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THREE-INPUT INTEGRAL MODEL OF THE RECOVERY PROBLEM OF ANTENNA SIGNALS WITH INTERFERENCE

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Abstract: The problem of increasing the resolution power of an antenna through input signal recovery using the computer implementation of a mathematical model in the form of the system of three Fredholm integral equations of the first kind is examined. To solve the system of linear integral equations, regularizating algorithms and corresponding softwares based on generalized Tikhonov and Lavrentiev methods with determination of a regularization parameter by means of the model experiments technique have been developed. The algorithms are implemented in Matlab environment and can be used with other application packages. The efficiency of the developed computer tools has been confirmed by solving test and practical problems.

Key words: signal recovery, antenna, integral equation, regularization, model experiment.

1. Topicality

When modern surveillance systems being developed, the important problem in developing software systems for the recovery of distorted signals occures. In many cases, these problems are formulated in the form of integral equations of the first kind and their systems. For their solving special regularization methods can be used [1-8], which were focused and discussed in detail in a case of one integral equation. For solving systems of this type of integral equations there exist just some general recommendations. In this regard, this article examines numerical algorithms that utilize the provisions of the Tikhonov and Lavrentiev methods in combination with the method of model (computational) experiments. The algorithms are applied to solve the system of linear integral equations (SLIE) arising while solving the problem of increasing an antenna resolution and separating a weak signal from anisotropic background noise.

2. Statement of the problem

Let us assume that antenna has an directional characteristic (DC) R; its rotation allows us to measure a field U – a response at the output of the antenna - as a function of direction y. It is necessary to use the known R and U for determining the true field P at the antenna input as a function of y. Determining the field P gives us a possibility for increasing the antenna resolution power by mathematical processing of the measurements results, and this, in turn, allows us to reveal the fine structure of the field (to separate a weak signal C from the background noise, to classify components of the field P in the event of closely-spaced sources under observation).

Let us assume that the sources of field P are distributed only in the horizontal plane (if they have different elevation angles, then the following procedure will result in an elevation-averaged solution). Further, the field P is the sum of three components: a useful signal C, an interfering signal C_M, and an interference noise Π, wherein each of the components can be distributed or localized (lumped).

We assume that the components' spectra $S_C(f)$, $S_{C_M}(f)$ and $S_\Pi(f)$ (where f is frequency) are known and the scanning of the antenna, that is, measuring the field $U_i(y)$ as function of direction y in three different ($i = 1, 2, 3$) frequency bands, has been done. As a result, we get the amplitudes of the useful signal, interfering signal, and noise interference $P_C(y)$, $P_{C_M}(y)$, $P_\Pi(y)$ and so obtain a system of 3 linear integral Fredholm equations of the first kind:

$$\begin{aligned}&\int_{-p}^{p} R_{iC}(y,y')P_C(y')dy' + \int_{-p}^{p} R_{iC_M}(y,y')P_{C_M}(y')dy' + \\ &+\int_{-p}^{p} R_{i\Pi}(y,y')P_\Pi(y')dy' = U_i(y), \\ &c \le y \le d,\ i = 1,2,3,\end{aligned} \tag{1}$$

where

$$R_{iC}(y,y') = \int_{f_{1_i}}^{f_{2_i}} g(f)R_f(y,y')S_C(f)df, \tag{2}$$

$$R_{iC_M}(y,y') = \int_{f_{1_i}}^{f_{2_i}} g(f)R_f(y,y')S_{C_M}(f)df, \tag{3}$$

$$R_{i\Pi}(y,y') = \int_{f_{1i}}^{f_{2i}} g(f) R_f(y,y') S_\Pi(f) df \qquad (4)$$

and $g(f)$ is the sensitivity of transducers constituting the antenna as function of frequency f; $[f_{1i}, f_{2i}]$, $i = 1, 2, 3$ are different frequency bands.

In practice, the sources are usually located in a limited area. So, to reduce the calculations without significant losses of accuracy [10] the equation (1) may be written as:

$$\int_a^b R_{iC}(y,y') P_C(y') dy' + \int_a^b R_{iC_M}(y,y') P_{C_M}(y') dy' +$$
$$+ \int_a^b R_{i\Pi}(y,y') P_\Pi(y') dy' = U_i(y), \qquad (1')$$
$$c \le y \le d, \ i = 1, 2, 3,$$

where $[a,b]$ is a solution search area; $[c,d]$ is a measurement area.

3. Modernization of Lavrentiev and Tikhonov regularization methods for SLIE cases

The most well-known methods for solving ill-posed problems are Tikhonov [1] and Lavrentiev [2] regularization methods, Ivanov and Bakushinsky methods of quasi-solutions, the methods of statistical regularization, iterations, piece-wise integration etc. [1- 8]. However, these methods were discussed in details only for their application to scalar linear integral equations of the first kind. But for solving the systems of *n* equations, such as (1) and (1'), these methods were discussed in very general form, when corresponding equations were written in an operator notation

$$R_{uo} P = U ,$$

where R_{uo} is an integral operator. For a more detailed approach these methods require further modification if being applied to solving the SLIE.

Let us consider the application of the Tikhonov and Lavrentiev regularization methods in case of the SLIE.

We write the SLIE as

$$A_i P \equiv \sum_{j=1}^N \int_a^b R_{ij}(y,y') P_j(y') dy' = U_i(y), \qquad (5)$$
$$c \le y \le d, i = \overline{1, N},$$

where N is a number of the equations (equal to the number of the sought functions, $P_j(y')$); $R_{ij}(y,y')$ are the kernels of the integral equations; $U_i(y)$ are right sides of the equations, A_i are integral operators. In (5) the functions $R_{ij}(y,y')$ and $U_i(y)$ are considered to be known, and the functions $P_j(y')$ are sought.

The Lavrentiev method applied to the SLIE (5) looks like:

$$a_i P_i(y) + \sum_{j=1}^N \int_a^b R_{ij}(y,y') P_j(y') dy' = U_i(y),$$
$$a \le y \le b, i = \overline{1, N}, \qquad (6)$$

where a_i are the parameters of regularization. The SLIE (6) solution with correctly selected a_i is stable.

In the Tikhonov method applied to the SLIE (5) a stable solution is obtained as follows. A regularizing functional is formed:

$$\Phi = \sum_{\mathbf{l}=1}^N q_\mathbf{l} \int_c^d \left[A_\mathbf{l} P - U_\mathbf{l}(y) \right]^2 dy + \sum_{\mathbf{l}=1}^N a_\mathbf{l} \int_a^b P_\mathbf{l}^2(y') dy' =$$
$$= \sum_{\mathbf{l}=1}^N q_\mathbf{l} \left\{ \begin{array}{l} \int_c^d \left[\sum_{j=1}^N \int_a^b R_{\mathbf{l}j}(y,y') P_j(y') dy' - U_\mathbf{l}(y) \right]^2 dy + \\ + a_\mathbf{l} \int_a^b P_\mathbf{l}^2(y') dy' \end{array} \right\},$$

where $q_\mathbf{l} > 0$ are weighting factors (which are usually assumed to be equal to 1 in variational problems if there is no indication of their more exact values).

Writing down the first variations of the functional Φ and transforming them, we obtain:

$$d\Phi_i = 2q_i \int_a^b a_i P_i(y) d P_i(y) dy +$$
$$+ 2 \int_a^b \sum_{j=1}^N \left\{ \begin{array}{l} \int_a^b P_j(y') * \\ * \sum_{\mathbf{l}=1}^N q_\mathbf{l} \left[\begin{array}{l} \int_c^d R_{\mathbf{l}i}(j, y) * \\ * R_{\mathbf{l}j}(j, y') dj \end{array} \right] dy' \end{array} \right\} d P_i(y) dy - \qquad (7)$$
$$- 2 \int_a^b \sum_{\mathbf{l}=1}^N q_\mathbf{l} \left[\int_c^d R_{\mathbf{l}i}(j, y) U_\mathbf{l}(j) dj \right] d P_i(y) dy = 0$$

thuswise

$$a_i q_i P_i(y) +$$
$$+ \sum_{j=1}^N \int_a^b P_j(y') \sum_{\mathbf{l}=1}^N q_\mathbf{l} \int_c^d R_{\mathbf{l}i}(j, y) R_{\mathbf{l}j}(j, y') dj \, dy' =$$
$$= \sum_{\mathbf{l}=1}^N q_\mathbf{l} \int_c^d R_{\mathbf{l}i}(j, y) U_\mathbf{l}(j) dj$$

or

$$a_i q_i P_i(y) +$$
$$+ \sum_{j=1}^N \int_a^b \left[\int_c^d \sum_{\mathbf{l}=1}^N q_\mathbf{l} R_{\mathbf{l}i}(j, y) R_{\mathbf{l}j}(j, y') dj \right] P_j(y') dy' =$$
$$= \int_c^d \sum_{\mathbf{l}=1}^N q_\mathbf{l} R_{\mathbf{l}i}(j, y) U_\mathbf{l}(j) dj, \ a \le y \le b, \ i = \overline{1, N}$$

or, finally,

$$a_i q_i P_i(y) + \sum_{j=1}^{N} \int_a^b K_{ij}(y, y') P_j(y') dy' = w_i(y), \quad a \le y \le b, \; i = \overline{1, N}, \tag{8}$$

where

$$K_{ij}(y, y') = \int_c^d \sum_{\mathbf{l}=1}^{N} q_{\mathbf{l}} R_{\mathbf{l}i}(j, y) R_{\mathbf{l}j}(j, y') dj, \tag{9}$$

$$w_i(y) = \int_c^d \sum_{\mathbf{l}=1}^{N} q_{\mathbf{l}} R_{\mathbf{l}i}(j, y) U_{\mathbf{l}}(j) dj. \tag{10}$$

Thus, the system of N linear integral equations of the second kind has been obtained for the function $P_j(y')$, and its solution is stable at the properly chosen values a_i. The stability of the Lavrentiev and Tikhonov equations for linear integral equations of the first kind is explained in [10].

4. LSLIE and TSLIE programs for solving SLIE with regularization methods

Computational algorithms and programs LSLIE and TSLIE have been developed as *m*-files in Matlab software enwironment to solve SLIE (5) using the Lavrentiev and Tikhonov generalized regularization methods by solving SLIE (6) and (8). At $q_i = 1$ the integrals in (6) and (8) are calculated by a trapezoidal rule with a step $h = \Delta y' = const$ and then SLIEs of nN order are solved regarding to the values $P_j(y')$ in the nodes $y' = a, \; a + h, \; \ldots, \; b$.

Separately, there have been developed the programs for solving the equations (5), (6) and (8) - (10) written in the most commonly used form:

$$A_i y \equiv \sum_{j=1}^{N} \int_a^b K_{ij}(x, s) y_j(s) ds = f_i(x), \quad c \le x \le d, \; i = \overline{1, N}, \tag{5'}$$

where $K_{ij}(x, s)$ are kernels; $f_i(x)$ are the right sides being known functions (and usually with errors); $y_j(s)$ are sought functions,

$$a_i y_i(x) + \sum_{j=1}^{N} \int_a^b K_{ij}(x, s) y_j(s) ds = f_i(x), \quad a \le x \le b, \; i = \overline{1, N}, \tag{6'}$$

$$a_i q_i y_i(x) + \sum_{j=1}^{N} \int_a^b k_{ij}(x, s) y_j(s) ds = w_i(x), \quad a \le x \le b, \; i = \overline{1, N}, \tag{8'}$$

$$k_{ij}(x, s) = \int_c^d \sum_{\mathbf{l}=1}^{N} q_{\mathbf{l}} K_{\mathbf{l}i}(t, x) K_{\mathbf{l}j}(t, s) dt, \tag{9'}$$

$$w_i(x) = \int_c^d \sum_{\mathbf{l}=1}^{N} q_{\mathbf{l}} K_{\mathbf{l}i}(t, x) f_{\mathbf{l}}(t) dt. \tag{10'}$$

The integrals in (9') and (10') are calculated by Simpson's rule with the step h ($(d - c)/h$ is to be even). The resulting linear algebraic equation is solved using a module entitled *sistema*.

5. Determination of the regularization parameters by the method of model experiments

The most difficult task while using the regularization method is determining the optimal values a_{iopt} of the parameters a_i, i.e., those values a_i whereby the obtained solution $P_i(y) = P_{a\,i}(y)$ most closely reflects the exact solution $\bar{P}_i(y)$.

A rather large group of methods for determining a_{iopt} [1-9] was developed to solve scalar integral equations. Those methods are generally time-consuming. In the considered problem the antenna directional characteristic remains unchanged over the time that is sufficient for setting up and solving a so-called model example [7-11], i.e. high efficiency in such conditions was demonstrated by the method of model experiments [10], according to which a significant part of the computation can be made in advance, and after the measurement of function $U(y)$ the problem is solved by multiplying a matrix by a vector of discrete values, which does not take long. This method provides the simplicity and efficiency of computer problem solving and consists of the following steps.

In advance (before measuring the function $U_i(y)$) the mode is determined, in which the measurements and their processing is to be performed. The m**ode** is a set of values *a, b, c, d, h,* $R_{ij}(y, y')$; $\|\Delta U_i\| / \|U_i\|$ are relative measurement errors (according to a norm); $\|\Delta R_{ij}\|$ are the errors of directional characteristics. Then a **model** experiment *W* is set which has the same mode as an upcoming problem *V*. In the experiment *W* an approximate solution $P_j(y)$ is chosen on the basis of antecedent information about the upcoming problem *V*. Then, the problem *W* is solved for different a_i; out of their set the values $a_i = a_{iopt}$ that provide the most exact solutions to the problem *W* (close to the exact solution $\bar{P}_{jW}(y)$) are selected for their next utilization while solving the problem *V*.

In [10, 11] it was demonstrated, that for linear integral equations of the first kind a_{optW} approaches a_{optV}. Similarly, it can be shown that a_{optW} is close to a_{optV} for a SLIE. In a model experiment W it is not necessary to choose an exact solution that is very close to that of the real problem V (which is unknown as well), since the relative error estimate does not depend directly on the right sides $U_i(y)$ (which, in turn, depend on the sought solution), and depends only on their relative errors $\|\Delta U_i\|/\|U_i\|$. Therefore, in the problem W it is sufficient to specify that, for example, the powers of the sources C and C_M differ by a factor of ten, P_C is less than the average value of Π by a factor of ten, C and C_M are within the main lobe of the directional characteristic.

Several modes can be set and a_i can be identified for each of them by setting up and solving the model experiment ptoblem.

"Fast" solving algorithm. In the case when the regularization parameter values a_i are determined using a model experiment, the inverse matrix method can be used to accelerate the solution of SLIE (6) or (8) by a quadrature method. According to it, the solution of (6) or (8) after their algebraization can be presented as

$$P = TU, \tag{11}$$

or, correspondingly,

$$P = LU,$$

where T and L are the precalculated inverse matrices of the linear algebraic equation system (depending on the mode). Then the solving process is reduced to a simple multiplication of T or L by the vector U, which will require minimal amount of machine time.

6. Solving time estimation

As in the method described in [10, 11], most of the time is spent while determining a_{iopt} for each mode (by solving a model experiment problem), as well as computing the matrix T or L. However, that calculation is performed before a real experiment (although possibly limited in time, e.g., one day before the experiment). During the real experiment itself it is required to measure $U_i(y)$, induce the matrix T or L and multiply T or L by U (it takes much less time).

If $n = (b-a)/h + 1$ is the number of samples during scanning and $N = 3$, the order of the matrix T or L equals to $3n$ and multiplying T or L by U requires about $(3n)^3$ operations such as multiplication, addition and assignment.

7. Numerical examples

To illustrate the method, we consider a problem close to a practical one.

A sought field $P(y)$ is presented as a sum of 3 components (i.e. $N = 3$): a useful signal C, an interfering signal C_M, and a distributed interference noise Π; C and C_M being represented in the form of d-functions $p_1 d(y - y_1)$ and $p_2 d(y - y_2)$, where p_1 and p_2 are their amplitudes, y_1 and y_2 are their coordinates and Π is a smooth function

$$\Pi(y) = \begin{cases} \Pi = const, & y \in [a,b], \\ 0, & y \notin [a,b]. \end{cases} \tag{12}$$

The energy spectra C, C_M and Π are chosen as

$$S_C(f) = 1/f^2 \cdot 10^{-b f^{3/2}}, \tag{13}$$

$$S_{C_M}(f) = 1/\sqrt{f} \cdot 10^{-b f^{3/2}}, \tag{14}$$

$$S_\Pi(f) = 1/f, \tag{15}$$

where f is frequency in kHz, $b = 0.5$ kHz$^{-3/2}$.

Nine directional characteristics with $g(f) = 1$ have such a form:

$$R_{iC}(y - y') = \sqrt{\frac{\int_{f_{1i}}^{f_{2i}} R_f^2(y-y') S_C(f)df}{\int_{f_{1i}}^{f_{2i}} S_C(f)df}}, \tag{16}$$

$$R_{iC_M}(y - y') = \sqrt{\frac{\int_{f_{1i}}^{f_{2i}} R_f^2(y-y') S_{C_M}(f)df}{\int_{f_{1i}}^{f_{2i}} S_{C_M}(f)df}}, \tag{17}$$

$$R_{i\Pi}(y - y') = \sqrt{\frac{\int_{f_{1i}}^{f_{2i}} R_f^2(y-y') S_\Pi(f)df}{\int_{f_{1i}}^{f_{2i}} S_\Pi(f)df}}, \tag{18}$$

where

$$R_f(y - y') = \frac{\sin[p sf/v \cdot \sin(y - y')]}{p sf/v \cdot \sin(y - y')}, \tag{19}$$

$s = 10$ m, $v = 1500$ m/s;

1st frequency band: $f_{11} = 1{,}5$ kHz, $f_{21} = 2{,}9$ kHz (approximately an octave);

2nd frequency band: $f_{12} = 1{,}2$ kHz, $f_{22} = 2{,}4$ kHz (an octave);

3rd frequency band: $f_{13} = 2$ kHz, $f_{23} = 2{,}8$ kHz (approximately a half-octave);

The right sides of the integral equations are:

$$U_i(y) = p_1 R_{iC}(y - y_1) + p_2 R_{iC_M}(y - y_2) + \\ + \Pi \int_a^b R_{i\Pi}(y - y')dy', \quad i = 1, 2, 3. \qquad (20)$$

The integrals in (16)-(18) and (20) are numerically calculated by the Gauss formula with the accuracy up to 9 digits.

In *the model experiment W* $p_1 = 10$, $p_2 = 15$, $y_1 = -1.9^{\mathbf{0}}$, $y_2 = -2.1^{\mathbf{0}}$ (the distance between C and C_M averaged after R_{iC} and R_{iC_M} equals approximately to $\Delta\Theta_{0,7}$, where $\Delta\Theta_{0,7}$ is the main lobe width at the level 0.7, i.e. C and C_M are within the main lobe of the directional characteristic,), $\Pi = 100$.

After solving the equations (7) for different a_1, a_2, a_3 their optimal values can be found:

$$a_{1opt} = 10^{-4,0}, \; a_{2opt} = 10^{-4,3}, \; a_{3opt} = 10^{-4,5}. \qquad (21)$$

Fig. 1 and Fig. 2 show the solution of the equations (7) for $a_i = a_{iopt}$.

We can see that the obtained solutions are close to the real signals C and C_M (although their peaks are not steep enough), as well as the distributed interference Π (although with small fluctuations around the mean).

Further, the real problem V with the error $\Delta U_V = 0.028 a_4 u_{mid}$ (where a_4 is a normally distributed random variable with rms=1, u_{mid} is an average response value) was solved with the found values a_1, a_2, a_3 (21). The results are as follows: $p_1 = 10$, $p_2 = 100$, $\Pi = 130$, $y_1 = -2.96^{\mathbf{0}}$, $y_2 = 3.12^{\mathbf{0}}$.

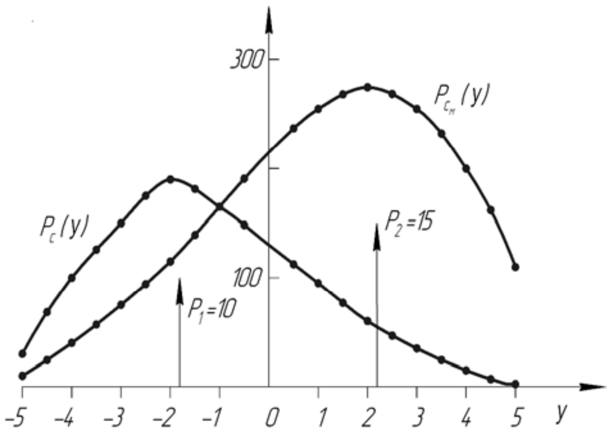

Fig. 1. Model experiment: graphs of the functions $P_C(y)$ and $P_{C_M}(y)$.

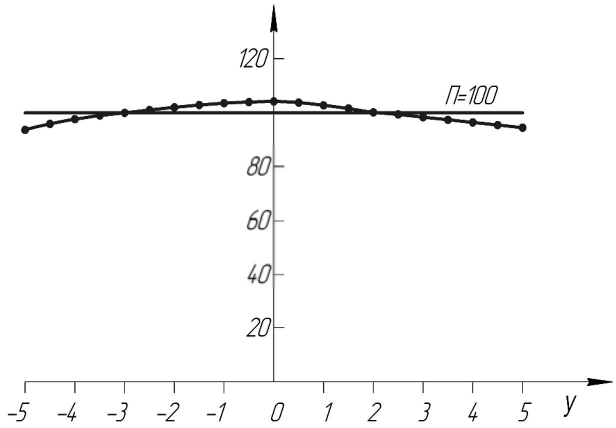

Fig. 2. Model experiment: graph of the function $P_\Pi(y)$.

Fig. 3 and Fig. 4 show the obtained solutions $P_C(y)$, $P_{C_M}(y)$, and $P_\Pi(y)$ for the real problem. It is evident that all three components C, C_M and Π can be clearly differentiated.

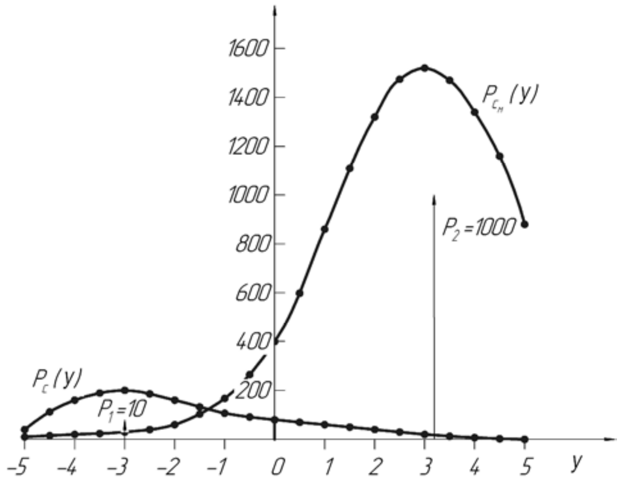

Fig. 3. Real problem: graphs of the functions $P_C(y)$ and $P_{C_M}(y)$.

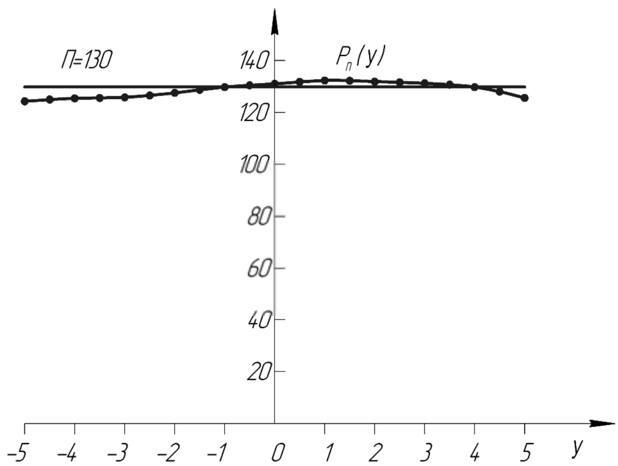

Fig. 4. Real problem: graph of the function $P_\Pi(y)$.

8. Conclusion

Numerical experiments show that the method gives correct solution for closely-spaced sources for the distance between them up to $\approx \Delta\Theta_{0,8} \div \Delta\Theta_{0,9}$ with the powers P_C and P_{C_M} that are fairly similar. When P_{C_M} increases by a factor of ten, the method is able to differentiate the sources at a distance up to $\approx \Delta\Theta_{0,5}$; increasing the distance between C and C_M up to $\approx \Delta\Theta_{0,2}$ makes it possible to resolve sources with a proportion $P_{C_M} / P_C \approx$ 40-60. All these statements are valid even in the presence of a distributed interference noise Π exceeding P_C in magnitude by a factor of ten or hundred at the antenna input (for averaging time about 5 sec). Thus, the considered regularization methods are functional, and the proposed programs are sufficiently effective, flexible, and easy to use. They can be used together with other application program packages, included in the software complex Matlab. Thus, the method of model experiments has shown its high efficiency for the task of increasing the resolution of an antenna.

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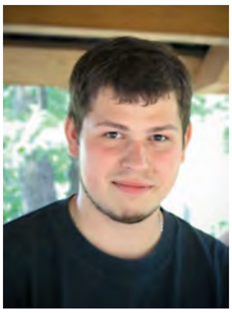
ТРИВХОДОВА ІНТЕГРАЛЬНА МОДЕЛЬ ЗАДАЧІ ВІДНОВЛЕННЯ СИГНАЛІВ АНТЕНИ З УРАХУВАННЯМ ЗАВАД

Дмитро Верлань, Наталія Костьян

Розглянуто задачу підвищення роздільної здатності антени шляхом відновлення вхідного сигналу за допомогою комп’ютерної реалізації математичної моделі у вигляді системи трьох інтегральних рівнянь Фредгольма I роду. Для розв’язання системи лінійних інтегральних рівнянь створено регуляризуючі алгоритми та відповідні програмні засоби на основі узагальнених варіантів методів Тихонова і Лаврентьєва з визначенням параметра регуляризації за методом модельних експериментів. Алгоритми реалізовано в середовищі Matlab та їх можна застосувати разом з іншими пакетами прикладних програм. Працездатність розроблених комп’ютерних засобів підтверджується розв’язанням тестових і практичних задач.



Dmytro Verlan – Postgraduate student at Taras Shevchenko National University of Kyiv, Ukraine. Main research areas are: integral equations, optimization, signal recovery, regularization, model experiments.

Nataliia Kostian – Graduated from Cherkasy Engineering and Technological Institute, Ukraine (major “Computer systems and networks”). She has been working at Kyiv National University of Technologies and Design since 2010 as Senior Lecturer at the Department of Information and Computer Technologies and Sciences.