

Research of dynamic system with unstable subsystem that has one root in the right half-plane

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A two-mass system is investigated. It is considered as a set of dynamical subsystems. To switch between these subsystems, fuzzy controller is synthesised. The research is held for the case when one subsystem is unstable having one root in the right-hand side of the plane. The stability of the overall system, depending on the stability of the subsystems of which it is composed is examined. For this, a 3D hodograph is used. Quality indexes of the system with unstable subsystem are calculated. The case when the system operates under external perturbations is consider. The influence of perturbations value and controller parameters of the transient of the system is investigated. The gain in the case of usage of the unstable subsystem is pointed out.

Keywords: *dynamical system, automatic control, fuzzy control, unstable subsystem*

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1. Introduction

At synthesis of electromechanical systems's controller an improve of performance play an inportant role. Traditionally, to configure the feedback control system to the desired behavior, one uses controls that are synthesized to one of standard distribution of polynomials roots (see. [1–4]).

Now, the use of the apparatus of fuzzy sets (see. [5,6]) for the synthesis of electromechanical systems controllers gained a considerable popularity. Indeed, the combination of fuzzy logic and classical control theory, can provide significant gains for the main quality indexes and speed of operation (see. e.g. [7–9]).

The objectives of this paper is to study the possibility of unstable subsystem, with one root in the right half-plane, usage. Also the impact of this subsystem on the dynamic characteristics of the overall system is investigated. Besides, study the behavior of the system with unstable subsystems in the case when the system is exposed to external disturbances is conducted. Studies in article form the theoretical foundation to improve dynamic performance electromechanical systems and ensure the relevance of this article.

2. Problem statement

We use mathematical tools from [1]. Let us consider two-mass system with the following transfer function

$$W(p) = \frac{c_{13}/T_{M1}}{H(p)},$$

$$H(p) = p^3 + p^2 \frac{k_{11}}{T_{M1}} + p \left(\frac{1}{T_C T_{M2}} + \frac{k_{12} + 1}{T_C T_{M1}} \right) + \left(\frac{k_{11} + k_{13}}{T_C T_{M1} T_{M2}} \right),$$

where c_{13} , T_{M1} , T_{M2} , T_C are coefficients which characterise the system and are described in [1]. k_{11} , k_{12} , k_{13} are unknown coefficients of feedback controller.

In classical control theory for determination of the controller's coefficients, characteristic polynomial of the system is set up for one of the standard linear forms (see. [4]). In this paper study is conducted for the case when the desired characteristic polynomial has the following form

$$H_{CT}(p) = p^3 + 3p^2\omega_0 + 3p\omega_0^2 + \omega_0^3,$$

where ω_0 is frequency of the system.

To improve the performance of the investigated system it is considered as a family of subsystems. The controller of each subsystem is set to other desired characteristic polynomial. The aim of this paper is to study a system consisting of two subsystems, controller of one of which is configured to characteristic polynomial with a root in the right half-plane. It turns out that if this subsystem is operating in the area of large deviations, the overall system performance increases.

To switch between the subsystems the theory of fuzzy sets is used. Based on the results of work [10,11] where only system's output signal was fuzzified and the number of rules is limited to two. Centre of mass defuzzification method is used. For fuzzification the following standard linear function (see [5]) is used.

$$S(u; \alpha, \beta) = \begin{cases} 0, & u < \alpha \\ (u - \alpha) / (\beta - \alpha), & \alpha \leq u \leq \beta \\ 1, & u > \beta \end{cases}$$

For comparison of the system with proposed controller and classical one, the values of following integral indicators were calculated

$$I_1 = \int e^2(t) dt;$$

$$I_2 = \int t e^2(t) dt;$$

$$I_3 = \int |e(t)| dt;$$

$$I_4 = \int t |e(t)| dt, \quad e(t) = x_{704} - x(t).$$

3. Results

The study was conducted for the case when the desired characteristic polynomial of the unstable subsystems has the following form

$$H(p) = (p - \omega_{0_2}) (p + \omega_{0_2})^2,$$

where $\omega_{0_1} = \omega_{0_2} = 4$,

$$k_{11_2} = T_{M1}\omega_{0_2},$$

$$k_{12_2} = \left(-\omega_{0_2}^2 - \frac{1}{T_C T_{M2}} \right) T_C T_{M1} - 1, \quad (1)$$

$$k_{13_2} = -\omega_{0_2}^3 T_C T_{M1} T_{M2} - T_{M1} \omega_{0_2}.$$

Results of simulation when $\alpha = 0.2$, $\beta = 0.8$ is at (Fig. 1).

The output signal of each of three integrators is given below (Fig. 2).

Let us show the dependence of maximal overshoot value on parameters α and β .

Using the reasoning from [9] for the study of stability will use three-dimensional hodograph is used. The dependence of the phase-frequency characteristics on time is shown in (Fig. 5).

Similarly as in the case of two roots in the right half-plane (see [8]) this figure shows that when the value of membership function increases the system turns from unstable to a stable state.

As a result of the experiments held one can argue that desired system's performance is obtained at $\alpha = 0.7$, $\beta = 1.0$. At this time of entering five percent area equals 0.773 s., and settle time is 0.544 s.

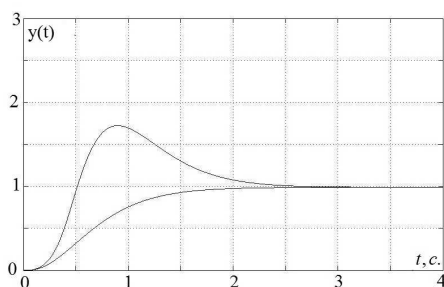


Fig. 1. Results of simulation of the system with coefficients (1).

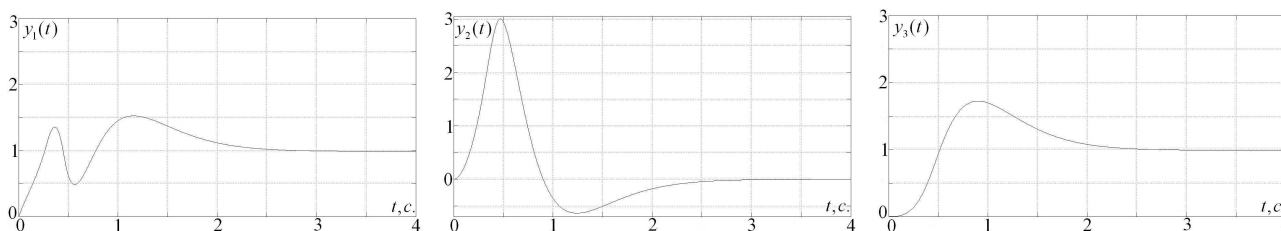


Fig. 2. Output signal of integrators in the case of one root in the right-hand side.

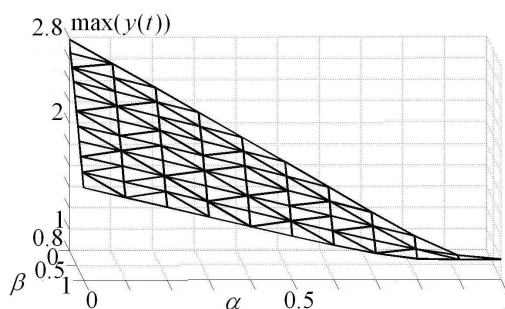


Fig. 3. Dependence of maximal overshoot value on parameters α and β value.

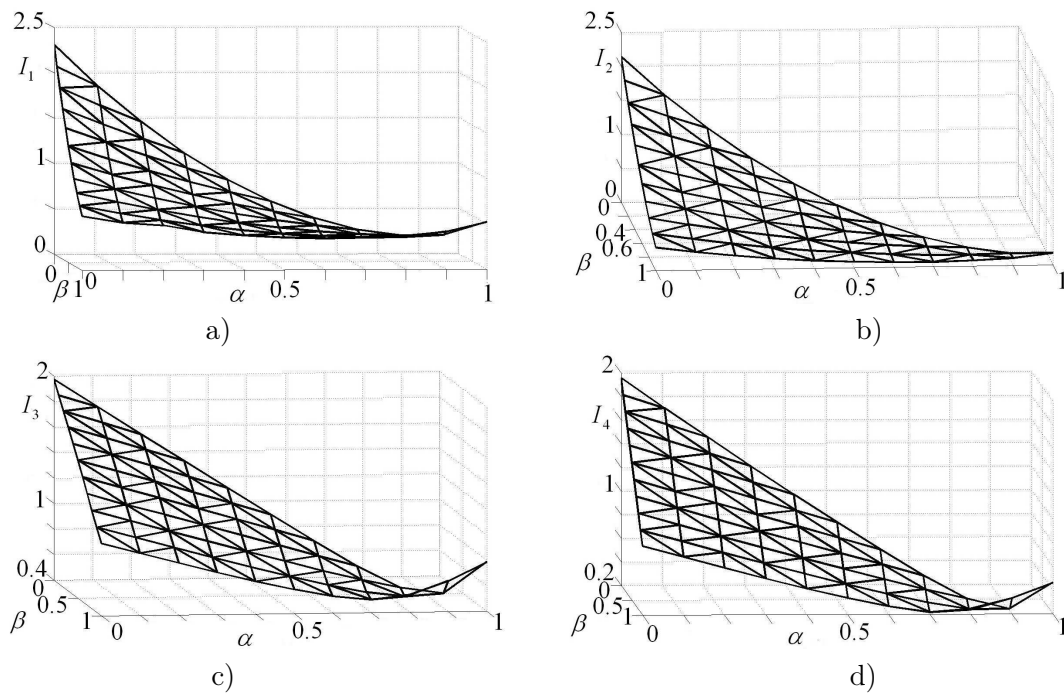


Fig. 4. Values of quality indexes at different values of α and β a) I_1 , b) I_2 , c) I_3 , d) I_4 .

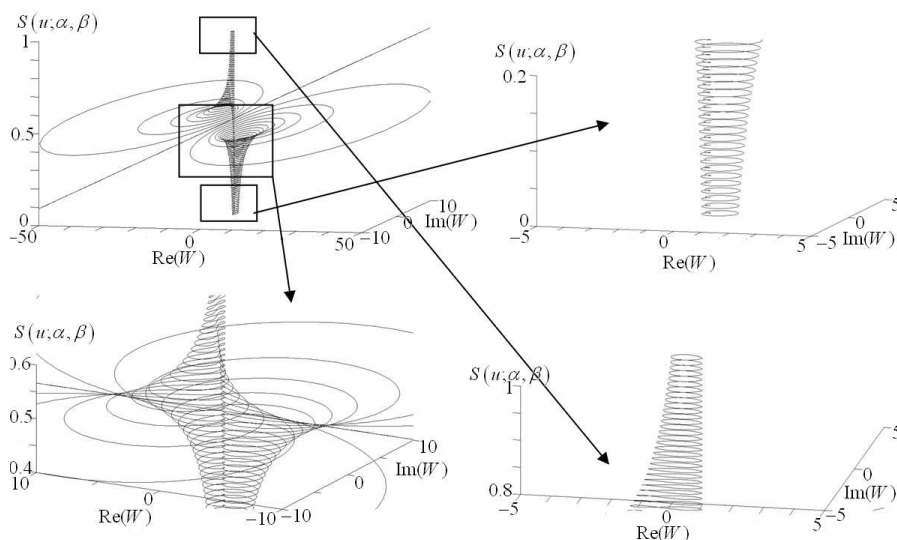


Fig. 5. Dependence of phase-frequency characteristics on time for the system with coefficients (1).

4. Analysis of the system under the perturbations

Consider the case when studied system is subject to external disturbances (Fig. 6).

As one can see, the system under study remains stable under disturbances. In addition, depending on the size of the perturbation and the parameters α and β of the system it may experience fluctuations including attenuating. This can be very useful when developing mechanisms which are based on

vibration, because it can be achieved not through additional mechanical devices, but only thanks to the electric motor.

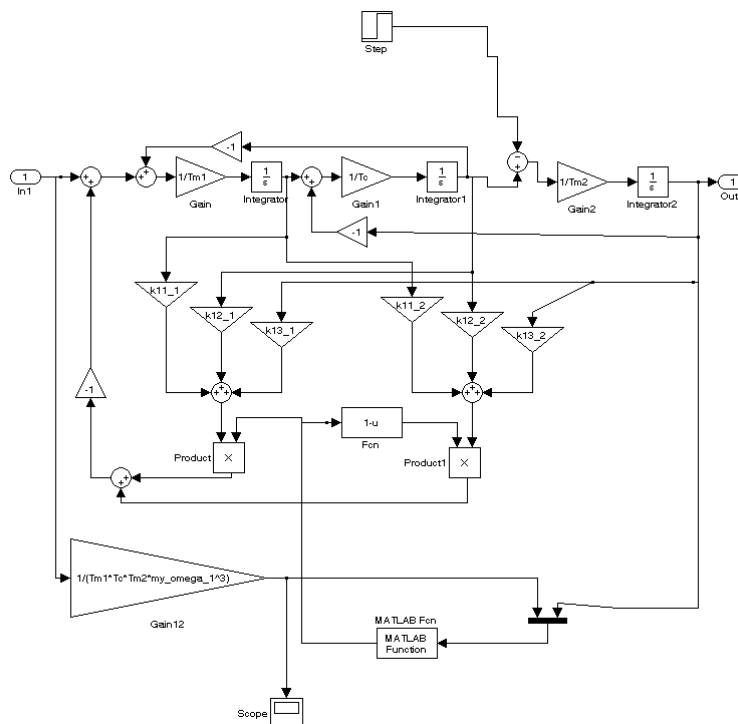


Fig. 6. Model of the system which is subject to external disturbance.

Obviously, the behavior of the system also depends on the value of the external perturbation. Different variants of the system, with coefficients (1), behaviour is considered (Fig.7–Fig.10):

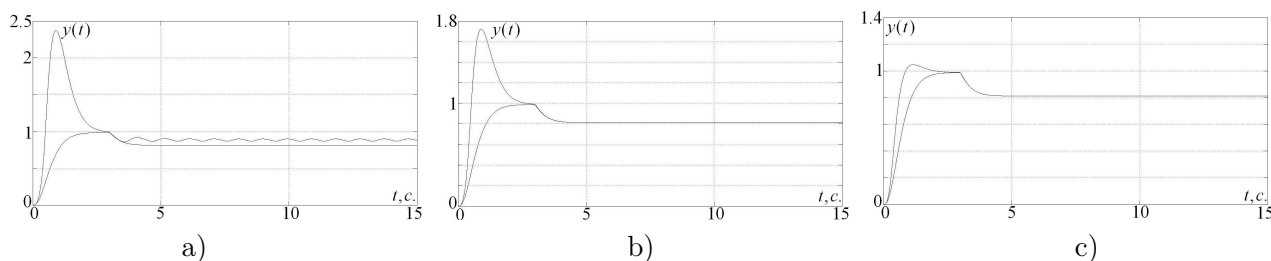


Fig. 7. Simulations at a value of disturbance equal 0.25 and following values of parameters a) $\alpha = 0.1$, $\beta = 0.3$, b) $\alpha = 0.2$, $\beta = 0.8$, c) $\alpha = 0.7$, $\beta = 1.0$.

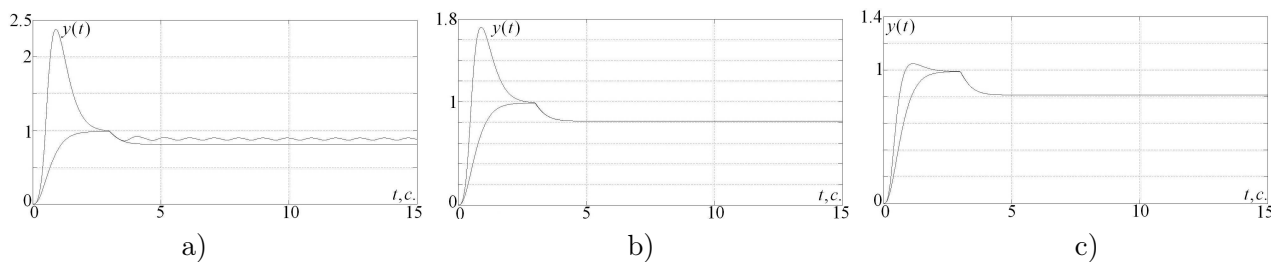


Fig. 8. Simulations at a value of disturbance equal 0.5 and following values of parameters a) $\alpha = 0.1$, $\beta = 0.3$, b) $\alpha = 0.2$, $\beta = 0.8$, c) $\alpha = 0.7$, $\beta = 1.0$.

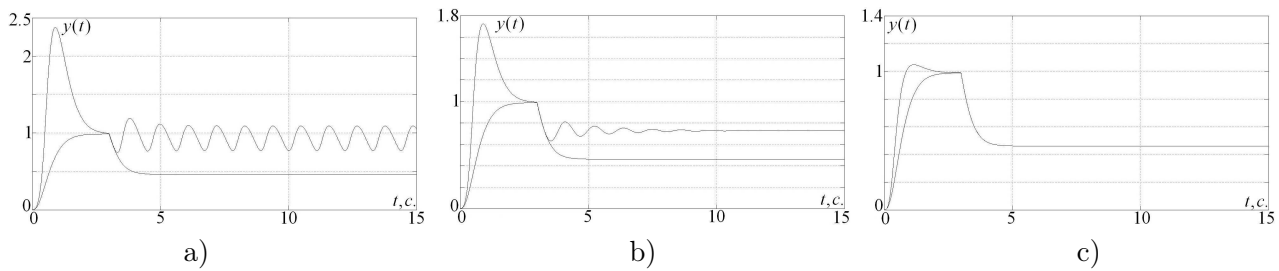


Fig. 9. Simulations at a value of disturbance equal 0.75 and following values of parameters a) $\alpha = 0.1$, $\beta = 0.3$, b) $\alpha = 0.2$, $\beta = 0.8$, c) $\alpha = 0.7$, $\beta = 1.0$.

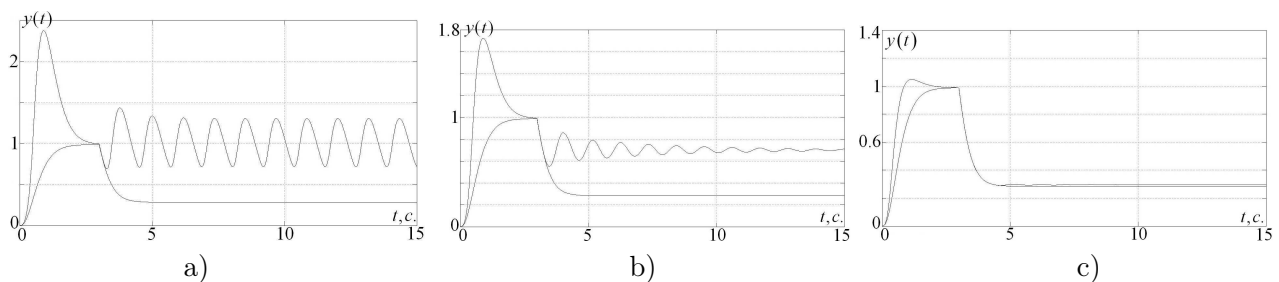


Fig. 10. Simulations at a value of disturbance equal 1.0 and following values of parameters a) $\alpha = 0.1$, $\beta = 0.3$, b) $\alpha = 0.2$, $\beta = 0.8$, c) $\alpha = 0.7$, $\beta = 1.0$.

5. Conclusions

Thus, analyzing the results of this paper it can be concluded that the requirement of subsystem's stability is not a prerequisite to ensure the stability of the whole system. This fact makes it possible not only wider application of known systems and mechanisms based on electrical motors in practice but the introduction new which contain an unstable subsystem and obtain at the same time winning the main quality indexes and speed of operation. In addition the device can provide versatility thanks only to change the value of the external load on the system.

The results obtained in this article show that based on the values of quality indexes can be achieved to 2.5 times when the first subsystem is unstable.

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Дослідження динамічної системи з нестійкою підсистемою, що має один корінь в правій півплощині

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У роботі розглянуто двомасову систему, що складається з підсистем. Для перемикання між цими підсистемами синтезовано нечіткий регулятор. Дослідження проведено для випадку коли одна з підсистем має один корінь в правій півплощині, тобто є нестійкою. Досліджено стійкість загальної системи в залежності від стійкості підсистем з яких вона складається, для чого використано тривимірний годограф. Обчислено інтегральні показники якості роботи системи з нестійкою підсистемою. Крім того розглянуто випадок коли на систему діють зовнішні збурення та досліджено їх вплив на перехідні процеси в системі. Встановлено переваги від застосування системи з нестійкою підсистемою.

Ключові слова: динамічна система, автоматичне керування, нечітке керування, нестійка підсистема

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