДПС ПМ побудована на основі розгалуженої заступної схеми магнітного кола машини з високим рівнем деталізації магнітопроводу. Описано послідовність розроблення заступної схеми, методик визначення її елементів. Представлено систему рівнянь електричної, магнітної та механічної рівноваги ДПС ПМ. Розроблена математична модель дає змогу розрахувати перехідні процеси та статичні характеристики ДПС ПМ з достатньою для інженерної практики точністю.



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