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THE DEVELOPMENT OF THE MATHEMATICAL MODEL OF THE FINE DISPERSE MATERIAL BEHAVIOR UNDER VIBRATION IN THE CONICAL HOLE OF THE DISPENSER

В статті розробляється математична модель поведінки дрібнодисперсного матеріалу у конічній лунці дозатора на основі рівнянь Нав'є-Стокса, досліджується вплив силових факторів на рух продукту та конструктивних параметрів лунки на процес вібраційного дозування, а також наводяться початкові і граничні умови, що однозначно визначають рух матеріалу у дозуючому обладнанні.

The mathematical model of the fine-disperse material behavior under vibration in the conical hole of the dispenser, based on the Navier-Stokes equations, is developing; the influence of the power factor on the movement of the product and the design parameters of the hole on the vibrating dosing process is investigating; the initial and boundary conditions, which uniquely determine the movement of the product in dispensing equipment, are provided.

Setting of the problem. The market of packaging products today is rich in granular materials (GM), which differ in chemical composition, properties, particle size and other parameters. The products of the fine particle size constitute considerable interest. The dosage of these materials creates many difficulties associated with the formation of lumps, unstable bulk density, uneven leaks etc. These issues are essential for volumetric type dispensers, since the dose formation accuracy depends on the leakage uniformity of the GM through the outlet. Considering the advantages of the volumetric dispensers over weight ones, primarily high performance, it is important to investigate the behavior of the product in the dosage and the influence of external loads on the performance. An effective way to improve the material fluidity is the use of vibration, in particular, the substantiation of the GM to the vibro-boiled condition. But there are many difficulties during investigation of the fluidized product $d \le 50$ micrometers. Therefore, the investigation of the fine disperse material movement in the vibro-boiled condition is an important task, especially for the conical holes dispensers, which are characterized with high outflow complexity of the GM.

Analysis of recent research and publications. The problems of GM dosage has long been engaged. A variety of models that describe the behavior of the product during dispensing are developed. The influence of the properties of fine disperse material on the outlet size of the dispenser and the necessary technological parameters of dosage are investigated [1]. However, this model is based on researching the forces, which act on a separate piece of GM, and cannot use this method in the case of vibro-liquided material, since the array of material in this case behaves differently than its separate parts. This model neglects the forces arising between the particles during their movement. Russian researchers I. Fedorenko, D. Pirozhkov, S. Sorokin [2, 3, 4] developed a model of the behavior of GM in the dispenser, based on Navier-Stokes equations. Held investigations allow us to describe the movement of the vibro-liquided material with high precision. However, these developments are limited to consideration of the GM with granular composition of particles $d \ge 50$ micrometers and dosing cylindrical hole.

Formulation of the research objectives. The purpose of the investigations is the development of the mathematical model of the fine disperse material movement in the conical hole of the dispenser using the Navier-Stokes equations.

Statement of the main research material. All models of the GM behavior can be divided into 3 types [3]:

- a model of a single particle;
- special models;
- continuous medium models.

The first type of models is specializing in the investigation of the behavior of a particle material; considering its motion, then, taking into account certain assumptions, make a decision, that all product in the hopper will behave a similar behavior. The disadvantage of such developments is ignoring the forces, which appear between particles. These investigations should be used only when the motion of the material in the absence of any mechanical influences on the product are considering, since during the external load the fine disperse material begins to behave similarly to fluid and motion of one particle is significantly different from the behavior of the entire material flow. Models of the single particle can be effectively used when determining the size of the dozing equipment outlet in case of impossible using of aids that improve the flowability of the GM. Also sufficient accuracy of this method is possible investigating material, consisting of particles of large size.

Special models are designed for investigation the behavior of GM in specific conditions, which can be provided only using a certain kind of equipment. Using the models of the second type is high-efficiency, but their major disadvantage is the point, that changing process conditions or equipment the models incorrectly describe the essence of the process of dosage.

The third type of models is consider the bulk material as a continuous body, viscous liquid or ideal gas. The use of this model is especially effective during the investigation of the vibro-boiled layer of fine disperse material, since this material is characterized by low permeability and its characteristics are similar to liquid in the vibro-liquided condition. Continuum models allow to consider the interaction of particles with each other and with the environment, surrounding them.

To investigate the behavior of fine-disperse material in the conical hole of the dispenser in the vibro-boiled state the continuum model, that most adequately captures the essence of the process of dosage, is used. Considering all existing models of this type, there is established, that the most suitable one in this case is the Lorentz model, which describes three possible states of the fine-disperse material under the influence of vibration: a preliminary consolidation, circulation and chaotic motions of the GM. Since the fine-disperse product behaves similarly to a liquid in the vibro-boiled state, the most effective way to obtain the Lorentz model is its deducing from the Navier-Stokes equations.

Modeling the behavior of the GM in the vibro-boiled state, there were made the following assumptions:

1. The vibration is transmitted through the bulk material vibro-floor; the wall of the conical hole is fixed.
2. The material under the vibration impact does the the vertical axis symmetric movement.
3. Only the layer of material, located in the conical hole of the dispenser, is exposed to the impact of vibro-boiling.
4. The movement of the material in the hopper in a state of vibro-boiling is considered as steady (inertial forces are negligible, that's why they are neglected in the modeling).
5. The circulation of particles under vibration occurs along circuits similar to Benard cells considering the bunker configuration.
6. There are forces of adhesion between the particles of the material.
7. Deformation of the GM in the vibro-boiled state is low (the material behaves similarly to a liquid).
8. The pressure of the material in the hopper varies only with the height of the product layer.

For studying the motion of GM we distinguish the elementary volume of the fluidized material (Fig. 1). Considering the assumptions, that were used in modeling the behavior of fine-disperse material, the movement of the product in the dispenser hole can be described using the equation of motion of Navier-Stokes, which look like in the vector form [2]:

$$\frac{d\overline{V}}{dt}+\left(\overline{V}\nabla\right)\overline{V}=\frac{1}{\rho}\operatorname{grad} p+\nu\Delta\overline{V}+\overline{f}\,, \tag{1}$$

where $\overline{V}$ – vector velocity field, t – time, ∇ – operator Nabla, Δ – vector Laplace operator, ρ – bulk density of the GM, p – GM pressure, ν – kinematic viscosity coefficient of the GM, $\overline{f}=\frac{\overline{F}}{m}$ – density of the forces, acting on the particles of the GM in the vibro-boiled condition, $\overline{F}$ – force, acting on the particle of the GM, m – particle mass product.

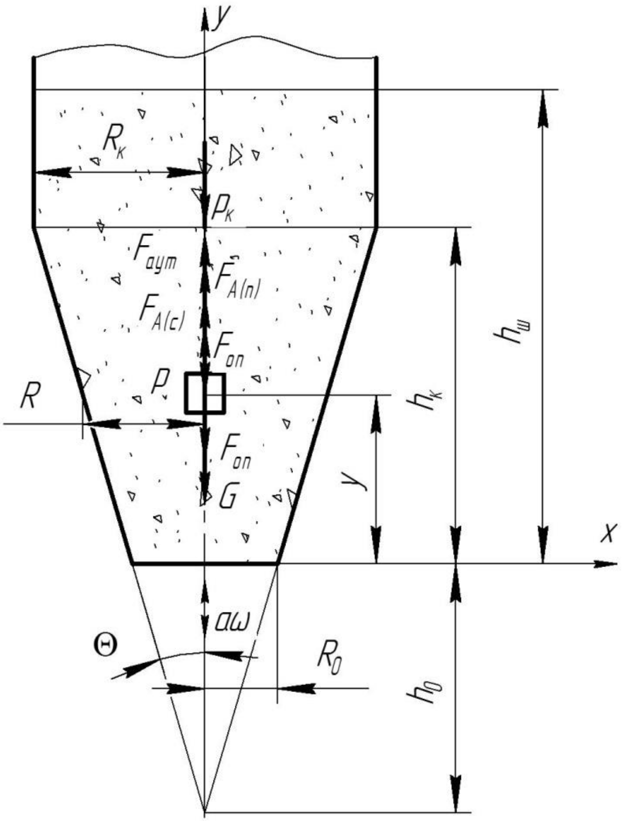

Fig. 1. Forces acting on the element of the vibto-boiled GM

As you can see, the GM pressure is an important component of the equation (1). It is dependent not only on the thickness of the material in the conical hole of the dispenser, which is above the investigated area, but also on the geometry of the hole: height, radius and angle:

$$p=\frac{\left(p_{\kappa}-\rho\cdot g\cdot y\right)\cdot\left(\frac{R}{R_{\kappa}}\right)^{2}}{\left(1+\left(\frac{R}{R_{\kappa}}\right)^{2}+\left(\frac{R}{R_{\kappa}}\right)^{4}\right)} \tag{2}$$

where p_{κ} – the pressure of the material on the boundary of transition from conical to cylindrical part of the hole (depending on the thickness of the cylindrical part of the hole $h_{ш}-h_{\kappa}$), g – acceleration of gravity, y – coordinate of the center of the selected volume element, $R=R_0+y\cdot\tan(\Theta)$ – radius of the conical hole in the volume element place , R_0 – a radius of the outlet, Θ – the angle of the hole, R_{κ} – the radius of the hole on the boundary of transition from conical to cylindrical part.

Since the most effective way to investigate the behavior of fine-disperse GM is to consider it as a continuous medium, then we consider the forces, acting on an elementary volume of the material. The importance of the investigation of the products is the force of gravity, which stimulates the bulk material outflow through the conical dispenser hole:

$$\overline{G}=\frac{\pi\cdot d_{ч}{}^{3}}{6}\cdot\rho_{ч}\cdot\overline{g}\,, \tag{3}$$

where $d_{ч}$ - the diameter of the GM particle, $\rho_{ч}$ - GM particle density.

During the vibro-boiled state the force of the dynamic pressure from the side of the air flow is acting the material, which is determined from the following equation:

$$\overline{F_n}=\frac{\pi\cdot d_{ч}{}^{2}}{8}\cdot\rho_n\cdot C_n\cdot\left(\overline{U}-\overline{V}\right)^{2}, \tag{4}$$

where ρ_n – the density of the air, C_n – the coefficient of air resistance, $\overline{U}$ – air velocity in the bunker.

The composition of the forces, acting the elementary volume of fine-disperse material in the vibro-boiled state, also include the Archimedes’ force, caused by the GM and the air:

$$\overline{F_{A(c)}} = \frac{\pi \cdot d_ч{}^3}{6} \cdot \rho \cdot \overline{g}, \tag{5}$$

where $\overline{F_{A(c)}}$ - the Archimedes' force caused by the environment.

$$\overline{F_{A(n)}} = \frac{\pi \cdot d_ч{}^3}{6} \cdot \rho_n \cdot \overline{g}, \tag{6}$$

where $\overline{F_{A(n)}}$ - the Archimedes' force caused by the air.

The significant effect on the behavior of fine-disperse material in the dispenser conical hole outflow is caused by the resistance force, that prevents free leakage of the product:

$$\overline{F_{on}} = \pi \cdot d_ч{}^2 \cdot \rho \cdot \overline{g} \cdot k \cdot f \cdot h, \tag{7}$$

where k –- coefficient of the material mobility, f – the coefficient of internal friction of the material, $h = h_к - y$ – the height of the GM layer above the particle.

As the movement of fine-disperse material is considered, the importance of adhesion force, i.e. adhesion between the particles of the product [5], is high. It can be caused by several forces acting simultaneously:

$$\overline{F_{аут}} = \overline{F_{кап}} + \overline{F_к} + \overline{F_м} + \overline{F_{м.з}}, \tag{8}$$

where $\overline{F_{кап}}$ – the capillary force, $\overline{F_к}$ – the cohesion force, $\overline{F_м}$ – the molecular force, – $\overline{F_{м.з}}$ the force of mechanical grip.

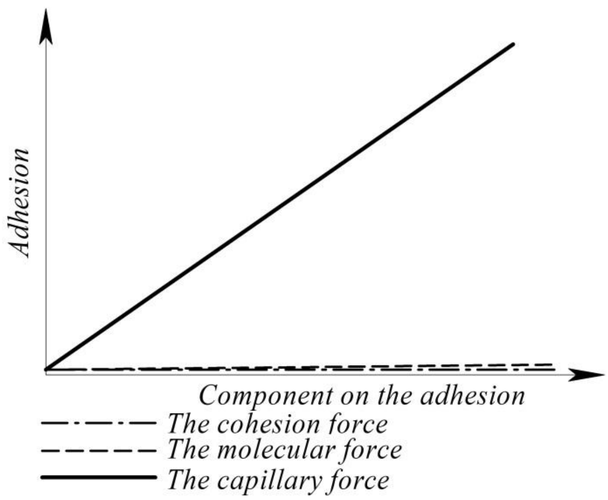

Fig. 2. The influence of the traction component on the adhesion

As we can see (Fig. 2), a significant effect on the growth of the adhesion power is caused only by capillary part $\overline{F_{кап}}$, because of increasing molecular $\overline{F_м}$ and cohesion $\overline{F_к}$ forces the traction between the particles remains almost unchanged $F_{кап} >> F_м, F_к$. Mechanical coupling forces $\overline{F_{м.з}}$ are typical for particles of the irregular shape. They usually occur when an external sealing material load acts, so in this case we neglect them. Thus, to simulate the movement of fine-disperse material it is enough to take into account the capillary force as the adhesion force [6]:

$$\overline{F_{аут}} \approx \overline{F_{кап}} = \frac{\pi \cdot \overline{\sigma_н} \cdot d_ч \cdot \cos(\Theta_к)}{1 + \frac{\beta}{2}}, \tag{9}$$

where $\overline{\sigma_н}$ – the surface tension of water, $\Theta_к$ – surface wetting marginal angle, β – the angle, that indicates the amount of water in the GM.

Considering the symmetry of the outflow bulk material with respect to the axis *Oy* and the forces acting the fine-disperse product, the equation (1) in the projections on the axis *Ox* and *Oy* looks like:

$$\begin{cases} \dfrac{\partial V_x}{\partial t} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + \nu\left(\dfrac{\partial^2 V_x}{\partial x^2} + \dfrac{\partial^2 V_x}{\partial y^2}\right) - V_x\dfrac{\partial V_x}{\partial x} - V_y\dfrac{\partial V_x}{\partial y}, \\ \dfrac{\partial V_y}{\partial t} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} + \nu\left(\dfrac{\partial^2 V_y}{\partial x^2} + \dfrac{\partial^2 V_y}{\partial y^2}\right) - V_x\dfrac{\partial V_y}{\partial x} - V_y\dfrac{\partial V_y}{\partial y} + g\left(\dfrac{\rho}{\rho_ч} + \dfrac{\rho_n}{\rho_ч} - 1\right) + \dfrac{6\cdot g\cdot\rho\cdot k\cdot(h-y)}{d_ч\cdot\rho_ч}\times \\ \times sign(V_y - a\omega\cos(\omega t)e^{-\varepsilon y}) + \dfrac{3\cdot C_n\cdot\rho_n}{4\cdot d_ч\cdot\rho_ч}\cdot\left|U_{0y}\cdot e^{-\delta y}\cdot\sin(\omega t) - V_y\right|\cdot sign\left(U_{0y}\cdot e^{-\delta y}\cdot\sin(\omega t) - V_y\right) + \\ + \dfrac{6\sigma_н\cdot\cos(\Theta_к)}{\left(1+\tan(\frac{\beta}{2})\right)\cdot d_ч{}^2\cdot\rho_ч}\cdot sign(V_y - a\omega\cos(\omega t)e^{-\varepsilon y}), \end{cases} \tag{10}$$

where ε – coefficient of the GM oscillations damping, δ – coefficient of the airflow velocity damping, a and ω – amplitude and frequency of the oscillations, U_0 – the amplitude of the velocity of the air in the bunker.

The continuity equation has to be added to the equation (10):

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y}, \tag{11}$$

To obtain the solution of Navier-Stokes equations we need to specify the initial and boundary conditions, which allow to select one of a plurality of possible solutions, that meets the real physical process described by this equation. The initial conditions indicate the behavior of the system at the initial time, and for this model they look like:

$$V_x\big|_{t=0} = 0,\ V_y\big|_{t=0} = 0,\ U\big|_{t=0} = a\omega. \tag{12}$$

where $U\big|_{t=0}$ - the initial velocity of the air flow.

The pressure in the dispenser hole varying with the vertical coordinate y is important for modeling the behavior of GM:

$$p\big|_{t=0} = p_0 - \rho\cdot g\cdot y. \tag{13}$$

The boundary conditions, specifying the behavior of the bulk material at the edge of the stream, are as follows for the model (Fig. 1):

$$y = 0,\ V_y = 0,$$
$$y = h,\ V_y = 0,\ p_к = 0,$$
$$x = \pm R,\ V_y = a_y\cdot\cos(\omega t)\ V_x = a_x\cdot\sin(\omega t). \tag{14}$$

This mathematical model with initial and boundary conditions allows to reduce the Navier-Stokes equation to the Lorenz system, as planned for the future in order to study the influence of process parameters of dosing and the dosing hole size on the uniformity of GM process leaks.

Conclusions. Thus, the fine-disperse material belongs to GM, causing considerable difficulties when dosing. This is due to their specific properties, the presence of adhesion forces, a tendency to caking and the formation of lumps. The hole geometry has a significant impact on the motion character of the product. The angle, diameter of the outlet, height of the metering hole determine the behavior of GM. This mathematical model gives a high degree of accuracy to describe the behavior of the bulk material in the vibro-boiled state and to investigate the influence of structural parameters of the hole on the dosing process. Using these investigations will improve the uniformity of leakage of material and thus will improve the accuracy and efficiency of the process of the fine-disperse material dosing through the dispensing hole with the arbitrary shape.

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