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High-Speed Computer Simulation of Electrical Circuits

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Abstract

The well-known numerical-analytic method based on zeroes/poles matching (as z-transform realization) of equivalent transfer function or complex impedance (conductivity) in Laplace domain was proposed for high-speed computer analysis of electric circuits' transients. The advantages of this method were illustrated on simple examples of the computer models of the first order electric circuit (RC-circuit that correspond to real pole) and second order electric circuit (RLC-circuit that correspond to complex pair of poles). These models were verified to the computer models based on the analytic method using the Laplace transform. The method can be applied to modern computer programs of power systems stability analysis, electrical circuits' transients' calculations and common systems dynamic analysis.

Keywords: computer simulation; electrical circuits; transient processes; zeros-poles matching method; z-transform.

1. Introduction

The computer simulations of the electrical systems and circuits are usually based on the numeric methods for the ordinary differential equations (ODEs). It is quite easy to solve the systems of the ODEs with smooth or low varying solutions if high-speed simulation are not required but problems appear when solutions are complicated with the very fast components of the process and must be implemented in real time or faster. The example to show one part of these problems using well-known environment MATLAB with Simulink can be found in [1] – the different methods for ODEs give the different results and very various calculation time for computer simulation of AC electric drive. Note that automatic step control of simulation can't to improve this situation. Different solutions (*only for second-order system of equations!*) obtained by means of different numeric methods intended for solving ODEs with the automatic step control strategy were described in [2] also as example for MATLAB ODEs suit. This phenomenon of ODEs solution using the numeric methods is the consequence of their basic principle – all numeric methods approximate the solution by the limited Taylor series that is suitable for continues smooth functions only. As the result, modern electric systems with pulse-width modulation (PWM) power electronics can't be simulated using the traditional approach because their signals are sampled (discontinued).

Such problem can be solved using analytic or semi-analytic (or semi-numeric) methods some of which are based on z-transform [3], [4], [5]. This method is suitable for nonlinear (linearized) systems ([4], [5], [6]) and is similar to Laplace operator method [7] but produces the recurrent simulation equations directly. This approach was well known during early-computer age (approx. 60-70 years of XX cent.) then computers were big, slow and had small memory capacity. But scientists and engineers solved many complex computing problems using

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the above method at that time (for example [8], [9]). Note that z-transform-based method is actual in the modern time [10]. The common problems such as:

- researcher must make the great amount of analytic transformations during the preparation process;
- researcher must understand the simulated problem well, made such an approach unpopular.

In the authors' opinion, this forgotten method is the best way to simulate the electric circuits and systems in real time or faster. It uses the recurrent simulation formulas based on zeros/poles matching method that is well known using as z-transform [3], [4]. This will be shown by two simple examples below.

2. Fundamentals

All of the real linear or linearized systems that can be described by transfer functions (for a control theory) or complex impedance (conductivity) in Laplace domain (for electric circuits' analysis) have the numerator polynomial order no greater than denominator polynomial order. We can decompose (residue) these systems to the elementary particles (fig. 1) using Heaviside theorem [11] in this cases:

- simple real poles (correspond to the first order block);
- complex conjugate of the poles (correspond to the second order block).

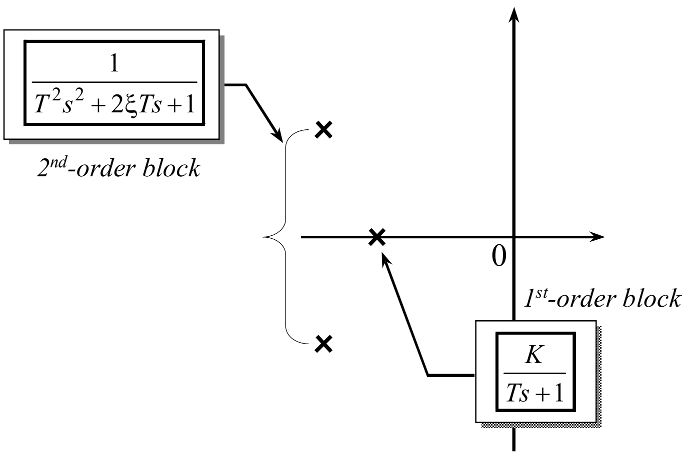


Fig. 1. The two types of the elementary dynamic blocks

These two blocks are the elementary parts to construct the whole designed computer model. The responses of every elementary dynamic block can be represented as the discrete transfer functions using zeros/poles matching method as the z-transform. This method corresponds to convolution integral using zero-order approximation of the signal (rectangles approximation) that produces simple but effective recurrent modelling equations. Some problems can be solved using this approach:

- absolute numerical stability – the theorem of the strong numeric stability of this formulas was proved in [11];
- simplicity of the obtained recurrent formulas – they are simple and comprehensible;
- obtained equations are quite effective – the operating step during the computer simulation is limited by the Nyquist-Shannon sampling theorem.

The basic principle of zeros/poles matched method is corresponding Laplace domain to unit circle (discrete-time equivalent or z-domain – asterisk marked).

$$\prod_{i=1}^{n}(s-R_i) \overset{\text{sampling}}{\Longrightarrow} \prod_{i=1}^{n}(z-R_i^*),$$

where R_i – i-th root of numerator/denominator of the system transfer function (it's zeros/poles); R_i^* – i-th root of numerator/denominator of the discrete transfer function of system that corresponds to a discrete-time $R_i^* = e^{R_i \cdot h}$ (it's discrete zeros/poles), where h is sampling time.

As the result, continuous-time transfer function (or impedance/conductivity) can be rewritten to discrete-time transfer function (or impedance/conductivity) with a sampling time h after this discretization procedure:

$$K\frac{\prod_{i=1}^{m}(s-Z_i)}{\prod_{j=1}^{n}(s-P_j)} \underset{P_i^*=e^{P_i\cdot h}}{\overset{Z_i^*=e^{Z_i\cdot h}}{\Rightarrow}} K^*\frac{\prod_{i=1}^{m}(z-Z_i^*)}{\prod_{j=1}^{n}(z-P_j^*)},$$

where K – the DC gain of the continuous-time system; K^* – the DC gain of the discrete-time system; m – order of numerator of the system (electrical circuit); n – order of denominator of the system (electrical circuit); Z_i – i-th root of numerator of the system transfer function (it's zeros); Z_i^* – i-th root of numerator of the discrete transfer function (it's discrete zeros); P_j – j-th root of denominator of the system transfer function (it's poles); P_i^* – j-th root of denominator of the discrete transfer function (it's discrete poles).

The last step is setting DC (or low-frequency) gain of the discrete transfer function equal to continuous-time system gain (or equal to impedance/conductivity on DC):

$$\lim_{s\to 0} K\frac{\prod_{i=1}^{m}(s-Z_i)}{\prod_{j=1}^{n}(s-P_j)} = \lim_{z\to 1} K^*\frac{\prod_{i=1}^{m}(z-Z_i^*)}{\prod_{j=1}^{n}(z-P_j^*)}.$$

There is after some elementary conversions:

$$K^* = \lim_{s\to 0} K\frac{\prod_{i=1}^{m}(s-Z_i)}{\prod_{j=1}^{n}(s-P_j)} \cdot \lim_{z\to 1}\frac{\prod_{j=1}^{n}(z-P_j^*)}{\prod_{i=1}^{m}(z-Z_i^*)}.$$

For example, first-order transfer function $\frac{1}{T\cdot s+1}$ or the simple electrical circuit with one reactance describe by the ordinary differential equation $T\cdot y' + y = x$, where y – output response, T – time constant, x – input signal or excitation. We perform discretization of this continuous-time system:

- the continuous pole of this system is $-\frac{1}{T}$;
- accordingly, the discrete pole of this system is $e^{-\frac{h}{T}}$;
- and discrete transfer function is $\frac{K^*}{z-e^{-\frac{h}{T}}}$;
- to equalize the gain of both systems (continuous and discrete) getting $K^* = 1-e^{-\frac{h}{T}}$ yield final discrete transfer function that corresponds to recurrent equation for simulation:

$$\frac{1}{T\cdot s+1} \Rightarrow \frac{1-e^{-\frac{h}{T}}}{z-e^{-\frac{h}{T}}} \Rightarrow y_{i+1} = y_i\cdot e^{-\frac{h}{T}} + \left(1-e^{-\frac{h}{T}}\right)\cdot x_i\,. \tag{1}$$

Using zeros/poles matching method, we get second order system with transfer function $\frac{1}{T^2s^2+2\xi Ts+1}$ (or equivalent second-order electrical circuit, RLC-circuit for example):

- this continuous system has a pair of complex conjugate poles: $\frac{-\xi \pm \sqrt{\xi^2-1}}{T}$;
- accordingly, the discrete poles of this system are $e^{\frac{h}{T}\left(-\xi \pm \sqrt{\xi^2-1}\right)}$;
- this pair of discrete complex conjugate poles produce discrete transfer function $\frac{K^*}{z-A+B\cdot z^{-1}}$,

where $A = 2e^{-\xi\frac{h}{T}}\cos\left(\frac{h}{T}\sqrt{1-\xi^2}\right)$, $B = e^{-2\xi\frac{h}{T}}$;

- to equalize the gain of both systems (continuous and discrete) getting $K^* = 1-A+B$ yield final discrete transfer function that corresponds to recurrent equation for simulation:

$$\frac{1}{T^2s^2+2\xi Ts+1} \Rightarrow \frac{1-A+B}{z-A+B\cdot z^{-1}} \Rightarrow y_{i+1} = y_i\cdot A - y_{i-1}\cdot B + x_i\cdot(1-A+B)\,. \quad (2)$$

Discrete transfer function for common-order continuous system can be obtained using Heaviside theorem (classic way to get z-transform) with sample step h:

$$W(z) = \sum_{i=1}^{n}\frac{P(s_i)}{Q'(s_i)}\cdot\frac{1}{1-e^{-h\cdot(s-s_i)}}\Bigg|_{z=e^{h\cdot s}} = \sum_{i=1}^{n}\frac{P(s_i)}{Q'(s_i)}\cdot\frac{1}{1-z^{-1}e^{-h\cdot s_i}},$$

where n – number of poles; z – z-transform operator; s_i – i-th root of denominator polynomial (pole); $P(s)$ – numerator of the continuous-time transfer function; $Q'(s)$ – first derivative of the denominator of the continuous-time transfer function.

3. Examples

The simple electric circuits are the good tests and illustrations to use for zeros/poles matching method.

The simple RC-circuit

The simple RC-circuit (fig. 2) is the good first object (example) to investigate the properties and behavior of the proposed modeling equations because it achievement is the possibility to find the analytic solution for all variables and compare it with simulation results (see [7], [14]). The capacitor's voltage $U_C(t)$ is unknown variable in this example.

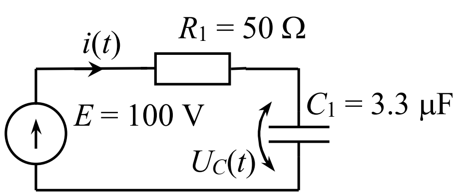


Fig. 2. The simple RC-circuite for the first example

The Kirchhoff's laws can be written for this circuit as equations:

$E = R_1\cdot i(t) + U_C(t)$ where $i(t) = C_1\frac{dU_C(t)}{dt}$. As the result, finally

$$R_1 C_1 \frac{dU_C(t)}{dt} + U_C(t) = E \,.$$

The solution of this differential equation can be written as recurrent formula based on the formula (1):

$$U_{C_{i+1}} = U_{C_i} \cdot e^{-\frac{h}{C_1 R_1}} + \left(1 - e^{-\frac{h}{C_1 R_1}}\right) \cdot E \,. \tag{3}$$

Symbolic solution for zero initial condition is known: $u_C(t) = E \cdot \left(1 - e^{-\frac{t}{C_1 R_1}}\right)$, and can be compared to zero/poles matching solution obtained by formula (3). Both solutions are identical and show in Fig. 3.

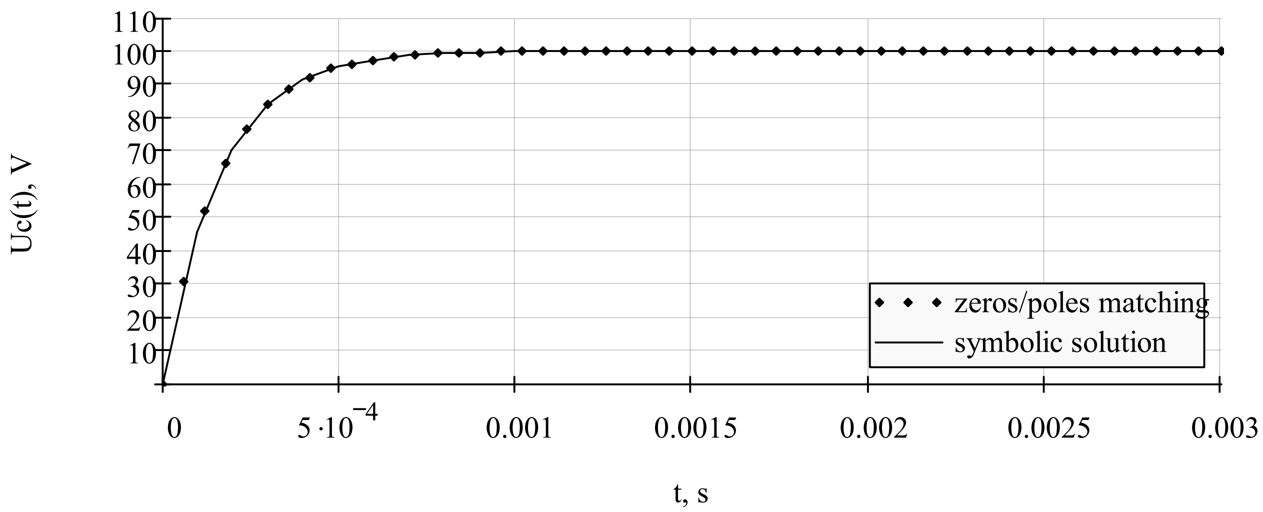
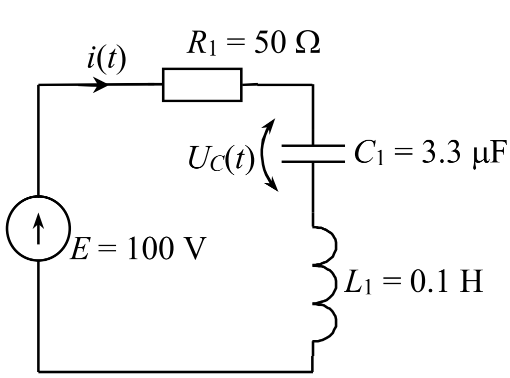


Fig. 3. The capacitor's voltage $U_C(t)$
for the first example for the zero initial condition

The simple RLC-circuit

The simple RLC-circuit (Fig. 4) is the good second object (example) to investigate the properties and behavior of the proposed modeling equation (2). The analytic solution for the unknown capacitor's voltage $U_C(t)$ can be found using Laplace transform for this example also (see [7], [14]).

Fig. 4. The simple RLC-circuite for the second example



The transient processes are described by the system of two equations of this circuit (4):

$$\begin{cases} U_C(t) = U_C(0) + \dfrac{1}{C_1}\int_0^t i(t)\,dt; \\ L_1\dfrac{di(t)}{dt} + R_1 \cdot i(t) + U_C(t) = E. \end{cases} \tag{4}$$

This equations can be rewritten to second-order differential equation (5) with zero initial conditions:

$$L_1C_1\frac{d^2U_C(t)}{dt^2} + R_1C_1\frac{dU_C(t)}{dt} + U_C(t) = E \text{ where } i(t) = C_1\frac{dU_C(t)}{dt}. \tag{5}$$

Denote $T = \sqrt{L_1C_1}$ and $\xi = \dfrac{R_1C_1}{2T}$ that produce recurrent equation (6) for computer simulation

$$U_{Ci+1} = U_{Ci} \cdot A - U_{Ci-1} \cdot B + E \cdot (1 - A + B), \tag{6}$$

where $A = e^{-h\frac{R_1}{2L_1}} \cos\left(\dfrac{h}{2L_1}\sqrt{4\dfrac{L_1}{C_1} - R_1^2}\right)$, $B = e^{-h\frac{R_1}{L_1}}$ in this example.

The computer simulation results can be verified with analytic solution using direct and inverse Laplace transform obtained from equation (5):

$U_C(s) = \dfrac{E}{s \cdot (L_1C_1s^2 + R_1C_1s + 1)}$, after inverse Laplace transform (using Mathcad or MATLAB for example) we get analytic solution for capacitor voltage for zero initial conditions and four digits precision:

$$U_C(t) = 100 - (100\cos(524.4t) + 4.334\sin(524.4t)) \cdot e^{-22.73t}. \tag{7}$$

Solution of the equation (6) can be compared to analytic solution (7) shown in fig. 5 for zero initial conditions. Both results are identical.

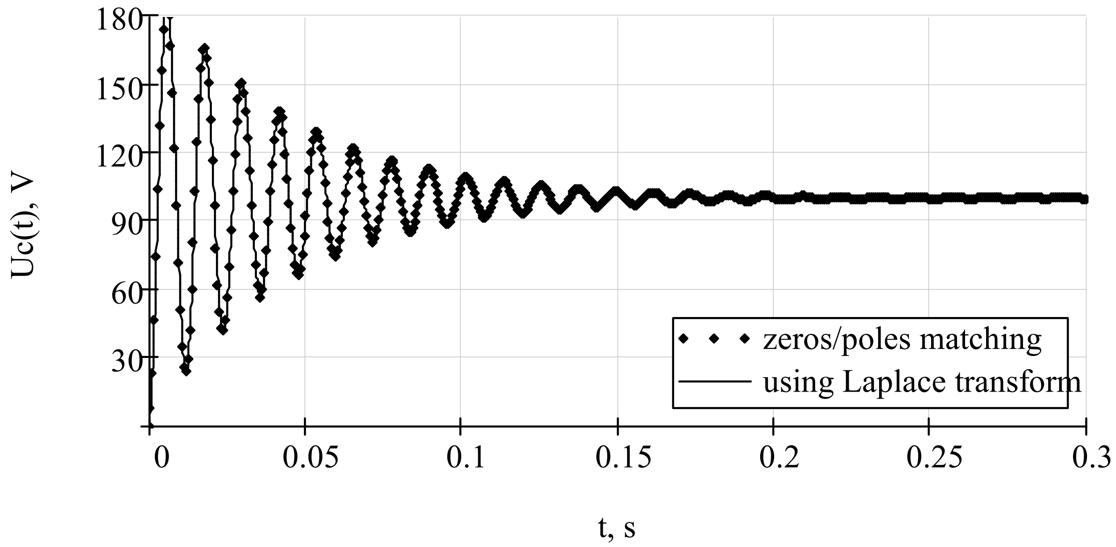


Fig. 5. The capacitor's voltage $U_C(t)$ for the second example for the zero initial condition

4. Conclusions

The overviewed method is suitable for various problems solution in the field of electrical engineering. The method is suitable for a wide range of linear and nonlinear dynamic systems [3], [11]. Main advantages of the proposed method are:

- obtained modeling equations are numerically stable and don't dependent on the step size;
- this method produces quite simple but very effective equations that are suitable for high-speed simulation of linear and nonlinear dynamic objects.

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Швидкісне комп'ютерне моделювання електричних кіл

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Анотація

Для високошвидкісного комп'ютерного моделювання перехідних процесів в електричних колах запропоновано використати давно відомий метод відповідності нулів/полюсів (як одну з реалізацій z-перетворення) відповідної неперервної передатної функції чи комплексного імпедансу (провідності) в області перетворень за Лапласом. Переваги цього методу показано на простих прикладах комп'ютерних моделей електричного кола першого порядку (RC-коло, яке відповідає одному дійсному полюсу) і кола другого порядку (RLC-коло, яке відповідає парі комплексно-спряжених полюсів). Отримані моделі були перевірені з використанням комп'ютерних моделей, які одержані аналітичним методом з використанням перетворення Лапласа. Метод може використовуватися в сучасних комп'ютерних програмах для аналізу стійкості електроенергетичних систем, розрахунку перехідних процесів в електричних колах та аналізу динаміки інших технічних систем.

Ключові слова: електричні кола; комп'ютерне моделювання; метод відповідності нулів і полюсів; перехідні процеси; z-перетворення.