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RECONSTRUCTION OF GIVEN SEQUENCE OF IMPULSES BASED ON MULTIINPUT SPIKE NEURON

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The problem of construction of idealizing sequence of spikes as the weighted sum of input sequences of spikes, that is important for learning procedure of spike neural network on the basis of methods of linear algebra with the use of idealizing impulses with a zero duration and single amplitude and conception of space of sequences of spikes is considered. It is proved that inner product of the weighted time series of spikes generates the proper metric which can be used for the estimation of neighborhood of two sequences. For determination of weight coefficients a least squares method is used. Resultant spike sequence is formed with the use of fuzzy numbers. Two examples of approximation of the sequence of spikes are demonstrated.

Keywords – spike neural network, spike-sequences, method of linear algebra, spike learning.

Introduction

Spike neural networks belong to the third generation of neural networks in which information is encoded by time between impulses (spikes) [1, 2]. Therefore the learning procedures of such networks are significantly different from the learning procedures of neural networks of previous generations with the threshold and continuous activation functions. Among the learning procedures of spike neural networks it can be distinguished [3]: 1) gradient methods (SpikeProp), close to the similar ones for neural networks with continuous activation functions; 2) statistical methods; 3) methods of linear algebra; 4) methods that are based on evolutionary strategies; 5) method ReSuMe and others. However, despite the large number of publications on this topic, some questions concerning the learning procedures of spike neural networks are not investigated enough.

Statement of the problem of spike sequence reconstruction

The method of spike neural network learning based on the concepts and methods of linear algebra: vector N -dimensional space, inner product of vectors, metrics and more, was developed in [4]. It provided the possibility to formalize, to certain extent, the process of learning. In this paper, unlike considered in [4], the method of least squares for determination of weighting coefficients of spike sequences is used and the reformation spike sequence with a large number of impulses which was received on the basis of the input sequences to given spike sequence with the using of method of fuzzy numbers is performed.

Spike neural network works with time series of idealized impulses which are described by the function:

$$x(t,t_i)=\begin{cases}1, & t=t_i\\ 0, & t\neq t_i\end{cases}. \qquad (1)$$

Thus time series of N impulses if described by the term:

$$w(t)=\sum_{i=1}^{N}x(t,t_i), \qquad (2)$$

where t_i - different among themselves time samples.

At the input of neuron in spike neural network weighted sum of time series is given:

$$g_{in}(t) = \sum_{j=1}^{k} c_j w_j = \sum_{j=1}^{k} c_j \sum_{i=1}^{N} x(t, t_{ji}) . \qquad (3)$$

At the output of neuron time series $g_{out}(t)$ by using of some spike neuron model is received.

As the learning problems of such simple neural structure two tasks can be formulated [4]:

1) determination the coefficients c_j, $j = \overline{1,k}$, such that the function $g_{in}(t)$ has been as close as possible to the given $g_{in}^*(t)$;

2) determination the coefficients c_j, $j = \overline{1,k}$, such that the function $g_{out}(t)$ has been as close as possible to the given $g_{out}^*(t)$.

The values $g_{in}(t)$, $g_{in}^*(t)$, $g_{out}(t)$, $g_{out}^*(t)$ can be considered as vectors in N-dimensional space. To assess the closeness of vectors must be entered in the space metric (norm). For this initially the concept of inner product of two spikes and sequence of spikes is introduced.

Inner product of two spikes $x(t,t_i)$ and $x(t,t_j)$ is defined by the expression:

$$< x(t,t_i); x(t,t_j) >= e^{-\alpha |t_i - t_j|}, \qquad (4)$$

where $\alpha > 0$ - some scaling parameter.

Inner product of two weighted time series is defined as:

$$\left\langle c_k \sum_{i=1}^{N} x(t,t_{ki}); c_m \sum_{j=1}^{N} x(t,t_{mj}) \right\rangle = c_k c_m \sum_{i,j} x(t,t_{ki}) \cdot x(t,t_{mj}) =$$

$$= c_k c_m \sum_{i,j} x(t,t_{ki}) \cdot x(t,t_{mj}) e^{-\alpha |t_{ki} - t_{mj}|} . \qquad (5)$$

Based on the introduced inner product the norm is determined:

$$\|\circ\| = \sqrt{\left\langle c \sum_{i=1}^{N} x(t,t_i); c \sum_{j=1}^{N} x(t,t_j) \right\rangle} = \sqrt{c^2 \sum_{i,j} e^{-\alpha |t_i - t_j|}} . \qquad (6)$$

It is obvious that in such a way introduced inner product and norm can be used as measure of closeness of two time series.

Metrics for weighted spike-sequences

Generalized conception of weighted time series with different weights for each of the components of the time series and the corresponding inner product conception and metrics are introduced in [4]. Let us consider the introduction of the metric in this case. Suppose the weighted time series in the form of spikes is given as:

$$w(t) = \sum_{i=1}^{N} c_i x(t,t_i) . \qquad (7)$$

Norm in this case will have the form:

$$\|\circ\| = \sqrt{< w(t); w(t) >} = \sqrt{\left\langle \sum_{i=1}^{N} c_i x(t,t_i); \sum_{j=1}^{N} c_j x(t,t_j) \right\rangle} =$$

$$= \sqrt{\sum_{i,j} c_i c_j \left\langle x(t,t_i); x(t,t_j) \right\rangle} = \sqrt{\sum_{i,j} c_i c_j e^{\alpha |t_i - t_j|}} . \qquad (8)$$

The radicand is a quadratic form with matrix:

$$T=\begin{bmatrix} 1 & \dots & e^{-\alpha|t_1-t_N|} \\ e^{-\alpha|t_2-t_1|} & \dots & e^{-\alpha|t_2-t_N|} \\ e^{-\alpha|t_3-t_1|} & \dots & e^{-\alpha|t_3-t_N|} \\ \dots & \dots & \dots \\ e^{-\alpha|t_N-t_1|} & \dots & 1 \end{bmatrix}. \quad (9)$$

Obviously that the matrix T is symmetric. In order for such a way introduced norm could be some metrics, it is necessary to prove that the quadratic form is positive definite. A necessary and sufficient condition for this is the positive value of all major minors of the quadratic form of matrix [5]. Not to reduce the generality of the problem, we assume that $t_1 < t_2 < \dots < t_N$. Then the expression is correct:

$$e^{-\alpha|t_i-t_{i+2}|}=e^{-\alpha|t_i-t_{i+1}|-\alpha|t_{i+1}-t_{i+2}|}=e^{-\alpha|t_i-t_{i+1}|}\cdot e^{-\alpha|t_{i+1}-t_{i+2}|}.$$

In the general case (at $k>0$):

$$e^{-\alpha|t_i-t_{i+k}|}=e^{-\alpha|t_i-t_{i+1}|-\alpha|t_{i+1}-t_{i+2}|-\dots-\alpha|t_{i+k-1}-t_{i+k}|}=$$
$$=e^{-\alpha|t_i-t_{i+1}|}\cdot e^{-\alpha|t_{i+1}-t_{i+2}|}\cdot\dots\cdot e^{-\alpha|t_{i+k-1}-t_{i+k}|}. \quad (10)$$

Let us denote $e^{-\alpha|t_i-t_{i+1}|}=a_{i,i+1}$. As a result, the matrix T will have the form:

$$T=\begin{bmatrix} 1 & \dots & a_{12}a_{23}\dots a_{N-1,N} \\ a_{12} & \dots & a_{23}a_{34}\dots a_{N-1,N} \\ a_{12}a_{23} & \dots & a_{34}a_{45}\dots a_{N-1,N} \\ \dots & \dots & \dots \\ a_{12}a_{23}\dots a_{N-1,N} & \dots & 1 \end{bmatrix}. \quad (11)$$

Let us consider an arbitrary major minor of k-th order:

$$M_k=\begin{vmatrix} 1 & \dots & a_{12}a_{23}\dots a_{k-1,k} \\ a_{12} & \dots & a_{23}a_{34}\dots a_{k-1,k} \\ a_{12}a_{23} & \dots & a_{34}a_{45}\dots a_{k-1,k} \\ \dots & \dots & \dots \\ a_{12}a_{23}\dots a_{k-1,k} & \dots & 1 \end{vmatrix}. \quad (12)$$

Let us subtract from the k-th row $(k-1)$-th row multiplied by $a_{k-1,k}$, from $(k-1)$-th row $(k-2)$-th row multiplied by $a_{k-2,k-1}$, etc., from the third row the second row multiplied by a_{23}, and from the second row the first row multiplied by a_{12}. In this case minor will increase at $a_{12}\cdot a_{23}\cdot\dots\cdot a_{k-1,k}$ times, that is the sign of minor will not change since $a_{i-1,i}>0$. As result we obtain the minor:

$$M^*_k=\begin{vmatrix} 1 & \dots & a_{12}a_{23}\dots a_{k-1,k} \\ 0 & \dots & a_{23}a_{34}\dots a_{k-1,k}(1-a_{12}^2) \\ 0 & \dots & a_{34}a_{45}\dots a_{k-1,k}(1-a_{23}^2) \\ \dots & \dots & \dots \\ 0 & \dots & 1-a_{k-1,k}^2 \end{vmatrix}. \quad (13)$$

The resulting minor has three-diagonal structure, its value is:

$$M_k^*=(1-a_{12}^2)(1-a_{23}^2)\dots(1-a_{k-1,k}^2). \quad (14)$$

Since $0 < a_{i,i+1} < 1$, the value $M_k^* > 0$, as it was required to prove. To prove that the introduced inner product generates relevant norm the construction of some function $F(w)$ that is interpreted as a hypothetical synaptic reaction to weighted spike sequence is used in [4].

Determination of weights

Let us given weighted time series $g^* = \sum_{i=1}^{N} a_i x(t, t_i)$ should be approximate by the linear combination of time series of spikes w_1, w_2, … , w_k:

$$g = \sum_{i=1}^{k} c_i w_i = \sum_{i=1}^{k} c_i \sum_{j=1}^{N} x(t, r_{ij}) . \quad (15)$$

To solve this problem two methods are used in [4]. The first one based on time series of spikes, w_1, w_2, … , w_k an orthogonal basis w_1^*, w_2^*, … , w_k^* is constructed using the procedure of Gram-Schmidt orthogonalization and projection of given sequence g on this basis is searched. Obtained by that expansion coefficients are the desired weights. The second method is iterative and consists in successive reduction of norm difference between sequences g and g^*.

Below this problem is solved using the method of least squares.

The difference between the time series:

$$g - g^* = \sum_{i=1}^{k} c_i \sum_{j=1}^{N} x(t, r_{ij}) - \sum_{i=1}^{N} a_i x(t, t_i) . \quad (16)$$

Square norm of the difference:

$$E = \left\langle g - g^*; g - g^* \right\rangle = \left\langle \sum_{i=1}^{k} c_i \sum_{j=1}^{N} x(t, r_{ij}) - \sum_{i=1}^{N} a_i x(t, t_i); \sum_{i=1}^{k} c_i \sum_{j=1}^{N} x(t, r_{ij}) - \sum_{i=1}^{N} a_i x(t, t_i) \right\rangle =$$

$$= \sum_{i=1}^{k} \sum_{p=1}^{k} c_i c_p \sum_{j=1}^{N} \sum_{m=1}^{N} e^{-\alpha |r_{ij} - r_{pm}|} - 2 \sum_{i=1}^{k} c_i \sum_{j=1}^{N} \sum_{e=1}^{N} a_e e^{-\alpha |r_{ij} - t_e|} + \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j e^{-\alpha |t_i - t_j|} . \quad (17)$$

Let us minimize this norm by the values c_i, $i = \overline{1;k}$ by the method of least squares. The condition of minimum E: $\frac{\partial E}{\partial c_i} = 0$, $i = \overline{1;k}$.

As a result, we obtain a system of k linear algebraic equations as to k unknown c_i, $i = \overline{1;k}$:

$$\sum_{p=1}^{k} c_p \sum_{j=1}^{N} \sum_{m=1}^{N} e^{-\alpha |r_{ij} - r_{pm}|} = \sum_{j=1}^{N} \sum_{m=1}^{N} a_m e^{-\alpha |r_{ij} - t_m|} . \quad (18)$$

After solving this system of equations we obtain the weights for the input sequences of spikes. The output sequence of spikes with using these weights generally contains of $k \times N$ impulses that need to be reduced to N impulses.

These impulses are distributed in such a way that most of them are concentrated in the vicinity of the given sequence of impulses. The transformation of the number of impulses $k \times N$ to the number of impulses N (or close to this value) is carried out in two stages. At the first stage the “fuzzyfication” of each of impulses in obtained sequence is carried out, that is the transition from clear to fuzzy numbers is used. At that Gaussian membership function $\exp(-\beta (t - t_i)^2)$, where $\beta > 0$, is used. After summing the fuzzy numbers continuous function of time $f(t)$ with the maxima concentrated in a neighbourhood of a given sequence of impulses is obtained.

The next step is the reverse transition from fuzzy to clear information. To do this temporal coordinates t_i^* of maxima of function $f(t)$ that exceed a certain threshold h are defined. These coordinates define the resulting spike sequence.

Examples of reconstruction of spike sequences

For practical implementation of the algorithm MATLAB was used.

Using the generator of pseudorandom numbers on the interval [0, 1] sample of 20 random input spike sequences with amplitude 1, each of which contained 10 impulses, and given spike sequence of 10 impulses shown in Fig. 1,a) were formed.

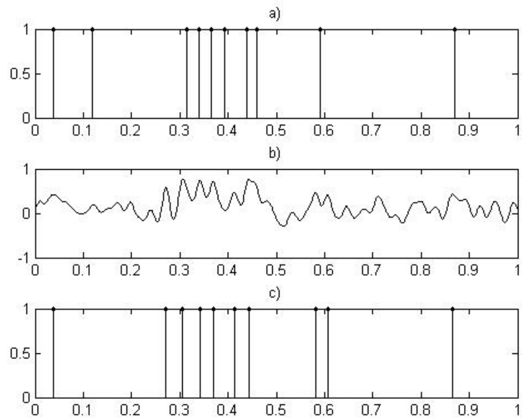

Figure 1. Example of reconstructing a spike sequence with 20 input sequences

After the transition to fuzzy numbers function $f(t)$ shown in Fig. 1,b) was obtained. As result of the transition to clear numbers by selecting maxima in the "fuzzy" output function and setting the threshold $h = 0{,}5$ we get output spike sequence as close as possible to the desired (Fig. 1,c)). As can be seen by comparing Fig. 1,a) and Fig. 1,c) the output sequence is not enough close to given one, because of the small number of input spike sequences.

To increase the accuracy approximation of the output spike sequences to given, sample of 100 random input spike sequences was formed. The results of reconstructing a sequence of impulses are shown in Fig. 2. As can be seen by comparing Fig. 2,a) and Fig. 2,c) received sequence is similar to that specified.

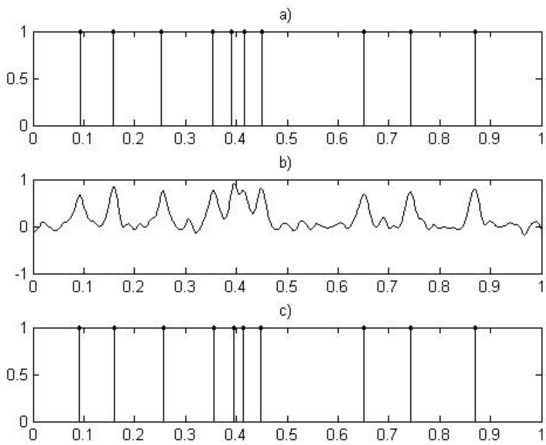

Figure 2. Reconstruction of spike sequence with 100 input sequences

Conclusions

In the learning procedures of spike-neural networks it is necessary to determine the weights such a way that approximate given spike sequence by the weighted sum of the input impulses sequences. To solve this problem at idealized spike-sequences introduction of metric, the method of least squares to determine weights, the procedure of transition to fuzzy numbers and on the back from fuzzy numbers to clear were used. Results of algorithm are illustrated by two examples approximation of given sequence of 10 impulses by 20 and 100 spike sequences.

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