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# FEATURES OF MONITORING THE TRACTION TRANSMISSION OF A RUNNING ELECTRICAL COMPLEX IN THE EVENT OF ITS DEVIATION FROM THE SCHEDULE

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**Abstract:** The purpose of this work is to mathematically describe the problem of traction transmission control to suppress deviations from a schedule. A mathematical description of the algorithm solves the problem of traction transmission control. The description of the problematic issue is based on the device variations. The object of the analysis is a specified performance criterion of the studied system, being considered as a function of the significant parameters of the system and the environment. The goal of optimization is to attain minimum operation per running unit at given trajectory values. The optimization resource comprises the control values of a traction motor, a parameter of complex traction control, and parameters for control of a braking force component.

**Key words:** schedule; control; running electrotechnical complex; traction transmission; optimization; functionality; variation; behind schedule.

## 1. Introduction

The category of running electrotechnical complexes include a great variety of the equipment available for use in the transportation process – trains, diesel trains and traction locomotives, electric cars, traction buses, autonomous electric vehicles, electrically driven agriculture vehicles (tractors, combines, self-propelled machines) and others. The main purpose of all these objects is to provide planned execution of transportation processes and certain related or major technical tasks (tillage, harvesting, spraying, etc.).

There are various circumstances - deviation of a true value of the moving object weight from the estimated, presence of strong head or side wind, road condition, failure to keep the time on certain areas of the path, boarding-deplaning delays or cargo-handling delays and other factors, that make it possible for the planned schedule to be deviated from. Therefore, it is necessary to adjust the magnitude of the average velocity to perform the schedule set [1–3].

## 2. Statement of the problem

When setting the problem of a rational control, we have to deal with competing indices of the operation of a running electrotechnical complex – time, speed, and energy costs. The problem of finding a rational solution is to choose a compromise for the given set of parameters.

Note the characteristic stages of the task to be solved:

1. The object of the analysis is a specified performance criterion of the studied system, considered as a function of the significant parameters of the system and the environment - minimizing the time of travelling along a certain route section at the least traction.

2. The optimization is aimed at a minimum amount of work done by the object at given point values of the trajectory. That is to say, an optimization criterion is the amount of work to be done to transport the complex from one trajectory point to another.

3. The optimization resource comprises the values of control variables of a traction motor, as well as the control parameter of tractive force, and the control parameters of a braking force value component, which is determined by the action of mechanical and electrical brakes.

Note that these values are considered continuous in terms of technical characteristics of modern systems for traction units control using on-board computers.

4. The optimization is performed taking into account the limitations imposed on phase motion coordinates (setting certain standard motion parameters – time, ranges of speed).

## 3. Analysis of recent research and publications

Many works are dedicated to the research into the process of motion parameter optimization and the process of traction control [1–7]. This confirms the relevance of the subject. However, almost all of them are devoted to operation of a railway rolling stock without including a wide range of characteristic differences of the class of running electrotechnical systems – absence of railway tracks, appearance of related technical problems along with the task of transportation, the autonomy of traction transmission of a centralized power supply system, etc.

In general, the problem of finding a rational control law can be formulated as the problem of searching an extremum (the largest or smallest value) of the function $f(x)$ of $n$-dimensional vector argument $x$ taking into account certain constraints. This task can be described by the following set of expressions

$$\min f(x),$$

where $x \in X$.

Here $X$ is a certain subset of $n$-dimensional Euclidean space $E_n$. That is, $X$ is the permissible set of the task, and the points belonging to $X$ are its valid points.

Electromagnetic variables are considered as control variables if the duration of electromagnetic transients can be neglected compared to electromechanical ones.

The solution to the stated problem should generally provide:

1. Optimal movement in performing the constraints imposed on the given values by the schedule and related technical problems. At the same time, it should be taken into consideration that in the process of operational traction transmission control, the determined motion parameters may vary within certain limits, also new parameters may be introduced, and old limitations may be abolished.

2. The best operation of the traction transmission elements in case of limitations that depend on the parameters of traction electric transmission and corresponding actuating mechanisms of the mechanical part of the electrical complex.

Where the duration of electromagnetic transients or parts thereof cannot be neglected in comparison with duration of electromechanical processes, it is necessary to consider a corresponding differential equation with appropriate variables. Then as control variables the characteristic coefficients are chosen that correspond to electromagnetic transients or their parts (that cannot be neglected in comparison with electromechanical processes) and, as well, to their differential equations; for example, voltages may be chosen as the characteristic coefficients.

The local constraints, which are primary in stating the problem, and originate from the traction asynchronous motor include:

– heat restriction, which is determined by a maximum allowable temperature curve of the traction machine elements;

– restriction due to the maximum value of the supply voltage modulus;

– restriction due to power consumption associated with the limit performance of a diesel generator or a primary source of energy;

– restriction of mechanical strength of a traction asynchronous machine rotor.

**The aim** of this article is a mathematical description of the problem of traction transmission control so that the running electrotechnical complex can suppress the deviations from a schedule provided the main technical task assigned to the object has been accomplished.

**4. Delivery of research material**

Let us suppose that a running electrotechnical complex left timely its starting point and came timely to the motion curve point $(t_1, S_1)$, but for the objective reasons described above, it did not arrive at the point with coordinates $(t_2, S_2)$, i.e. there is a deviation of the timetable curve from $t_2$ (Fig. 1).

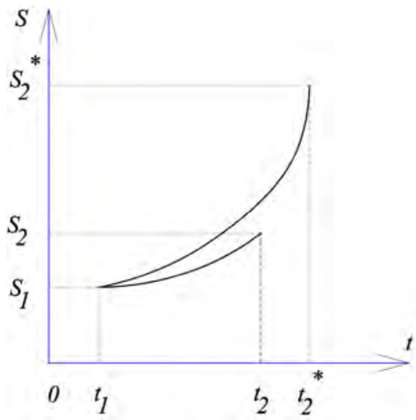


*Fig. 1. Change of the upper boundary condition of a motion curve.*

Let the object arrive at the point with the ordinate $S_2$ at $t_2^*$, i.e. the time deviation from the schedule is:

$$\Delta t_2 = t_2 - t_2^*. \quad (1)$$

We reduce the problem of changing the parameters of traction electric transmission control to overcome the actual deviation from the schedule to the problem of the calculus of variations with moving ends. The task of overcoming the actual deviation from the schedule will be solved by using the methods, the solution of which is in [8–12].

Let us suppose that the endpoint of a complex motion curve specified by certain boundary conditions can change its place. Then the class of acceptable curves connecting two points of the motion curve given by the boundary conditions is expanded – except the comparison curves that have common boundary points with the studied motion curve, according to research [11], it is also possible to take curves with displaced boundary points. So if for any curve $S = S(t)$ the condition of existing an extremum in the problem with moving boundary conditions is met, then the condition of existing an extremum is very likely to be met for a narrower class of the curves having common boundary points with the curve $S = S(t)$. Therefore, the basic condition for achieving an extremum in the problem with moving boundary points [10] is to be met – a function has to be a solution to the Euler equations.

That is to say, the curves $S = S(t)$ on which the extremum is implemented in the task set have to be extremums [9, 10].

According to the methods [12], we write down the functional variation as

$$\delta\phi = \left[\left(F - \frac{dS}{dt}\cdot\frac{\partial F}{\partial S'}\right)\delta t + \frac{\partial F}{\partial S'}\delta S\right]\Bigg|_{t_1}^{t_2} + \int_{t_1}^{t_2}\delta S\left[\frac{\partial F}{\partial S} - \frac{d}{dt}\left(\frac{\partial F}{\partial S'}\right)\right]dt. \tag{2}$$

As, according to [11], the extremum can only be achieved by the following solutions

$$S = S(t, a_1, a_2), \tag{3}$$

where $a_1, a_2$ are the options for the Euler equation solution, then to solve the problem set, it is necessary to consider the functional value only with respect to the functions of the given family.

Then the functional $\phi\left[S(t, a_1, a_2)\right]$ becomes a function of the parameters $a_1, a_2$ and the boundary conditions $t_1, t_2$, and functional variation (1) coincides with the differential of this function.

Let an increment for the endpoint of a motion curve be specified within the schedule overrun, i.e. the increment of the endpoint from the boundary traffic conditions within the time deviation from the schedule (1):

$$\Delta S_2 = S_2 - S_2^*, \tag{4}$$

where $S_2^*$ is the final value of the motion curve ordinate on the corresponding motion curve.

In accordance with the accepted system of notations, we introduce the following substitution:

$$\begin{cases} \delta t_2 = \Delta t_2; \\ \delta S_2 = \Delta S_2. \end{cases}$$

The curves $S = S(t)$ and $S = S(t) + \delta S$ can be considered close if the following system of requirements is met [12]:

$$\begin{cases} \delta t_2 \to \delta t_{2min}; \\ \delta S_2 \to \delta S_{2min}; \\ \delta S \to \delta S_{min}, \end{cases} \tag{5}$$

where $\delta t_{2min}, \delta S_{2min}, \delta S_{min}$ stand for the minimum possible value of the deviation of curve parameters within the schedule overrun.

The relevant extremals that are possible solutions to the problem and pass through the initial boundary condition form the following set of extremals:

$$S = S(t, a_1). \tag{6}$$

Then the functional $\phi\left[S(t, a_1)\right]$ on the curves of the set (6) is transformed into a function of two variables – $t_2$ and $a_1$.

If the curves of the set (6) in the neighbourhood of the extremal which is the best solution to the problem, do not intersect, the functional $\phi\left[S(t, a_1)\right]$ can be considered as a unambiguous function of the upper boundary conditions for solving the problem [12]. This is because the upper boundary conditions of the motion curve determine an extremal of the curves set (6) and thus determine the value of the functional. The latter statement is illustrated in Fig. 2, which shows a set of curves (6).

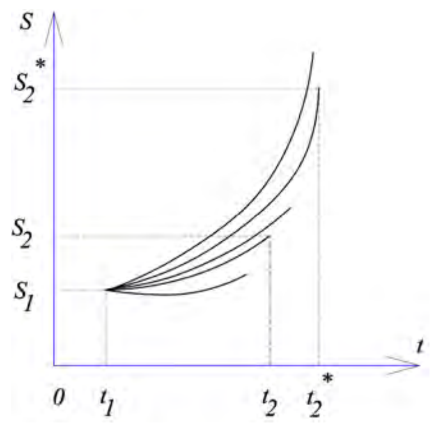


*Fig. 2. Bunch of curves in terms of (6).*

Let us perform the calculation of the variation of functional $\phi\left[S(t, a_1)\right]$ with respect to the extremals of the set $S = S(t, a_1)$ when moving the boundary point of the motion curve from coordinates $\left(t_2, S_2\right)$ to coordinates $\left(t_2^*, S_2^*\right)$.

Because the functional $\phi\left[S(t, a_1)\right]$ on the curves of the set is a function of the coordinates of the initial upper boundary condition fixed by coordinates $\left(t_2, S_2\right)$ its variation coincides with the differential of this function [9].

Let us extract the main part from the increment $\Delta\phi$, which is linear with restect to appropriate coordinate increments of the upper boundary conditions $\delta t_2, \delta S_2$. On the basis of (2), taking into account that $\Delta\phi = \delta\phi$, we have the following:

$$\Delta\phi = \int_{t_1}^{t_2+\delta t_2} F\left(t,\ S+\delta S, \frac{dS}{dt}+\frac{d(\delta S)}{dt}\right)dt + \tag{7}$$
$$\int_{t_1}^{t_2}\left[F\left(t,\ S+\delta S, \frac{dS}{dt}+\frac{d(\delta S)}{dt}\right) - F\left(t,\ S, \frac{dS}{dt}\right)\right]dt.$$

The augend of the second member of equation we transform, using the mean value theorem [12], into the following form:

$$\int_{t_1}^{t_2+\delta t_2} F\left(t,\ S+\delta S, \frac{dS}{dt}+\frac{d(\delta S)}{dt}\right)dt =$$
$$= F\big|_{t=t_2+\sigma\delta t_2}\,\delta t_2, \tag{8}$$

where the value $\sigma$ is limited as follows: $\sigma \in (0;1)$.

Given the continuity of the function $F$, it can be considered, based on study [8] that

$$F\big|_{t=t_2+\sigma\delta t_2} = F\left(t,\ S, \frac{dS}{dt}\right)\Bigg|_{t=t_2} + \varepsilon_2, \tag{9}$$

where $\varepsilon_2 \to 0$ in the fulfilment of the system of conditions (5). Then, based on (9), we write down expression (8) as follows:

$$\int_{t_1}^{t_2+\delta t_2} F\left(t,\ S+\delta S, \frac{dS}{dt}+\frac{d(\delta S)}{dt}\right)dt = \tag{10}$$
$$= F\big|_{t=t_2}\delta t_2 + \varepsilon_2\delta t_2.$$

In (7), the addend of the second member of equation will be transformed via the development of the integrand to a Taylor series causing the following results:

$$\int_{t_1}^{t_2}\left[F\left(t,\ S+\delta S, \frac{dS}{dt}+\frac{d(\delta S)}{dt}\right) - F\left(t,\ S, \frac{dS}{dt}\right)\right]dt =$$
$$= \int_{t_1}^{t_2}\left[\begin{array}{l} F_S\left(t,\ S, \frac{dS}{dt}\right)\delta S + \\ +F_{\frac{dS}{dt}}\left(t,\ S, \frac{dS}{dt}\right)\frac{d(\delta S)}{dt}\end{array}\right]dt + R_2, \tag{11}$$

where according to [10] the constant (in terms of a single problem with one set of initial parameters) quantity $R_2$ is imposed by the following system of restrictions:

$$\left\{R_2 \to 0;\ R_2 \ll \delta S;\ R_2 \ll \frac{d(\delta S)}{dt}.\right.$$

Let us integrate by parts the second term of the expression under the integral sign $\int_{t_1}^{t_2}\left[F_S\delta S + F_{\frac{dS}{dt}}\frac{d(\delta S)}{dt}\right]dt$ which is a writing of the linear part of (11). As a result of the integration, we obtain the following:

$$\int_{t_1}^{t_2}\left[F_S\delta S + F_{\frac{dS}{dt}}\frac{d(\delta S)}{dt}\right]dt =$$
$$= \left[F_{\frac{dS}{dt}}\cdot\frac{d(\delta S)}{dt}\right]\Bigg|_{t_1}^{t_2} + \int_{t_1}^{t_2}\left[F_S - \frac{d}{dt}F_{\frac{dS}{dt}}\right]dt. \tag{12}$$

According to [9], the value of a functional should be determined only on the extremals. Then we can write down the following identity:

$$F_S\left(t, S, \frac{dS}{dt}\right) \equiv \frac{d}{dt}F_{\frac{dS}{dt}}\left(t, S, \frac{dS}{dt}\right). \tag{13}$$

Based on the assumption on a constant quantity of initial coordinates of the starting route point, defined as $(t_1, S_1)$, we have the expression

$$\delta S\big|_{t=t_1} = 0.$$

That is to say, taking into account the last formula, we receive from the expression (12)

$$\int_{t_1}^{t_2}\left[F_S\delta S + F_{\frac{dS}{dt}}\frac{d(\delta S)}{dt}\right]dt =$$
$$= \left[F_{\frac{dS}{dt}}\cdot\frac{d(\delta S)}{dt}\right]\Bigg|_{t=t_1}. \tag{14}$$

According to the comparison of increments and differentials of the respective functions for classical calculus of variations problems in [11], $\delta S\big|_{t=t_2}$ is not equal to $\delta S_2$, which is an increment of the coordinates $S_2$. This is because $\delta S_2$ is the increment of the coordinates $S_2$ when a boundary condition (a boundary point in the graph of a motion curve) is shifted to the position with the coordinates $(t_2+\delta t_2, S_2+\delta S_2)$, and $\delta S\big|_{t=t_1}$ represents the increment in the ordinate point $t_2$ when transiting from the extremal passing through the points $(t_1, S_1)$ and $(t_2, S_2)$ to the extremal passing through the points $(t_1, S_1)$ and $(t_2+\delta t_2, S_2+\delta S_2)$. To investigate these statements, we denote the boundary conditions with the points that is shown in Fig. 3.

Then, on the basis of studies [10], we write down that $BD = \delta S\big|_{t=t_2}$, $FC = \delta S_2$, $EC \approx S(t_2)\delta t_2$ (the given approximated equality, according to [12], is valid to infinitesimals of higher order accuracy) $BD = FC - EC$.

The given geometric interpretation of the increment in the upper boundary condition allows the following equation to be ultimately added:

$$\delta S\big|_{t=t_2} = \delta S_2 - S(t_2)\delta t_2. \tag{15}$$

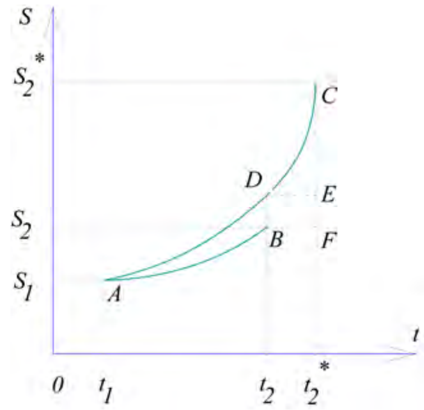


*Fig. 3. Interpretation of increment in upper boundary conditions.*

Then, finally, we can write down that

$$\int_{t_1}^{t_2+\delta t_2} F dt = F\Big|_{t=t_2} \delta t_2. \qquad (16)$$

Taking into account (15), we can write the following:

$$\int_{t_1}^{t_2}\left[F\left(t,\ S+\delta S, \frac{dS}{dt}+\frac{d(\delta S)}{dt}\right)-F\left(t,\ S,\frac{dS}{dt}\right)\right]dt \approx$$

$$\approx F_{\frac{dS}{dt}}\Big|_{t=t_2}\cdot\left(\delta S_2-\frac{dS(t_2)}{dt}\delta t_2\right), \qquad (17)$$

which, similarly to the above given approximated equality in [12], is valid with an accuracy up to infinitesimals of higher order than $\delta S_2$ and $\delta t_2$.

Then according to expression (7), we shall have

$$\Delta\phi = F\delta t_2 + F_{\frac{dS}{dt}}\Big|_{t=t_2}\cdot\left(\delta S_2-\frac{dS(t_2)}{dt}\delta t_2\right). \qquad (18)$$

After analytic transformations and using the idea of the identity of increment and functional differential (mathematical manipulation for their axes decomposing is shown in [11]), we write down (18) as

$$d\overline{\phi}(t_2,S_2)=\left(F-F_{\frac{dS}{dt}}\cdot\frac{dS}{dt}\right)\Big|_{t=t_2} dt_2 + \left(F_{\frac{dS}{dt}}\right)\Big|_{t=t_2} dS_2, \qquad (19)$$

where $\overline{\phi}(t_2,S_2)$ is the vector function, into which the extremal-established functional $S=S(t,a_1)$ is transformed, with the expressions for increments in corresponding coordinates of boundary points of a motion curve and differentials of the functions of the same variables being equivalent, that is the following equations being performed: $\begin{cases} dt_2=\delta t_2=\Delta t_2; \\ dS_2=\delta S_2=\Delta S_2. \end{cases}$

The main prerequisite of the existence of the extremum $\delta\phi=0$ has the following form [12]:

$$\left(F-\frac{dS}{dt}\cdot F_{\frac{dS}{dt}}\right)\Big|_{t=t_2}\delta t_2 + F_{\frac{dS}{dt}}\Big|_{t=t_2}\cdot\delta S_2 = 0. \qquad (20)$$

If the variations $\delta S_2$ and $\delta t_2$ are independent, it is necessary to add the following system of conditions [9]:

$$\begin{cases} \left(F-\frac{dS}{dt}\cdot F_{\frac{dS}{dt}}\right)\Big|_{t=t_2}=0; \\ F_{\frac{dS}{dt}}\Big|_{t=t_2}=0. \end{cases} \qquad (21)$$

Let us suppose that the upper boundary condition of a motion curve can move along a certain curve (Fig. 4)

$$S=\varphi(t). \qquad (22)$$

This curve passes through all possible locations of the boundary conditions of the extremal set that are the possible solutions to the problem set (Fig. 4).

Based on the presented mathematical steps for determination of an electric vehicle complex trajectory, we choose a monitoring system for the traction transmission.

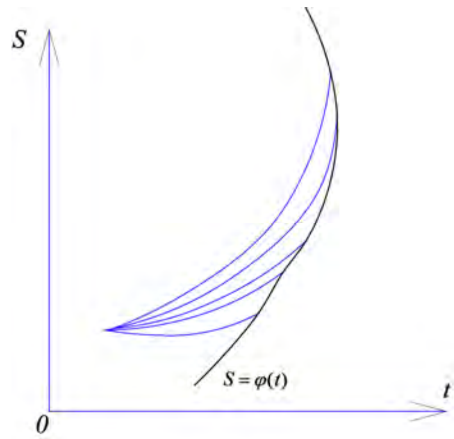


*Fig. 4. The curve along which the locus of the upper boundary conditions of a motion curve is located.*

## 5. Conclusions

Using the presented mathematical model of monitoring the motion modes allows an efficient change of traction for keeping the object schedule. To ensure the practical realization of the problem by using an information-control system of traction transmission performance, the proposed algorithmic problem solution sequence is to be used as a tool for implementing the traffic control algorithm. Further research in this direction consists both in developing a complete mathematical model of the controlled object and in simulating solutions developed on its basis.

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## ОСОБЛИВОСТІ ЗАДАЧІ КЕРУВАННЯ ТЯГОВОЮ ЕЛЕКТРОПЕРЕДАЧЕЮ РУХОМОГО ЕЛЕКТРОТЕХНІЧНОГО КОМПЛЕКСУ ПРИ ВІДХИЛЕННІ ВІД ГРАФІКА РУХУ

Дмитро Кулагін, Петро Андрієнко

Автори математично описали задачі керування тяговою електропередачею для подолання рухомим електротехнічним комплексом відхилення від графіка руху. В статті запропоновано математичне описання алгоритму вирішення поставленої задачі керування тяговою електропередачею. Описання проблемного питання здійснено на основі апарату варіаційного числення. Об’єктом аналізу заданий критерій ефективності досліджуваної системи.

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