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S. B. Maksymovych,
Lviv Polytechnic National University,
Department of Building Structures and Bridges

EXPERIMENTAL STUDY OF CRACK RESISTANCE AND STRENGTH OF OBLIQUE PLANES OF REINFORCED CONCRETE POINT-FORCE LOADED BEAMS WITH POSITIVE AND NEGATIVE BENDING MOMENT DIAGRAM

Розглянуто експериментальні дослідження похилих перерізів нерозрізних залізобетонних балок за одночасної дії постійної поперечної сили і двозначних моментів. Випробувані балки мали однакові характеристики і відрізнялися тільки видами армування похилих перерізів. Під час аналізу експериментальних досліджень виявлено специфіку утворення і ширини розкриття похилих тріщин у зоні дії постійної поперечної сили і двозначного згинального моменту. Зафіксована динаміка розвитку та ширина розкриття похилих тріщин залежно від навантаження. Виявлено також вплив різних видів армування похилих перерізів на несучу здатність балок.

Експерименти показали, що зародження похилих тріщин у балках з двозначною епюрою згинальних моментів за відношення a/d = 1 проходить у середині висоти балки. Розподіл напружень по довжині розтягнутої поздовжньої арматури балок не відповідає теоретичній епюрі згинальних моментів. У разі двозначної епюри згинальних моментів та відношенні a/d = 1 найефективнішою поперечною арматурою, з огляду на несучу здатність похилих перерізів і ширину розкриття похилих тріщин, є горизонтально розміщена поперечна арматура.

Ключові слова: залізобетонні балки, похилі перерізи, двозначна епюра згинальних моментів.

The paper discusses experimental study of oblique planes of reinforced concrete solid beams at simultaneous exertion of constant transverse force and positive and negative bending moments. The tested beams had similar characteristics and differed only in types of reinforcement of oblique planes. Analysis of experimental results revealed the specifics of development and width of oblique cracks in the zone of effect of constant transverse force and positive and negative bending moments. The dynamics of development and width of oblique cracks depending on the load was recorded. The study also assessed the effect of various types of oblique planes reinforcement on bearing capacity of the beams.

The experiments showed that nucleation of oblique cracks in beams with positive and negative bending moment diagram for the relationship a/d = 1 occurs at mid-height of the beam. Distribution of stresses along the tension longitudinal reinforcement of beams is different from the theoretical bending moment diagram. In beams with positive and negative bending moment diagram at the relationship a/d = 1, horizontal (transverse) reinforcement is the most effective type of transverse reinforcement in terms of bearing capacity of oblique planes and width of oblique cracks.

Key words: reinforced concrete beam, oblique planes, positive and negative bending moment diagram.

Statement of the research problem

Reinforced concrete solid beams are widely used in modern construction applications. Although work of oblique planes of such elements and that of simply supported beams have much in common, there are also significant differences between them. Change of bending moment sign in the area of application of constant transverse force suggests that the compressed zone of the beam has changed its position. Therefore, stress strain behaviour of oblique planes in solid beams and that in simply supported beams differ, and this is bound to affect crack resistance and strength of their oblique planes.

Analysis of recent research and publications

Experimental research into solid beams [1–3] revealed a peculiarity of nucleation and development of cracks in the above-mentioned areas, as well as substantial influence on the length of anchoring of the longitudinal reinforcement in the tensile zone. A typical pattern of nucleation and development of cracks in the area of simultaneous actions of constant transverse force and bending moments passing through the zero point can be seen on Leonhardt beams (batch HH) [1] (Fig. 1).

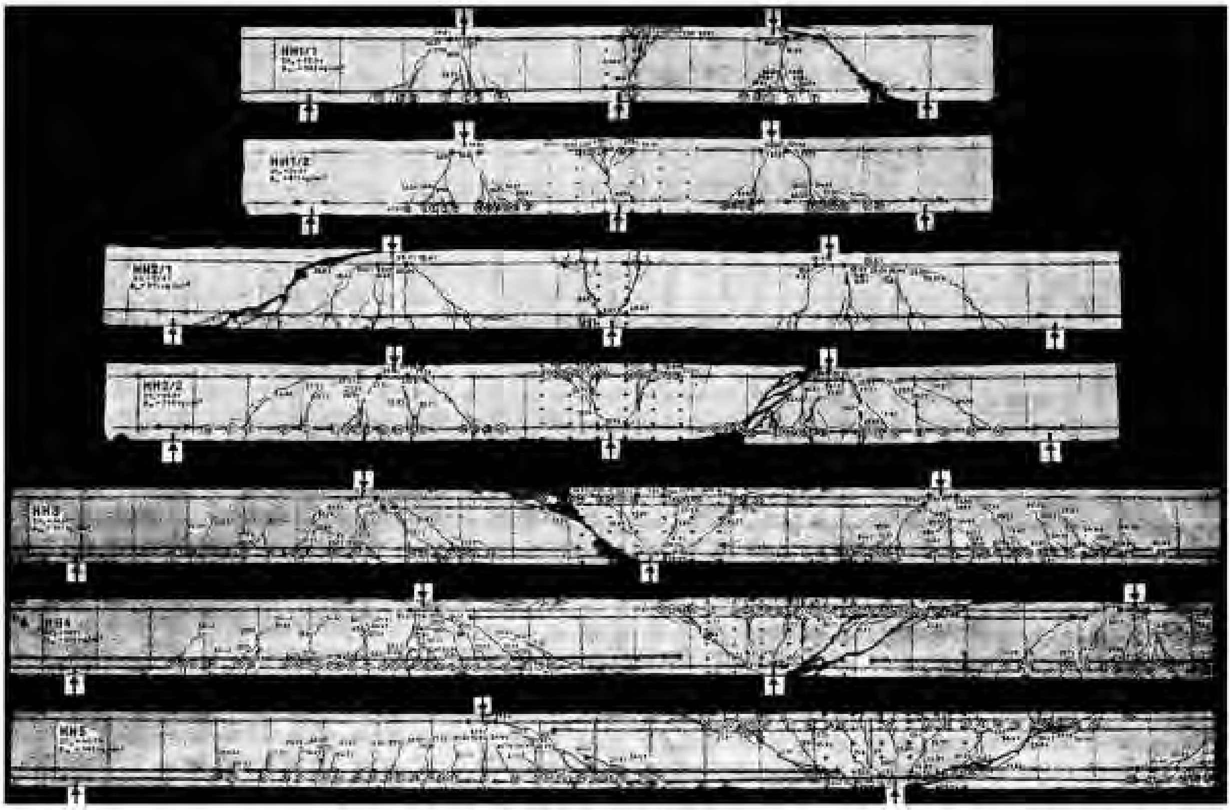

Fig. 1. Pattern of nucleation and development of cracks and destruction of HH beams [1]

On different stages of loading, it can also be traced in beam 66/1 [2] (Fig. 2) in the site of the constant transverse force and bending moment diagram going through the zero point.

For loads being lower than the operation one, zones of positive and negative bending moments developed only vertical cracks. The load increasing, new vertical cracks appeared, and those which developed initially transformed into diagonal cracks in the areas of bending moment sign change.

At loads exceeding the operation one, beams' lateral surfaces got covered with diagonal cracks, except for areas of a certain length flanking the zero point of bending moments. No cracks in areas on both sides of the zero point were recorded in the experiments [3] when point force was applied at a distance from the interior support exceeding $(2-2{,}5)h_0$.

Analysis of some studies [1, 2, 3, 8] suggested that the transverse force is not a criterion to describe crack formation and work of oblique planes in solid and simply supported beams. Therefore, according to the effective Ukrainian standard ДСТУ Б В.2.6-156:2010 [4] and EUROCODE 2 [5], transverse reinforcement for these two cases is to be designed using one and the same method.

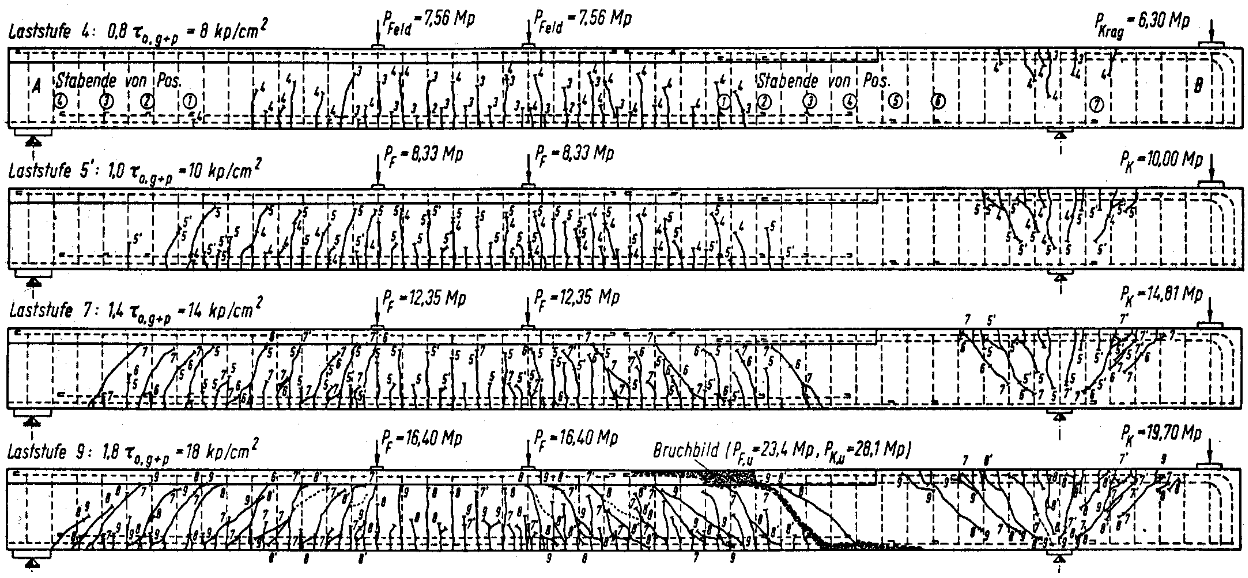

Fig. 2. Pattern of nucleation and development of cracks in 66/1 beam [2]

Goal of the research

Taking into account that work strain of reinforced concrete beams with both positive and negative bending moment diagrams has not been sufficiently studied, the task was set to explore the work of beams in the area where bending moment changes sign for cases when force is applied close to the interior supports for small shear spans ($a/d \le 1$).

Summary of the main information

In accordance with the task, a program of experimental research was developed. It included testing 8 simple overhanging reinforced concrete beams loaded with point force.

Samples with total length of 2900 mm were testes as simple overhanging beams (l = 1050 mm, $l_к$ = 700 mm) loaded with two point forces applied to the spans ($a = d$) and with one force per corbel. The diagram of testing the beams is shown in Fig. 3.

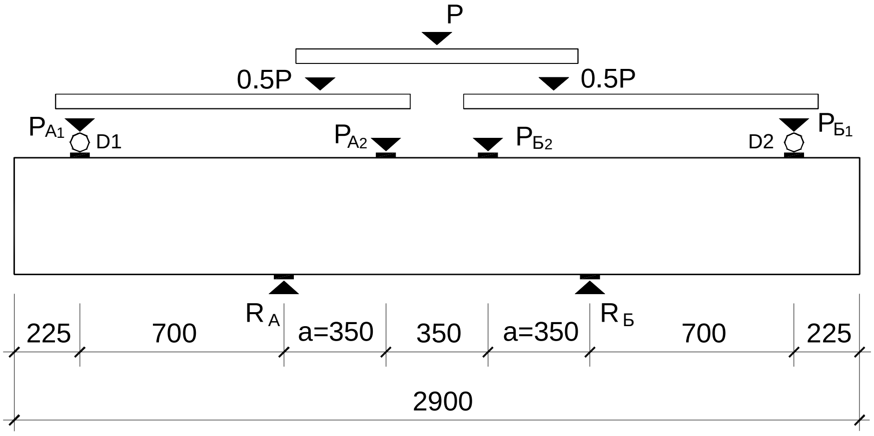

Fig. 3. Beams testing diagram

All the test samples had similar cross-section of $b \times h$ = 120×400 mm and the same main longitudinal lower and upper reinforcement (4∅12 A400C). The beams differed only in type of oblique planes reinforcement.

The experiment involved gauging strain of concrete, longitudinal and transverse reinforcement and observing cracks nucleation and width. Strain values were taken using a comparator with gauged length 50 mm.

Characteristics of the test beams are summarized in Table 1.

Table 1

Characteristics of test beams

Beam Grade	l, mm	$l_к$, mm	b, mm	h, mm	d, mm (support/span)	f_{cm}, MPa	Longitudinal Reinforcement				Transverse Reinforcement			
							Reinforcement (upper and lower)	A_s, cm^2	$\sigma_т$, MPa	ρ, % (upper/lower)	Reinforcement	s, mm	A_{sw}, cm^2	$\sigma_т$, MPa
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Б-1-1	1050	700	120	400	347/ 350	47,4	4∅12 A400C	4,52	410,4	1,09/ 1,08	–	–	–	–
Б-1-2	1050	700	122	402	349/ 347	47,4	4∅12 A400C	4,52	410,4	1,06/ 1,07	∅12 A400C	–	2,26	410,4
Б-1-3	1050	700	123	410	358/ 352	47,4	4∅12 A400C	4,52	410,4	1,03/ 1,04	∅12 A240	70	2,26	254,1
Б-1-4	1050	700	123	400	348/ 349	47,4	4∅12 A400C	4,52	410,4	1,06/ 1,05	∅10 A240	60	1,57	258,0
Б-1-5	1050	700	122	402	342/ 359	47,4	4∅12 A400C	4,52	410,4	1,08/ 1,03	∅6,5 A240	60	0,664	262,0
Б-1-6	1050	700	123	398	339/ 342	47,4	4∅12 A400C	4,52	410,4	1,08/ 1,07	∅6,5 A240	–	0,664	262,0
Б-1-7	1050	700	123	400	345/ 350	47,4	4∅12 A400C	4,52	410,4	1,07/ 1,05	∅6,5 A240	50	0,664	262,0
Б-1-8	1050	700	123	398	345/ 343	47,4	4∅12 A400C	4,52	410,4	1,07/ 1,07	∅6,5 A240	80	0,664	262,0

Patterns of nucleation and development of cracks and destruction of beams are shown in photos presented in Fig. 4.

The results of experimental testing of the beams are grouped in Table 2 using the symbols applied in Fig. 3.

Destruction of all beams took about the same course, although in some beams there were some peculiarities (Fig. 4). The first vertical cracks in the beams formed above the supports A and Б at loading P = 300 kN. Later, when P = 400 kN, span vertical cracks developed, and then, contiguously, appeared oblique cracks. Oblique cracks formed in the vicinity of supports A and Б in the middle of the beam section height, i.e. they did not reach the lower and upper longitudinal reinforcement. With load increasing up to P = 500 kN, there was observed formation of new vertical cracks, which were more and more approaching the supports, and oblique cracks continued to develop and increase in width. At P = 600 kN, oblique cracks developed more intensively both towards the support and towards the applied force, whereas vertical cracks in the middle of the beam span remained unchanged. At P = 700 kN oblique cracks reached the bottom and top surfaces of the beam. Further increase in the load to P = 800÷1000 kN led to intensive growth of width of oblique cracks and their combination with other cracks, which ultimately caused destruction of the beam along one of these oblique cracks. All the beams failed along the oblique cracks (Fig. 4).

Assessing width of oblique cracks in reinforced concrete beams is one of the least studied issues. It is discussed only in several special studies [6–8], and only some of them provide results of measuring width of oblique cracks. Experimental research found that width of oblique cracks is much larger than that of vertical ones. Often due to excessive widening of oblique cracks in real structures of buildings and constructions the latter have to be strengthened. The emergency state of Dnister Hotel in Lviv [9] is an obvious example of such situation, and this is what prompted the within research.

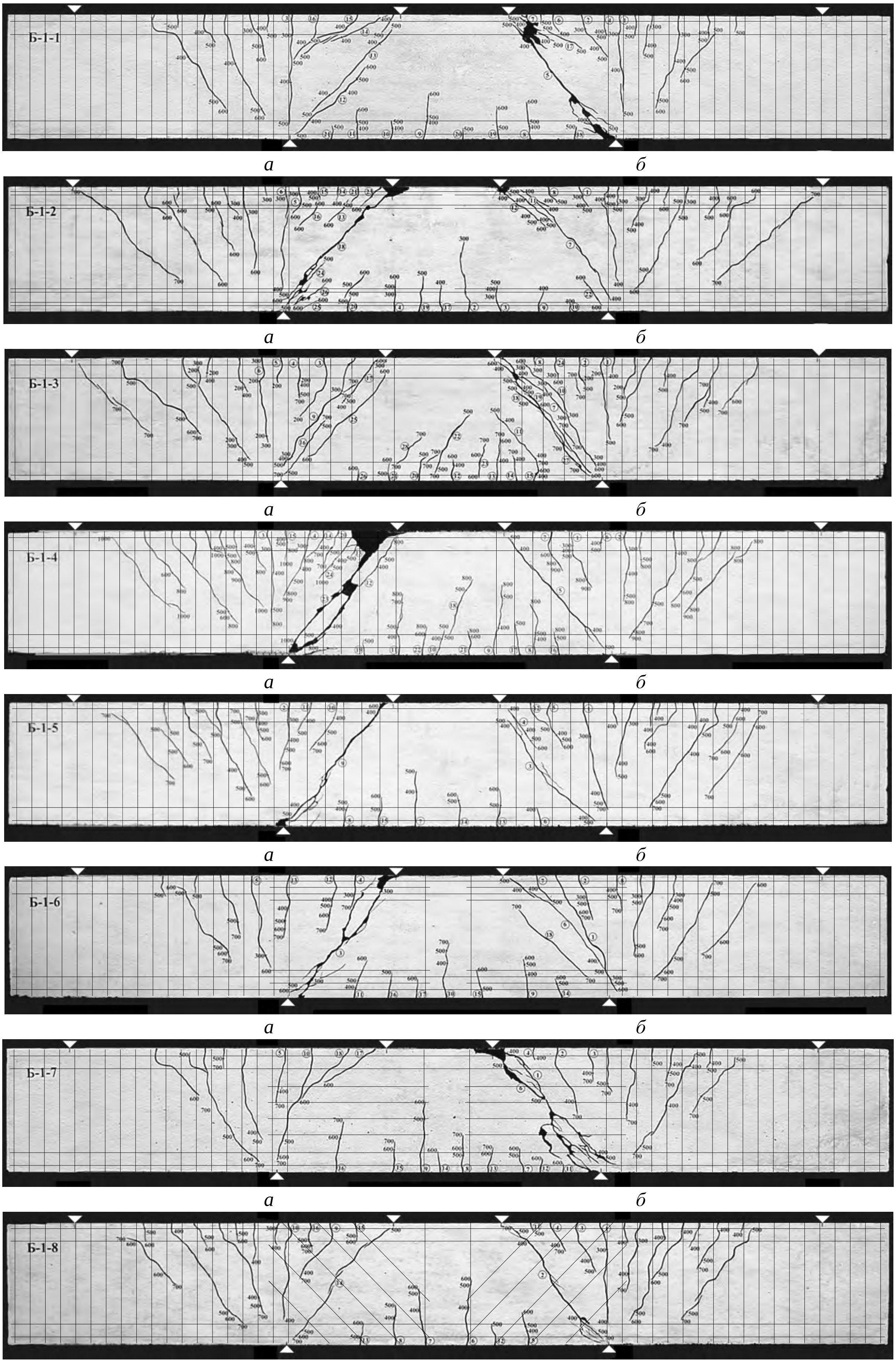

Fig. 4. Pattern of nucleation and development of cracks and destruction of Б-1-1 – Б-1-8 beams

Table 2

Results of beams testing

Beam Grade	*a/d*	P, кН	$R_А$, кН	$R_Б$, кН	$P_{А1}$, kN	$P_{А2}$, kN	$P_{Б1}$, kN	$P_{Б2}$, kN	$M_{А\ оп.}=M_{А\ пр.}$, kNм	$M_{Б\ оп.}=M_{Б\ пр.}$, kNм	$V_А$, kN	$V_Б$, kN	Failure Location	Failure type	Crack Number
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Б-1-1	1,00	1317	655,0	662,1	131,0	521,0	132,4	532,6	91,7	92,7	524,0	529,7	near support Б	along the oblique crack	5
Б-1-2	1,01	1411	712,1	698,5	142,4	575,0	139,7	553,4	99,7	97,8	569,7	558,8	near support А		18
Б-1-3	0,99	1318	645,0	672,9	129,0	505,0	134,6	549,4	90,3	94,2	516,0	538,3	near support Б		19
Б-1-4	1,00	1328	652,9	675,0	130,6	513,5	135,0	548,9	91,4	94,5	522,3	540,0	near support Б		23
Б-1-5	0,97	1330	675,0	655,0	135,0	548,0	131,0	516,0	94,5	91,7	540,0	524,0	near support А		9
Б-1-6	1,02	1345	665,7	679,3	133,1	527,2	135,9	548,8	93,2	95,1	532,6	543,4	near support Б		3
Б-1-7	1,00	1317	640,0	677,0	128,0	497,2	135,3	556,5	89,6	94,8	512,0	541,7	near support Б		1
Б-1-8	1,02	1355	670,0	685,0	134,0	530,0	137,0	554,0	93,8	95,9	536,0	548,0	near support Б		2

This problem has gained particular topicality with application of steels with enhanced mechanical behaviour for reinforcing concrete structures, which results in reduced density of longitudinal and transverse reinforcement in their sections, in increased strain of such reinforcement and, therefore, in larger width of vertical and oblique cracks.It was experimentally established that the width of oblique cracks is most substantially affected by transverse and longitudinal reinforcement, offset bends, strength of concrete, ratios *a/d* and areas of transverse sections.

Dynamics of development and growth of oblique cracks which led to the beams failure is shown in Fig. 5–7. As seen from these figures, oblique cracks were nucleated in mid-height of the beam, and this is where they had maximum width ending up in their destruction.

The performed analysis of beams testing showed that the type of reinforcement of oblique planes has no significant impact on the moment of oblique cracks development, i.e. oblique cracks formed approximately at the same load P = 400 kN. Up to P = 800 kN, values of oblique crack width in all the beams of the tested batch were almost the same, and only further growth in load rendered them considerably different.

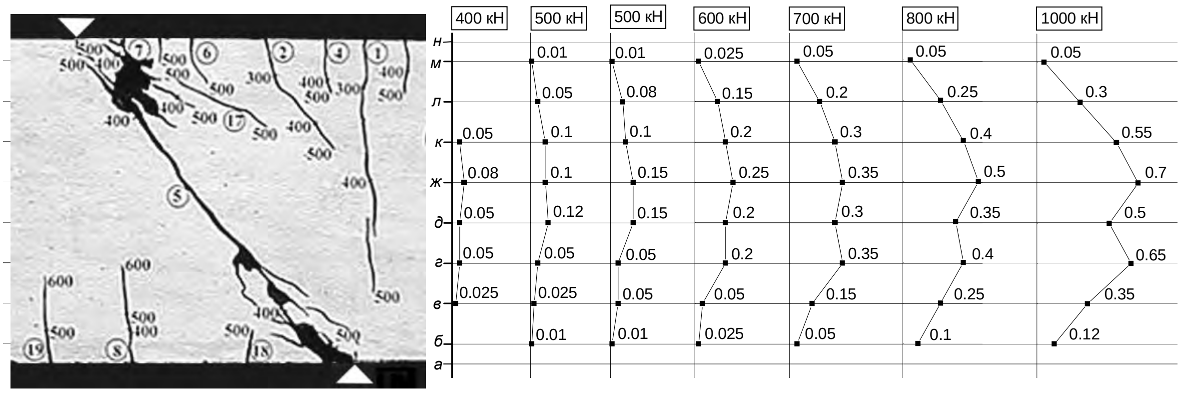

Fig. 5. Dynamics of development and growth of oblique crack 5 in the beam Б-1-1vs. load

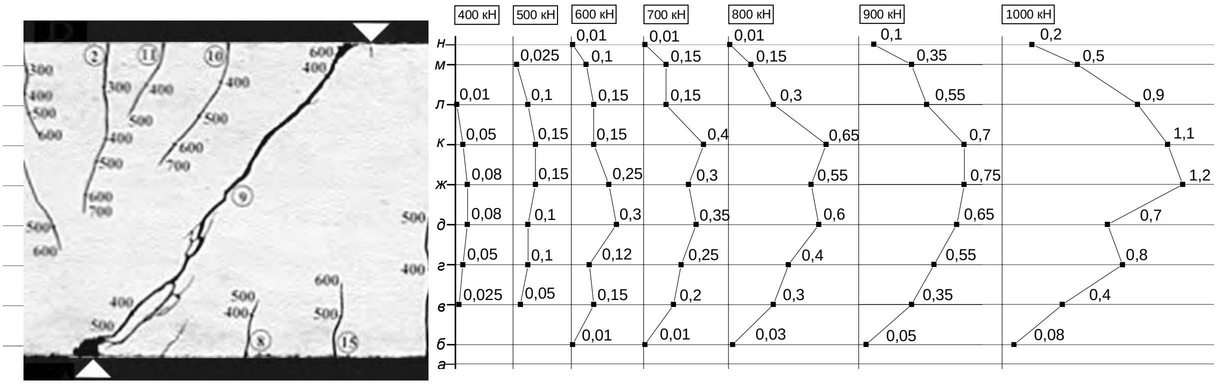

Fig. 6. Dynamics of development and growth of oblique crack 9 in the beam Б-1-5 vs. load

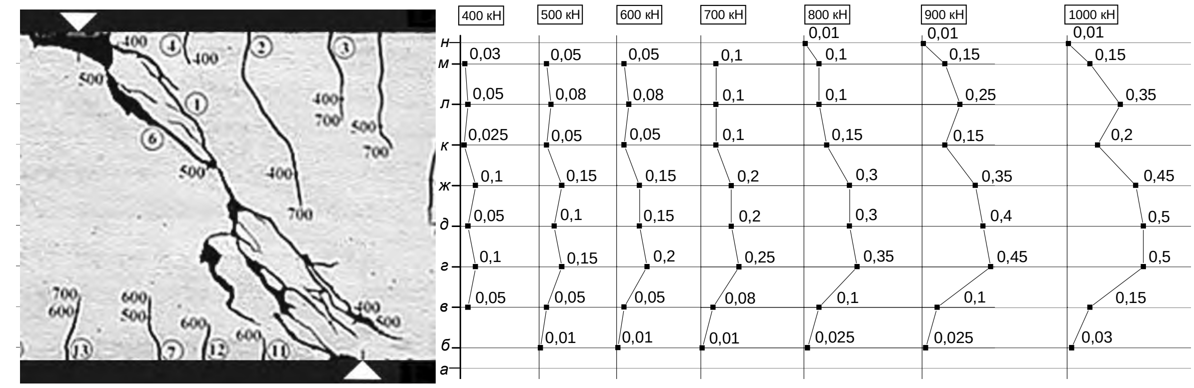

Fig. 7. Dynamics of development and growth of oblique crack 1 in the beam Б-1-7 vs. load

At 1000 kN, maximum width of oblique cracks along which the beams failed, amounted to the following values:

Б-1-1	crack 5 – 0,7 mm;	Б-1-4	crack 23– 0,6 mm;
Б-1-2	crack 18 – 0,5 mm;	Б-1-5	crack 9 – 1,2 mm;
Б-1-3	crack 19 – 0,6 mm;	Б-1-6	crack 3 – 0,8 mm;
Б-1-7	crack 1– 0,5 mm;	Б-1-8	crack 2 – 0,7 mm.

As it can be seen, the smallest width of oblique failure cracks of 0,5mm was observed in beams Б-1-2 and Б-1-7, whose oblique planes were strengthened with horizontal reinforcement.

Comparison of the values of oblique failure cracks width in the beams Б-1-5, Б-1-6, Б-1-7 and Б-1-8 with different types of oblique planes reinforcement but the same amount of transverse reinforcement reveals that transverse reinforcement is most effectively located in the beam Б-1-7 with the smallest oblique failure crack width of 0,5 mm.

As it was noted above, all the beams of the batch under study collapsed along oblique planes. Values of collapsing force P at which failure of each beam occurred are adduced in Table 2 column 3.

By comparing bearing capacity of beam Б-1-1 (without transverse reinforcement) with beams Б-1-2; Б-1-3; Б-1-4; Б-1-5; Б-1-6; Б-1-7; Б-1-8, one can determine the impact of each type of transverse reinforcement on oblique planes strength at $a/d=1$.

The beams collapsed at the following loads:

Б-1-1 – $P=1317$ kN; Б-1-2 – $P=1411$ kN; Б-1-3 – $P=1318$ kN; Б-1-4 – $P=1328$ kN;
Б-1-5 – $P=1330$ kN; Б-1-6 – $P=1345$ kN; Б-1-7 – $P=1317$ kN; Б-1-8 – $P=1355$ kN.

Comparison of bearing capacity of the beams of the tested batch with that of Б-1-1 beam (without transverse reinforcement) shows bearing capacity of the former to be higher by:

Б-1-2 $(1411-1317)/1317\cdot 100\ \% = 7{,}1\ \%$; Б-1-3 $(1318-1317)/1317\cdot 100\ \% = 0{,}08\ \%$;

Б-1-4 $(1328-1317)/1317\cdot 100\ \% = 0{,}84\ \%$; Б-1-5 $(1330-1317)/1317\cdot 100\ \% = 0{,}99\ \%$;

Б-1-6 $\left(1345-1317\right)/1317\cdot 100\ \%=2{,}13\ \%$; Б-1-7 $\left(1317-1317\right)/1317\cdot 100\ \%=0\ \%$;

Б-1-8 $\left(1355-1317\right)/1317\cdot 100\ \%=2{,}88\ \%$.

The analysis suggests that the beam with the oblique plane reinforced with horizontal reinforcement located at the top and at the bottom of the oblique plane has the largest bearing capacity. Bearing capacity of the beam Б-1-2 is by 7,1 % higher than that of Б-1-1. Comparison of bearing capacity of the beams Б-1-5; Б-1-6; Б-1-7; Б-1-8 which have different types of oblique planes reinforcement but the same amount of transverse reinforcement shows that transverse reinforcement is most effectively located in the beams Б-1-6 and Б-1-8.

Results of processing the values of strain of longitudinal top and bottom reinforcement in the area of simultaneous exertion of constant transverse force and bending moment are presented in Fig. 8, 9.

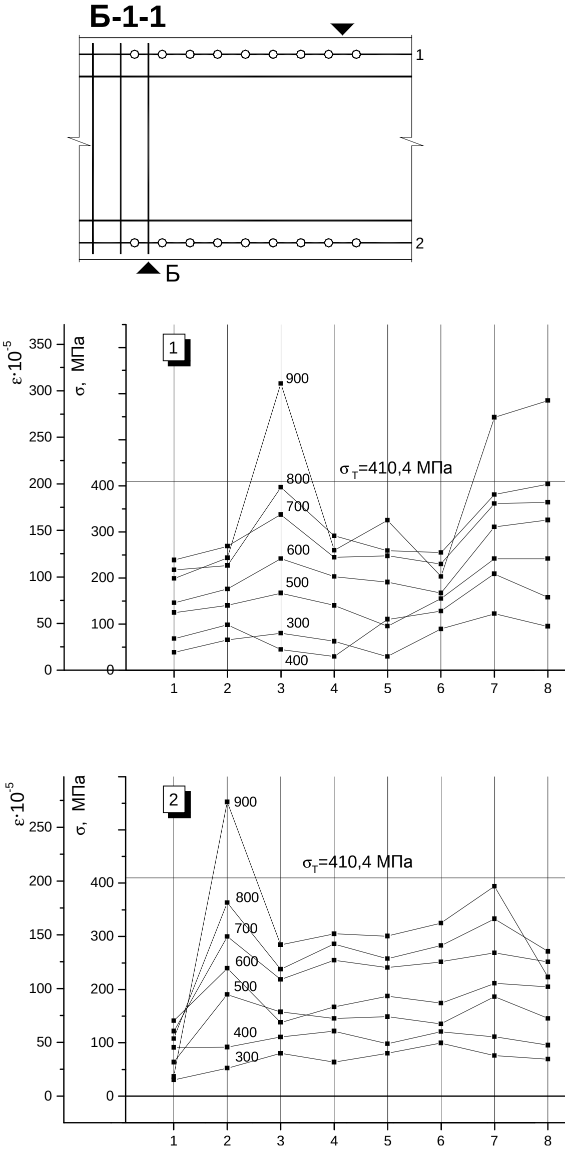

Fig. 8. Strain (stress)
in reinforcement 1, 2 of Б-1-1 beam vs. load

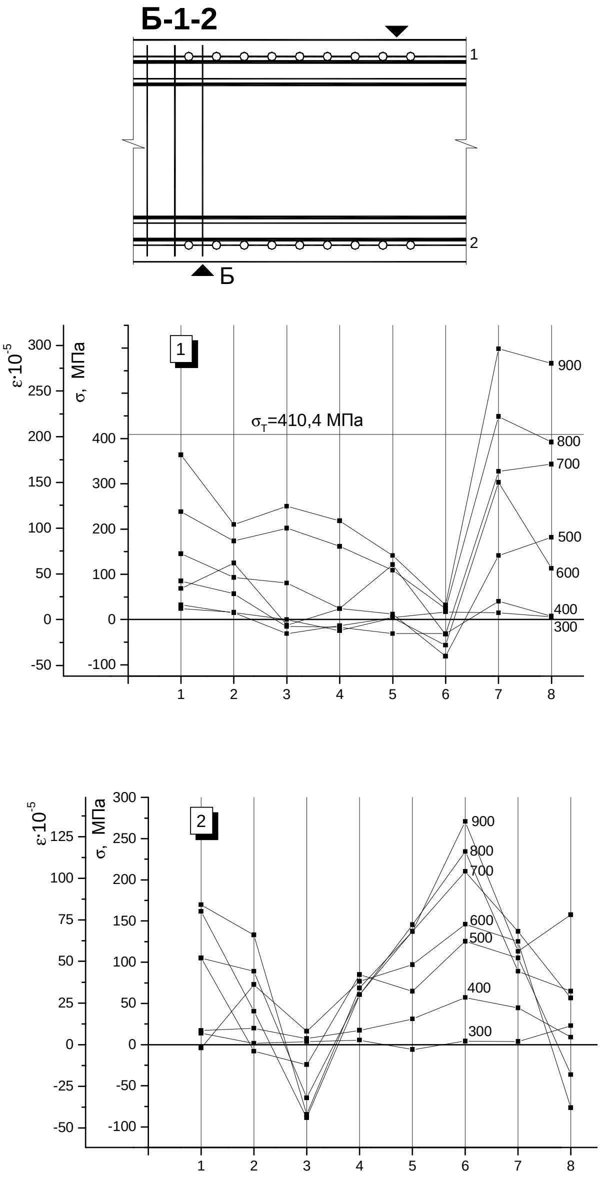

Fig. 9. Strain (stress)
in reinforcement 1, 2 of Б-1-2 beam vs. load

As a rule, there was a particular pattern of growth of strain and, respectively, of stress. At low initial loads, distribution of strains (stresses) in longitudinal bottom and top reinforcement mainly corresponds to bending moment diagram. As load increases, the plot of strains (stresses) distribution flattens, approaching a straight line. In sites of cracks nucleation, intensive growth of strains (stresses) in the longitudinal reinforcement causes hikes on the plot.

When going farther from the site of maximum moment to the support, tensile stress in the longitudinal reinforcement is declining due to adhesive force between the reinforcement and the concrete. However, with load increase and emergence of cracks, adhesion between the reinforcement and the concrete lessens, and tensile forces in the reinforcement become increasingly different from the theoretical bending moments diagram, namely they are equalizing. Maximum equalizing of tensile force values in the longitudinal reinforcement is achieved before the failure of the beam. The phenomenon of stress equalizing in longitudinal reinforcement in the section from the support to the point of force application implies increasing weakening of concrete-to-steel bonds in this area.

Figures 8 and 9 show that in the beams Б-1-1, Б-1-2 at the load $P = 900$ kN, stress in some areas of the longitudinal reinforcement seems to have reached the yield strength. However, this is not the case, because the plots show longitudinal reinforcement stress in the bottom row. Calculation of total stress values for the two rows of longitudinal reinforcement revealed that before the beam failure they had not reached the yield strength.

Conclusions

1. The type of reinforcement of oblique planes has no significant impact on the moment of oblique cracks development, i.e. oblique cracks formed approximately at the same load.

2. Nucleation of oblique cracks in beams with positive and negative bending moment diagram for the relationship $a/d = 1$ occurs at mid-height of the beam.

3. Maximum width of oblique cracks, from their emergence and to beam failure, is observed in sites of their nucleation.

4. Distribution of strains (stresses) along the tension longitudinal reinforcement of the beams is different from the theoretical bending moment diagram, i.e. at simple supports and in zero points of strain (stress) moments it is not equal to zero.

5. In beams at the relationship $a/d = 1$ with positive and negative bending moment diagram, horizontal (transverse) reinforcement is the most effective type of transverse reinforcement from the point of view of bearing capacity of oblique planes and width of oblique cracks.

6. Longitudinal reinforcement has a significant impact on the strength of oblique planes of reinforced concrete beams, both with singled-signed bending moment diagrams and with positive and negative ones.

1. F. Leonhardt, R. Walther Beiträge zur Behandlung der Schubprobleme im Stahlbetonbau. Beton und Stahlbetonbau 1, 2/1965. 2. K. Herbert Kupfer Zusammenhang Zwischen Momenttendeckung und Schubsicherung beim Schlanken Plattenbalken. BetonundStahlbetonbau 10/1967. 3. Залесов А. С. Исследование прочности по наклонным сечениям элементов с двузначной эпюрой изгибающих моментов / А. С. Залесов, Х. А. Зиганшин // Поведение бетонов и элементов железобетонных конструкций при воздействии различной длительности : сб. науч. трудов. – М. : НИИЖБ, 1980. 4. ДСТУ Б В.2.6-156:2010. Конструкції будинків і споруд. Бетонні та залізобетонні конструкції з важкого бетону. Правилапроектування. – К. : МінрегіонбудУкраїни, 2011. – 118 с. 5. Eurocode 2: Design of Concrete Structures. EN 1992 – 1.1: General Rules and Rules for buildings. – Brussels : CEN, 2004. – 226 p. 6. Карабаш В. Г. Скалывание при изгибе железобетонных балок. Исследования – Железобетонные конструкции / В. Г. Карабаш. – М. : ВНИИС, 1955. 7. Боришанский М. С. Образование косых трещин в стенках предварительно напряженных балок и влияние предварительного напряжения на прочность под действием поперечных сил. Прочность и жесткость железобетонных конструкций (к 100-летию со дня рождения Лолейта А. Ф.) / М. С. Боришанский, Ю. К. Николаев. – М., 1968. 8. Шостак Б. А. Исследование трещиностойкости наклонных сечений ненапряженных железобетонных балок прямоугольного сечения при кратковременном действии сосредоточенных нагрузок : дисс. … канд. техн. наук / Б. А. Шостак. – Львов, 1971. 9. Дорошкевич Л. О. Аналіз причин аварійного стану залізобетонних ригелів рам готелю “Дністер” у м. Львові / Л. О. Дорошкевич, С. Б. Максимович, Б. Ю. Максимович // зб. наук. статей “Проблеми теорії і практики будівництва.” Т. IV. Проектування, обстеження і експлуатація будівель і споруд. Основи і фундаменти. – Львів : Держ. ун-т “Львівська політехніка”. 1997. – С. 34–39.