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ADDING TO THE SYNTHESIS OF THE KINEMATICS METHOD PROFILE’S TOOTH IN THE PLANAR GEARING SYSTEM

Запропоновані теоретичні положення для синтезу профілів зубців зубчастих коліс у площинному зачепленні, що доповнюють кінематичний спосіб визначення рівняння зачеплення. Новизна полягає у тому, що вираз, який в нерухомій системі координат описує лінію зачеплення профілів, отримують шляхом визначення найменшої відстані від миттєвого центра швидкостей (полюса зачеплення) до точки контакту профілів. Записуючи лінію точок контакту у відповідній рухомій системі координат визначають шуканий профіль зубців колеса або рейки.

The offered theoretical positions for synthesis the tooth’s profiles of gear-wheels in the planar gearing system. A novelty consists in determination of the least distance from the instantaneous centre of speeds to the gear contact point on transverse line of action. Writing down the equalization of transverse line of action in the corresponding movable coordinates system determine the tooth profile of wheel or basic rack .

Raising of problem. The functional, operating and economic indexes of far or modern transport, power and production machines determine gearings which enter in the complement of their drive. The gearings loading ability, reliability, noise descriptions depend directly on hooking formed of tooth of wheels lateral active surfaces. Therefore, perfection of gearings belongs to the actualing scientific and technical tasks which can be decided by the synthesis of tooth type workings surfaces. Synthesis of tooth type it is also enough important for realization of technological processes of the toothed surfaces on details, which utilized for formation of transmissions or toothed connections, directed on primary tooth forming. Cylinder gear-wheels, toothed rails, spline shaft, ratchet wheels, belong to the list of the mentioned details, asterisks of chain-drives and others like that. In this case actuality is become by tasks directed on the high-quality and reliable planning of workings types of cuttings instruments which apply in technological processes tooth cutting on the proper details. Cuttings of tooth carry out the generating method or method tooth cutting in which use the continuous rotation of blank [3, 4].

Analysis the last researches and publications. A synthesis and analysis of the planar gearing system is accompanied the decision of such basic tasks: the first task is determination of tooth type this envelope to a family of profile lines of primary tooth type on driving gear or gear rack, the second task is establishment the equalization of transverse line of action for conjugate profile [2, 3].

The tooth conjugate profile of driving and driven wheels set at relative rolling of their centroid without sliding. With gears which link the co-ordinates systems $S1(x1, y1)$ and $S2(x2, y2)$ return accordingly on the corners of φ_1 and φ_2 . Equalization $F_1(x_1, y_1)= 0$ in the co-ordinates system S_1 determines the set type of tooth. By expressions of transition from the co-ordinates system S_1 to the system S_2 get the family of profile lines equalization of the set type as equalization $F2(x_2, y_2, \varphi_1)= 0$.

In obedience to differential geometry positions envelope to this family of profile lines set after the exception of parameter of c1 from the equalizations system :

$$F_2 (x_2 , y_2 , \varphi_1) = 0 ; \qquad (\partial F_2 / \partial x_2)\cdot(\partial x_2 / \partial \varphi_1) + (\partial F_2 / \partial y_2)\cdot(\partial y_2 / \partial \varphi_1) = 0 . \qquad (1)$$

This expressions allow to get the co-ordinates of plural points of contact tooth profiles. They enable to define equalization of transverse line of action or for the processes of tooth cutting - equalization of profiling line in the immobile co-ordinates system S (x, y). In same queue, type of tooth of the driven wheel determine in the mobile co-ordinates system S_2 , using the formulas of connection between the immobile and mobile co-ordinates system.

The kinematics method of determination the engagement gearing equalization is based on determination of vectors scalar work – is normal $\vec{N}_i$ and relative speed $\vec{v}_i$ which that sets tooth contact condition with conjugating profiles [3]:

$$\vec{N}_i \cdot \vec{v}_i = 0 \,. \qquad (2)$$

The resulted methods are basic, constantly common during the analysis of engagement gearing, determination of type of conjugating tooth or profiling of gear cutting instruments which use for realization of generating method at making of details with the certain type tooth. However, achievement procedure of necessary eventual result is accompanied difficult enough mathematical expositions, necessity to carry out both differentiation of difficult functions and determine necessary parameters from mostly transcendent gear edge contact equalization. All of it induces to the continued searching of ways from the improvement other methods of synthesis and analysis of the planar gearing system.

Purpose of researches. Development the theoretical positions from improvement the synthesis kinematics method of the engagement gearing and profiling of metal-cutting instruments for tooth cutting on details by the generating method. It is got results for the planar gearing system due to consideration of certain basic terms [1, 2, 3]. (Fig.1). Rotary motion round the centers O_1 and O_2 that belong to the gear-wheels accompanied mutual rolling without relative sliding by centroid of these wheels. Centroid it circles which have the radiuses R_1 and R_2 . Size them back proportional angular speeds of ω_1 and ω_2 of these wheels. Centroid touch each other in the point P . This point is the instantaneous center of rotation (pitch point) . Profiles tooth types of in the planar gearing system the guilty belong to envelope.

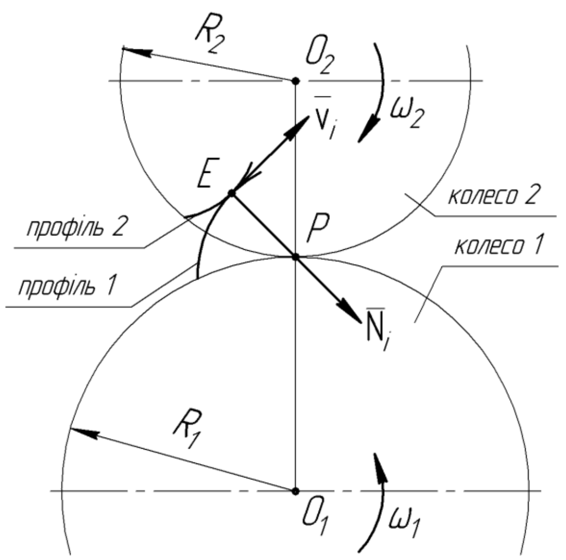

Fig. 1. General chart of the planar gearing system *for two gear-wheels*

In obedience to the Villis theorem in the point E of gear edge contact the general normal $\vec{N}_i$ must pass through point *of P* the instantaneous center of rotation (pitch point) .

Vector of relative speed $\vec{v}_i$ of contact point E of wheel tooth profile 2 in relation to the contact point E of wheel tooth profile 1 directed along general tangent to the profiles in the gear edge contact point.

Therefore cutting in or departure of tooth profile absents one from other.

Except for it, in obedience to the proper theorem of theoretical mechanics, eventual moving of flat type in the plane it is possible to carry out by only one rotation on some corner round the instantaneous rotation center of P with the proper instantaneous relative angular speed of ω_{12} .

Consequently, relative speed $\vec{v}_i$ in the contact point E is equal to work of instantaneous angular speed of sch12 on the rotation radius PE of instantaneous circulating motion. Therefore instantaneous rotation radius of PE is on are general normal $\vec{N}_i$ to the tooth profiles in the instantaneous contact point E .

From the geometrical standpoint of view instantaneous radius of rotation PE even the least distance from the instantaneous center of rotation P to the gear edge contact point E.

This least distance *is PE* conducted along are general normal $\vec{N}_i$ to the gear edge contact point E.

Distance determine PE in the co-ordinates immobile system S, which dispose so, that its beginning coincided with the instantaneous center of rotation P. Then expressions for determination of distance from the instantaneous center of rotation P to the gear edge contact point E it is possible considerably to simplify because co-ordinates of point P levels to the zero ($x_P = y_P = 0$).

The conducted analysis allowed to set that in the planar gearing system, such rule is carried out: *in the* planar gearing system *, for certain instantaneous position of conjugating types of tooth, distance from the instantaneous center of speeds to the gear edge contact general point is minimum.*

Use the resulted rule the offered method to the synthesis (analysis) of the planar gearing system, which is represented in the following algorithm:

1. To develop a layout of the co-ordinates systems by mobile chart S_1, S_2, S_3 and immobile S.
2. To write down is known profile tooth equalization in the mobile co-ordinates system S_1 or S_3 - $F_1(x_1, y_1) = 0$ or $F_3(x_3, y_3) = 0$.
3. To set centroid of rail or wheels R_1 and R_2 and define the location of instantaneous center of rotation P.
4. To define mutual correlations between the co-ordinates of points in the co-ordinates systems S, S, S_1, S_2 or S_3, that depend on the turn corners φ_1 and φ_2.
5. To write down the equalization of the set tooth profile in the immobile co-ordinates system S, for example, $F_{01}(x, y) = 0$ or $F_{03}(x, y) = 0$ або $y = f_{01}(x)$ or $y_0 = f_{03}(x)$).
6. To write down expression which determines the square of distance from the instantaneous center of rotation P (pitch point) to the point E on to the set profile:

$$r_E^2 = (x_E - x_P)^2 + (y_E - y_P)^2, \tag{3}$$

where r_E^2 is a square of length of vector $\vec{r}_E$; (x_E, y_E); (x_P, y_P) are the proper co-ordinates of points E and P.

7. In the immobile co-ordinates system S for the turn corner φ_1 or φ_2 to set the least distance between the points P and E for this purpose to differentiate expression (3) :

$$\mathrm{d}(r_E^2)/\mathrm{d}(x_E) = 0, \tag{4}$$

8. To set equalization of transverse line of action in the immobile co-ordinates system S after determination of co-ordinates contact point $E(x_E, y_E)$.
9. To define profile tooth equalization in the mobile co-ordinates system S_2 or S_3. For this purpose to use equalization of transverse line of action in the immobile co-ordinates system S and correlation between the co-ordinates of points in the co-ordinates systems S, S_1, S_2 or S_3.

Example. To define profile tooth equalization of gear shaper cutter for cutting of generating method of spline shaft tooth (Fig. 2).

1. Use three co-ordinates systems - two mobile - S_1 and S_2, that stockings with a spline shaft and gear shaper cutter, one immobile co-ordinates system - S.
2. Equalization of type of spline shaft tooth is in the mobile co-ordinates system S_1 :

$$F_1(x_1, y_1) = x_1 - h = 0. \tag{5}$$

3. Spline shaft centroid and gear shaper cutter are circles by radiuses accordingly R_1 and R_2, the instantaneous center of rotation (pitch point) located in the point P
4. Equalization of co-ordinates of points is in the different co-ordinates systems:

- transition from the system $S_1(x_1, y_1)$ to the system $S(x, y)$:

$$x_1 = x \cdot \cos\varphi_1 + y \cdot \sin\varphi_1 + R_1 \cdot \sin\varphi_1; \qquad y_1 = -x \cdot \sin\varphi_1 + y \cdot \cos\varphi_1 + R_1 \cdot \cos\varphi_1. \tag{6}$$

- transition from the system *of* $S(x, y)$ to the system *of* $S_2(x_2, y_2)$:

$$x_2 = x \cdot \cos\varphi_2 - y \cdot \sin\varphi_2 + R_2 \cdot \sin\varphi_2; \qquad y_2 = x \cdot \sin\varphi_2 + y \cdot \cos\varphi_2 - R_2 \cdot \cos\varphi_2. \quad (7)$$

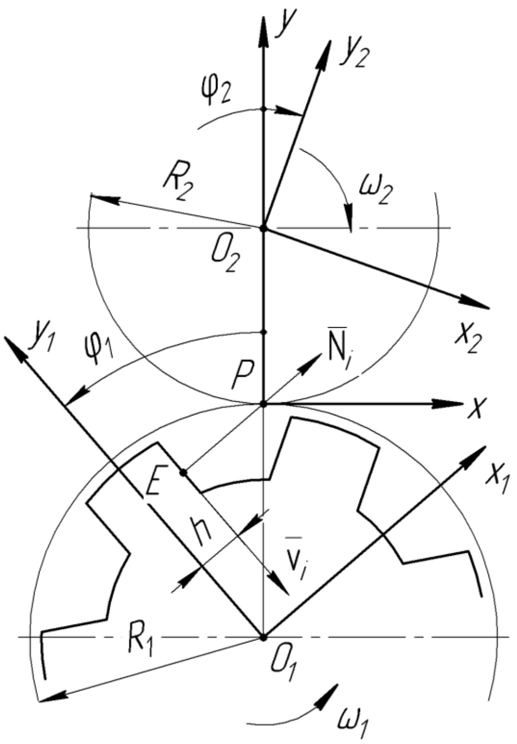

Fig. 2. Chart of determination profile tooth of gear shaper cutter for cutting of spline shaft tooth

5. Equalization of the set type of spline shaft tooth is in the immobile co-ordinates system S (to use (5)):

$$x \cdot \cos\varphi_1 + y \cdot \sin\varphi_1 + R_1 \cdot \sin\varphi_1 - h = 0 \;;$$

$$y = \frac{h}{\sin\varphi_1} - R_1 - x \cdot \mathrm{ctg}\,\varphi_1 . \quad (8)$$

6. Square of distance from the instantaneous center of rotation $P(x_P, y_P)$ to the point $E(x_E, y_E)$ in the immobile co-ordinates system S, if $x_P = y_P = 0$:

$$r_E^2 = (x_E - x_P)^2 + (y_E - y_P)^2 ;$$

$$r_E^2 = (x_E)^2 + \left(\frac{h}{\sin\varphi_1} - R_1 - x_E \cdot \mathrm{ctg}\,\varphi_1\right)^2 . \quad (9)$$

7. The least distance is between the points P and E :

$$\frac{\mathrm{d}(r_E^2)}{\mathrm{d}x_E} = 2x_E + 2 \cdot \left(\frac{h}{\sin\varphi_1} - R_1 - x_E \cdot \mathrm{ctg}\,\varphi_1\right) \cdot \mathrm{ctg}\,\varphi_1$$

$$x_E \cdot (\mathrm{tg}\,\varphi_1 - \mathrm{ctg}\,\varphi_1) + \frac{h}{\sin\varphi_1} - R_1 = 0 . \quad (10)$$

8. Equalization of transverse line of action in the immobile co-ordinates system S (to define the co-ordinates x_E, y_E of contact points E from expressions (10) (8)):

$$x_E = \left(R_1 - \frac{h}{\sin\varphi_1}\right) \cdot \left(\frac{1}{\mathrm{tg}\,\varphi_1 - \mathrm{ctg}\,\varphi_1}\right); \quad y_E = -\left(R_1 - \frac{h}{\sin\varphi_1}\right) \cdot \left(\frac{\mathrm{tg}\,\varphi_1}{\mathrm{tg}\,\varphi_1 - \mathrm{ctg}\,\varphi_1}\right) \quad (11)$$

9. Equalization of tooth profile in the mobile co-ordinates system S_2 (to apply (7)):

$$x_2 = x_E \cdot \cos\varphi_2 - y_E \cdot \sin\varphi_2 + R_2 \cdot \sin\varphi_2; \quad x_2 = x_E \cdot \sin\varphi_2 + y_E \cdot \cos\varphi_2 - R_2 \cdot \cos\varphi_2 . \quad (12)$$

Conclusions. Offered new method of synthesis or analysis of the planar gearing system. In this method for the engagement gearing set the location of instantaneous center of rotation and determine minimum distance from him to the gear edge contact point along are general normal. It will allow to get equalization of transverse line of action in the immobile co-ordinates system, and sweat to define the conjugating tooth profile of pair gear. Equalizations are got simple and allow to avoid difficult procedures of kinematics analysis for determination of co-ordinates of instantaneous normal vectors and relative speed in the planar gearing system.

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