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O. TADYEYEV

Department of Geodesy and Cartography, National University of Water and Environmental Engineering,
Soborna str., 11, 33028, Rivne, Ukraine, tel. +38(096)7488449, e-mail: oleksandrtad@gmail.com

EVALUATION OF THREE-DIMENSIONAL DEFORMATION FIELDS OF THE EARTH BY METHODS OF PROJECTIVE DIFFERENTIAL GEOMETRY; RIGID ROTATIONS OF THE EARTH

Aim. The aim is this research is the evaluation of the Earth's rigid rotation as a component of global deformation fields in interconnection with angular distortions of the geocentric spatial coordinate system. **Methodology**. Solutions will be achieved by methods of projective differential (metric) geometry based on the differential presentation of transformations of Riemannian space images in the form of its complicated diffeomorphic manifolds. Based on the essence of the International Terrestrial Reference System (ITRS).in which the input data are given, and on a global scale the deformation fields, as a Riemannian manifold it is defined as a the tangent of Euclidean space. To solve the problem are used the methods of description the change of the Riemannian metric in the tangent Euclidean space, is parameterized by the Cartesian coordinate system. **Results**. The basis of the methods used enabled the results, which are the terms of its content. The practical application has a dual interpretation. In the former , an expression of angular distortions for needs of the deformation analysis is derived from formulas for angles of the rigid Earth's rotation into projections on ITRS coordinate planes. At the same time, it is proven that these angles are indicators of the coordinate system distortion. The hypothesis of probable deformations of the spatial geocentric coordinate system is substantiated by the geophysical content of the ITRS concept. The identity of conditions of the Earth's parameterization by ITRS and of the tangent Euclidean space parameterization by the Cartesian coordinate system has been proven. On this basis, the truthfulness of the hypothesis can be verified by empirical values of angles that are defined from results of GNSS-observations. In this case of significant importance, they are indicators of angular distortions of the ITRS system or an expression by deviations from the axes orthogonality in its ITRF version as measures of the oblique-angled Cartesian system into the any epoch of observation that follows. Using methods of projective differential geometry the formulas are obtained for the coordinate axes directions of the deformed system. **Scientific novelty.** It is proven that the approach for solving the problem of the deformation analysis in geodynamics based on the Riemannian geometry it is generalizing relative to its use. On this basis, prospects for filing of deformation fields by nonlinear functional models are substantiated. **Practical significance.** The obtained results are designed to be used for the evaluation of global deformation fields of the Earth and solving problems of the modern geodesy in its interconnection with geodynamics in the context of reference frame research. All analytical expression of angular distortions is given in general form, which is able to transfer the nonlinear deformation tendencies. A methodology of the deformation analysis is adapted to be used as input data for the results of the Global Navigation Satellite System (GNSS) monitoring station coordinates, taking into account the probable ITRS angular distortions.

Key words: Riemannian diffeomorphic manifolds; space metric tensor; deformation analysis; rigid rotation of the Earth; deformation of the coordinate system.

Introduction

Evaluation and analysis of deformation fields of the Earth is one of urgent problem solving in modern geodynamics. Solving the problem has a complex character and is achieved by methods from various natural sciences. Among them, geodetic methods occupy a special place as such that they are able to quantitatively estimate the movement of the Earth's physical surface and provide its study and interpretation within the various mathematical models. The purpose and strategic direction of research is defined by resolutions of the International Association of Geodesy (IAG) in the framework of the activities of Sub-Commission 3.2 "Crystal Deformation" and of Commission 3 "Earth Rotation and Geodynamics". Among the priority directions envisaged "to study the deformation of the crust at all scales from global plate tectonics to local deformation, to contribute reference frame related work in order to better understand deformations, to improve reference frames, and to develop and coordinate international programs related to observations, analysis, and fields data interpretation" [International Association…]. The main source of quantitative information for this research is defined as the data of continuous monitoring of station coordinates, which are determined using the GNSS method. There is also a natural relationship with the IAG Commission 1 “Reference Frames”, as the reference frame definition must be consistent with the actual crystal deformation.

Analysis of the research and unresolved parts of the general problem

Results of the GNSS-monitoring of station coordinates are expressed in the ITRS. This is such a type of reference system that is tied to the body, which moves unevenly, not in straight lines, and with the acceleration under the forces of the nonzero resultant. ITRS is a non-inertial system that moves and rotates together with the Earth. Taking into account that the inertia of any real reference system in general is approximate and any point can be selected as the beginning of the coordinate system. It is the non-inertia that makes a certain uneven motion. The starting point of ITRS was placed in the center of mass of the solid Earth, the oceans, and the atmosphere. ITRS system is burdened by a condition of No-Net-Rotation (NNR). This condition corresponds to the conception of the conservation of the angular momentum of the Earth as a whole, which coincides with a zero aggregate angular momentum of all lithosphere plates according to their kinematical model.

Monitoring of the ITRS system is carried out jointly with Earth monitoring using such methods of satellite geodesy as Very Long Baseline Interferometry (VLBI), Satellite Laser Randing (SLR), Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS), and GNSS. The combined data processing of monitoring results obtained by these methods, and also Earth Orientation Parameters (EOPs), which was provided by the International Earth Rotation Service (IERS), the time and factors of the physical origin (the scale factor is taken into account) allows the setting of the International Terrestrial Reference Frame (ITRF) versions of the ITRS. Defined on a given epoch of observations, the ITRF version is identified as a datum. The position of the each current datum is calculated taking into account the displacements T_x, T_y, T_z of the system starting point relative to its pre-implementation, the scale factor D, changes of the coordinate axes orientation R_x, R_y, R_z, and velocities $\dot{T}_x, \dot{T}_y, \dot{T}_z, \dot{D}, \dot{R}_x, \dot{R}_y, \dot{R}_z$. ITRF solutions based on this data are achieved using the well-known Helmert transformation. IERS recommends the use of its linearized form [IERS Conventions…??]. Transformation parameters for the latest ITRF solutions are presented in Table. Parameters were obtained under the coordination of the ITRS Centre of the IERS, hosted by the National Geographic Institute (IGN) of France. According to a UN General Assembly resolution dated 26 February 2015, the program sought to support and improveg the geodetic infrastructure and ensure the sustainable development this ITRS Centre that is recognized as a responsible institution in determining the content of ITRS and achieving ITRF solutions for the creation of Global Geodetic Reference Frame (GGRF).

The choice of the ITRS starting point, which corresponds to the center of mass of the solid Earth, oceans, and atmosphere today, is debatable. Thus, according to the arguments [Argus et al., 2010], such an approach is not always confirmed in practice. The motion velocity of the geocentric system starting point, according to the authors, is not a constant as there are unacceptably large differences between ITRF versions. For example, the linear velocity of its motion in the ITRF2005 differs from ITRF2000 by 1.8 mm/year and from ITRF1997 by 3.4 mm/year. In addition, some ITRF versions give the position differently and then determine the starting point velocity accordingly. Thus, in ITRF1997, the starting point is defined by the joint processing of observation data using the GNSS, VLBI, and SLR methods as the geometric center of the solid Earth's shape. Its velocity is the average velocity of the Earth's surface determined under the hypothesis of the sustainability of its motion within the NUVEL-1A kinematical model. In ITRF2000 (and in the following ITRF versions), the starting point already has been taken as the center of mass of the solid Earth, oceans, and atmosphere under the NNR condition, and its velocity is determined by the SLR observation of a LAGEOS (LAser GEOdynamics Satellite) orbit. In order to eliminate these inconsistencies, the lithosphere plate’s movement velocities (GEODVEL) [Argus et al., 2010] are determined under the hypothesis that the starting point is the center of mass of the solid Earth. The components of geodetic velocities are computed taking into consideration the motion of the solid Earth’s center of mass regarding the starting point of ITRF2005, amended by 0.3, 0.0 and 1.2 mm/year in the directions of the coordinate axes x, y, z respectively. Station velocities in ITRF2000 are amended by -0.1, 0.1 and -0.6 mm/year and assigned to the same mass center. On such a basis the kinematical models MORVEL [DeMets et al., 2010] and NNR-MORVEL56 [Argus et al., 2011] are created. The reasoning for this kind of amendment used to eliminate systematic offset of an ITRS starting point, in regard to the center of spherical readout base of plate rotation (as the solid Earth center of mass), is presented by [Kogan, Steblov, 2008]. The research results presented in [Wu et al., 2011], show the coordination of ITRF2008 starting point with the solid Earth center of mass at the level of 0.5 mm/year and then, in terms of a numerical indicator, previous arguments are being questioned.

This is contradictory, concerning the presentation in Table, to results obtained by the analytical centers of the International GNSS Service (IGS). For example, the combined solutions relative to the ITRF2005 version showed the geocenter displacement estimates at the level 5 mm in components x, y and 10мм in the z component. The inconsistency of orientation parameters with the IERS data are estimated at the level (-0.04–+0.05)× 0.001″ with an accuracy ±0.05×0.001″ [Ferland, Piraszewski, 2009].

In [Altamini et al., 2012] a kinematical model ITRF2008-PMM was presented, under which conditions of the geocenter position corresponded to the center of mass of the solid Earth, oceans. and atmosphere. This substantiated its obtainable accuracy at the level 0.3 mm/year and also presented comparisons with other used kinematical models. The model has been coordinated with the ITRF2008 version and is taken as a basis of the NNR-condition in the ITRF2014 solution and is associated with it as the newest plate motion model version [Altamini et al., 2016].

The aforementioned facts certify the absence of an unequivocal approach to the choice of the starting point of the geocentric system and determination its displacement.

Transformation parameters amongst the latest ITRF solutions
[Altamini et al., 2007; 2011; 2016]

Transformations	T_x, mm $\dot{T}_x$ mm/year	T_y, mm $\dot{T}_y$ mm/year	T_z, mm $\dot{T}_z$ mm/year	D 10^{-9} $\dot{D}$ 10^{-9}/year	R_x ·0.001″ $\dot{R}_x$ ·0.001″/year	R_y ·0.001″ $\dot{R}_y$ ·0.001″/year	R_z ·0.001″ $\dot{R}_z$ ·0.001″/year
ITRF2005–ITRF2000	0.1±0.3 -0.2±0.3	-0.8±0.3 0.1±0.3	-5.8±0.3 -1.8±0.3	0.40±0.05 0.08±0.05	0.00±0.01 0.00±0.01	0.00±0.01 0.00±0.01	0.00±0.01 0.00±0.01
ITRF2008–ITRF2005	-0.5±0.2 0.3±0.2	-0.9±0.2 0.0±0.2	-4.7±0.2 0.0±0.2	0.94±0.03 0.00±0.03	0.00±0.08 0.00±0.08	0.00±0.08 0.00±0.08	0.00±0.08 0.00±0.08
ITRF2014–ITRF2008	1.6±0.2 0.0±0.2	1.9±0.1 0.0±0.1	2.4±0.1 -0.1±0.1	-0.02±0.02 0.03±0.02	0.00±0.06 0.00±0.06	0.00±0.06 0.00±0.06	0.00±0.06 0.00±0.06

Used as input data, the results of GNSS-measurements significantly enhanced the potential of geodetic monitoring of the Earth's deformation fields. At the same time the ambiguity of datum establishment has created a problem that is associated with the effect of the loss of the invariance of deformation parameters. This led to a rethinking of the theoretical foundations of the deformation analysis and the development of new models and methods of data processing.

Studies of the influence of the reference system changed the interpretation of the deformation fields that were started in the time of introduction into research practices of modern satellite navigation technologies. It is probable that such consequences of such influence have previously accented attention, for example, [Dermanis, Grafarend, 1993]. The analysis of the effects of loss of the invariance of deformation parameters is presented in [Vanicek et al., 2008; Dermanis, 2009, 2010]. The problem of transforming the observed data into a single epoch remains relevant even until now. In order to balance the GNSS-data, transformation and theoretical foundations of continuum mechanics in

[Dermanis, 2010; Hossainali et al., 2011a, 2011b] are proposed methods which take into account these effects as at the ITRF solution epoch. Methods are based on a singular value decomposition of the deformation gradient and the classical theory of elasticity in its simplest linear form.

If you need to conduct the deformation analysis as at observations epoch that do not coincide with ITRF solution epoch, the influence of the effects of the invariance loss increase significantly .

This vision of solving problems of evaluation of three-dimensional deformation fields of the Earth in [Tadyeyev, 2015] is substantiated. The deformation analysis problems are considered in the context of "external modeling", as formulated in [Grafarend, Voosoghi, 2003]. Taking into account the defined tasks statement, there is reason to consider the problem including those that consider the probable effects of the loss of the deformation parameter's invariance.

The problem in the estimation of the three-dimensional deformation fields is considered from the standpoint of the differential presentation of transformations (mappings) of space images and the use of projective differential (metric) geometry methods [Kagan, 1947; Finikov, 1937]. According to the hypothesis that transformations have a geophysical origin, they are identified with deformations of the Earth's topographic surface as a domain of space. Then, using the three-dimensional metric tensor of space reveals perspectives to describe the deformation with different numerical characteristic content. Considering the established practice of the deformation analysis, these characteristics are divided into three groups: 1) main linear deformations – parameters of the form with change in the specified direction; 2) angular distortion parameters; 3) dilatation – parameters of relative changes in the volume of the Earth or the area of its surface while preserving the overall form.

Aim

The aim of this research is based on the theory of differential presentation of space image transformations. This part of the research will focus on the evaluation of angular distortions associated with the effects of rigid rotation of the Earth as a space transformation domain. Taking into account the concept of creating an ITRS, expressed as the input data, try to associate such distortions with probable deformations of the coordinate system and with the effects of the loss of the invariance in the interpretation of deformation fields of the Earth. Also it will try to present the solution results in a general view from the perspective of expression of the deformation using nonlinear analytical forms.

Methodology

A mapping (or transformation) of the space is a process where each point M of space corresponds to a certain point M'. The point M' is a mapping (or a projection) of M. The totality of points M_i ($i = \overrightarrow{1,n}$) of a certain part or even the whole space, is subject to unambiguous mapping (or transformation), forms in the transformation domain Δ. The totality of point M'_i that corresponds to point M_i forms the transformed domain Δ'. If in the three-dimensional Euclidean space is installed the system of Cartesian coordinates (x, y, z) and the domain Δ is closed and continuous, the points $M_i(x_i, y_i, z_i)$ completely defines (or delineates) the domain Δ. If, due to the unambiguous transformation of space the domain Δ mapped on Δ' and latter retained properties of the closed and continuous domain, the point $M'_i(x'_i, y'_i, z'_i)$ is completely defined by the domain Δ'.

Let it be known that the transformation domain Δ of the Earth as a planetary scale spatial body has coordinates $x_i = X_i^1$, $y_i = X_i^2$, $z_i = X_i^3$ and that point M_i meets the conditions of Earth parameterization by the ITRS. Points M_i are the GNSS-stations which are located on its physical surface. Designations (X^1, X^2, X^3) are identical to the (x, y, z). They are introduced solely for the purpose of a compact presentation of the following intermediate and as the final results of the problem's solution. If coordinates $x'_i = X'^1_i$, $y'_i = X'^2_i$, $z'_i = X'^3_i$ define the position of point M'_i, which is the mapping point M_i and defined as a transformed domain Δ', the mapping of Δ on Δ' can always can be expressed analytically by equations

$$\left.\begin{aligned} X'^1 &= u(X^1, X^2, X^3) \\ X'^2 &= v(X^1, X^2, X^3) \\ X'^3 &= w(X^1, X^2, X^3) \end{aligned}\right\}. \quad (1)$$

The general theory of mapping imposes on the base functions of transformation (1) homeomorphism conditions to include: uniqueness, continuity, and differentiability (Jacobian different from zero), but do not limit their analytical forms. This allows one to describe and transmit the transformation by any smooth or piecewise smooth functions which can be determined by the displacements $X_i'^k - X_i^k$ ($k = \overrightarrow{1,3}$) along a certain means of a parametrically given curve. In terms of the formulated objectives, this provides the prospect of the transfer within the functional model (1), of the nonlinear transformations.

A formulated problem statement in the part of the functional presentation of the transformation, at first examination, should entail a combination of mathematical tools which must be used to address it. The fact, that a solution with these kind of tasks is carried out in the environment of Riemannian space R_n and in this case in the R_3 dimension. Our task is formulated in the Euclidean space E_3 which is only a partial case of R_3, in addition to the conditions of E_3 parameterization by the rectangular Cartesian system of ITRS type. However, any complications or contradictions are eliminated if we use the properties of Riemannian space in the form of its complicated diffeomorphic manifolds. Diffeomorphic manifolds are called a couple of non-isotropic manifolds of the same dimension $n = 3$ that are subject to mutually unambiguous and continuously differentiated (homeomorphic) mapping. In this formulation, Riemannian geometry considers a diffeomorphic manifold E_3 as tangent to the Riemannian space.

Overall, a Riemannian space R_n is any manifold of the n dimension where in the infinitely small scale around the point M with coordinates $X^1, \ldots, X^n$ can be set as the field of a twice covariant, symmetric, with no degenerate tensor $g_{ij}(M) = g_{ij}(X^1, \ldots, X^n)$, i.e. such as $\det g_{ij} \neq 0$ and $g_{ij} = g_{ji}$. As for the other tensor $g_{ij}(M)$, it is set arbitrarily – on one tensor however the same manifold can be differently imposed as a Riemannian metric. Riemannian manifold geometry does not have the strictly formalized and hard character, as in the Euclidean space, and when compared to the previous, is amorphous and plastic. The manifold geometry depends only on its parameterization by its varying coordinate system, in the domain of change which is carried in manifold mapping in the form of continuously differentiated transformations of the $n-1$ class. Such transformations respectively define the class of smooth or piecewise smooth functions that are able to transmit these transformations, i.e. C^{n-1}. In addition, it is the only manifold to be considered having the accuracy of its replacement by a diffeomorphic manifold as long as this replacement is presented with the functional dependence between the point's coordinates in both manifolds. THIS PREVIOUS SENTENCE IS VAGUE AND UNCLEAR. The domain of the function's definition is every point on the manifold. At each point these functions define a tensor $g_{ij}(M)$ and their totality forms a tensor field. Therefore there is a need to set a tensor field instead of a tensor as a separate point, which is enough to determine the appropriate functions. The manifold parameterization by the coordinate system is carried out by a local frame (basis) in the tangent space. As local frames they are themselves coordinate systems that are considered in the space of their infinitely small scale around the points. If the local frame system along the parameterized curve is in a certain way, a parameterized curve occurs as the infinitesimal displacement ds of the point M, then the differential of arc ds is expressed by the differential quadratic form $ds^2 = g_{ij} dX^i dX^j$ (designation of the sum by Einstein rule); ds^2 – the linear element of Riemannian space. From this standpoint, a Riemannian space is the manifold in which is set by the invariant differential quadratic form ds^2. Thus, a tensor field of the Riemannian space is identified with the linear element ds^2, which determines its metrics. The tensor g_{ij} is a geometric image of the manifold [Rashevsky, 1967].

In view of our objective as a manifold it is the Euclidean space, which is tangent to an every given point of Riemannian space in the form of a local three-dimensional orthonormal coordinate basis of the ITRS type system. Then a tensor field with the Riemannian metric, which is defined by functional model (1), is transformed into geometric images of tensors of the tangent Euclidean space corresponding to it metric. Thus, for solving the problem we need only to comply with the conditions of sufficiently smooth changes of the Riemannian metric in the transition from point to point. These conditions must ensure the adequately constructed model (1) by the continuous differentiability of its base functions. Since the tangential coordinate bases both before and after the space transformation meets the conditions E_3, the formulated problem statement remains unchanged. Riemannian geometry methods are needed only at the stage of construction of the functional model (1). It must be noted that such model is able to solve the problem of the deformation analysis in geodynamics at the hypothesis that changing the metric of space has a geophysical origin and is caused by its deformation.

Let the coordinate system in which it is specified that the position of point M_i of the domain Δ (the initial state of the Earth at the time moment t_0) is a rectangular Cartesian and is identified with the ITRS. The linear element of the domain Δ is

$$ds^2 = \delta_{ij} dX^i dX^j . \quad (2)$$

Since the coordinate axes are orthogonal, the metric coefficients δ_{ij} are such that

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} .$$

They form the identity matrix. Linear element ds identifies a metric form of the nodeformed domain Δ in the initial state.

Metric form of the deformed domain Δ' (the final state at the time moment $t_1 = t_0 + dt_1$) is associated with a linear element ds' that is a mapping of the ds:

$$ds'^2 = e_{ij} dX^i dX^j . \quad (3)$$

Metric coefficients e_{ij} of the quadratic form (3) generate a symmetrical matrix which is called the main bivalent metric tensor of the space transformation (deformation):

$$e_{ij} = \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{12} & e_{22} & e_{23} \\ e_{13} & e_{23} & e_{33} \end{pmatrix} . \quad (4)$$

Coefficients e_{ij} fully define the functional model (1). The algorithm of disclosure for the coefficients is the following:

$$e_{11} = \left(\frac{\partial u}{\partial X^1}\right)^2 + \left(\frac{\partial v}{\partial X^1}\right)^2 + \left(\frac{\partial w}{\partial X^1}\right)^2 ;$$

$$e_{22} = \left(\frac{\partial u}{\partial X^2}\right)^2 + \left(\frac{\partial v}{\partial X^2}\right)^2 + \left(\frac{\partial w}{\partial X^2}\right)^2 ;$$

$$e_{33} = \left(\frac{\partial u}{\partial X^3}\right)^2 + \left(\frac{\partial v}{\partial X^3}\right)^2 + \left(\frac{\partial w}{\partial X^3}\right)^2 ;$$

$$e_{12} = \frac{\partial u}{\partial X^1}\frac{\partial u}{\partial X^2} + \frac{\partial v}{\partial X^1}\frac{\partial v}{\partial X^2} + \frac{\partial w}{\partial X^1}\frac{\partial w}{\partial X^2} ;$$

$$e_{23} = \frac{\partial u}{\partial X^2}\frac{\partial u}{\partial X^3} + \frac{\partial v}{\partial X^2}\frac{\partial v}{\partial X^3} + \frac{\partial w}{\partial X^2}\frac{\partial w}{\partial X^3} ;$$

$$e_{13} = \frac{\partial u}{\partial X^1}\frac{\partial u}{\partial X^3} + \frac{\partial v}{\partial X^1}\frac{\partial v}{\partial X^3} + \frac{\partial w}{\partial X^1}\frac{\partial w}{\partial X^3} . \quad (5)$$

According to the general theory of tensor analysis, a tensor e_{ij} is the main carrier of information about the state and nature of the final deformation of the domain Δ and, equally, about the deformation of the coordinate system X^i $(i = \overrightarrow{1,3})$, in particular, regarding change in the orientation of the coordinate axes (rotation) and (or) infringement of their orthogonality. In case of equality $ds = ds'$ has no deformation, it can only be the parallel displacement of the transformation domain and coordinate system as a whole. If $ds \neq ds'$, then it is a sign that with such displacement the deformations are taking place simultaneously with numerical expressions and character that depend on the absolute values of e_{ij} coefficients in general and ratios between diagonal and non-diagonal e_{ij} coefficients. As a measure of the deformation a difference of quadratic forms (2) and (3) or their ratio, [Sokol'nikov, 1971] is most often used.

Results

Using the geocentric system ITRS in the form of her ITRF versions forces one to consider the following statement in solving geodynamics problems in general and for deformation analysis.

The initial state of the Earth as a space domain Δ that is parameterized by the rectangular Cartesian coordinate system at the time moment t_0 ascribe to the datum, which corresponds to the last ITRF version. If the domain Δ at the time moment $t_1 = t_0 + dt_1$ is transformed into the domain Δ', in the final state an appropriate transformation has also undergone the coordinate system. The system X'^i $(i = \overrightarrow{1,3})$ is deformed relative to her state in the datum. Thus, in addition to the parallel displacement of the starting point (due to the translational motion of the Earth), there can occur an infringement of coordinate axe's orthogonality. Then the X'^i system becomes an oblique Cartesian coordinate system.

The hypothesis of ITRS deformation may be acceptable along with the following logical opinions. They are based on the geophysical essence of the NNR condition, which is used when creating the system. This condition puts the system dependent on the global tectonic activity of the Earth. One of the effects of such activity is recent movements and deformations of lithospheric plates. A substantial expression and anomalous character of such phenomena can violate the NNR condition as the conception of coincidence of a zero aggregate angular momentum of all plates and angular momentum of the Earth as a whole. In such a case it will be inevitable that there will occur a deformation of the ITRS, which is used for the Earth parameterization. The hypothesis may be confirmed or, equally, denied by the empirical method if in its basis put the following indisputable geometric argumentations.

In general, for any two points $P_1(X^i)$ and $P_2(X^i + dX^i)$ of the three-dimensional space $(i = \overrightarrow{1,3})$ with its quadratic form $ds^2 = g_{ij} dX^i dX^j$ it follows that the lengths of the elements of the arc ds in projections on coordinate axes of the system which is used for the parameterization of space, are expressed by products $ds^{(i)} = \sqrt{g_{ii}} dX^i$. Superscript in projection $ds^{(i)}$ symbolizes her belonging to the respective axis X^i. Cosines of angles η_{ij} between couples of projections $ds^{(i)}$ and $ds^{(j)}$ as they are shown in Fig. 1, express the formula [Sokol'nikov, 1971]

$$\cos \eta_{ij} = \frac{g_{ij}}{\sqrt{g_{ii} g_{jj}}}. \qquad (6)$$

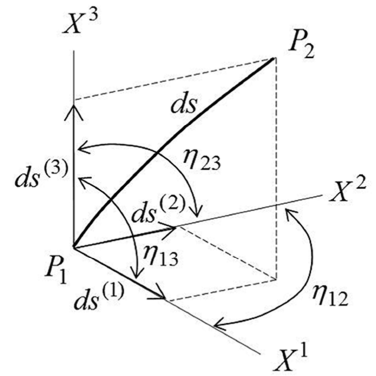

Fig. 1. Angles between projections of the arc ds on coordinate axes

If in the initial state at the time moment t_0 the coordinate system is rectangular, the metric coefficients $g_{ij} = \delta_{ij}$ and $\eta_{ij}^{(0)} = 90^\circ$ will follow the formula (6). Generally the formula (6) expresses angles between coordinate axes of the arbitrary are not-orthonormalized system in three-dimensional space with metrics that correspond to the tensor g_{ij}. If at the time moment t_1 a space has undergone a transformation that caused the change of its metric properties, then $g_{ij} = e_{ij}$. With metric coefficients e_{ii} associated the absolute linear elongations of space, which is directed along the coordinate axes. The coefficients e_{ij} provide $i \neq j$ are called shear (sliding). Assuming that $e_{ij} \neq 0$, we obtain $\eta_{ij}^{(1)} \neq 90^\circ$. Then $\eta_{ij}^{(1)}$ are absolute measures of the oblique coordinate system. Superscripts in angles $\eta_{ij}^{(0)}$ or $\eta_{ij}^{(1)}$ symbolize their belonging to time moments t_0 or t_1. Submitted facts must be recognized as sufficient argumentation of possible loss for the coordinate system's orthogonality according to the above formulated problem statement.

To express the deformation of the domain Δ for the period dt_1 with respect to its initial state takes as a basis the quadratic forms (2) and (3). If the measure of deformation has a difference $ds'^2 - ds^2$ then the change of arc length per unit length will be expressed in the formula

$$\frac{ds'^2 - ds^2}{ds^2} = \left(e_{ij} - \delta_{ij}\right)\frac{dX^i}{ds}\frac{dX^j}{ds} = \mu^2 .$$

Coefficients $\mu_i = \sqrt{e_{ii} - 1}$ are the relative linear elongations in projections $ds^{(i)}$. Then the formula (6) takes a form

$$\cos\eta_{ij}^{(1)} = \frac{e_{ij}}{\mu_i \mu_j} . \quad (7)$$

Let us assume that $\eta_{ij}^{(1)} = 90^\circ - \varepsilon_{ij}^{(1)}$, where $\varepsilon_{ij}^{(1)}$ is a change in the initial right angle between a couple of arc elements, which are directed along coordinate axes. Then from formula (7) follows:

$$\sin\varepsilon_{ij}^{(1)} = \frac{e_{ij}}{\mu_i \mu_j} . \quad (8)$$

If angles $\varepsilon_{ij}^{(1)}$ are small, then $\varepsilon_{ij}^{(1)} \approx e_{ij}$, that is considered practically permissible from the standpoint of the classical linear theory of deformation.

The submitted solution is based on the theory of tensor analysis in the presentation [Sokol'nikov, 1971]. In the context of solving the problem angles $\varepsilon_{ij}^{(1)}$ it can be interpreted in two ways. On the one hand, these are indicators of angular distortions of the coordinate system relative to its state in ITRF version. On the other, considering that the ITRS system is "tied" to the Earth, angles $\varepsilon_{ij}^{(1)}$ express her rigid rotation as an absolutely rigid body in directions between couples of coordinate axes $X^i X^j$ for the period dt_1 relative to the same ITRF version.

Consider the problem from another perspective – in projections of the arc ds on coordinate planes. Let us take as a basis professor G.A. Meshcheryakov's solutions for the definition of optimal projections while mapping various surfaces on a plane in mathematical cartography [Meshcheryakov, 1968]. On this basis the angular distortions of two-dimensional Cartesian coordinate system are expressed, as presented in [Tadyeyev, 2013].

Let the differential $dX^3 = 0$ and projection of the arc ds lay on the equatorial plane X^1OX^2. Here quadratic forms (2) and (3) have the appearance $ds_{12}^2 = \delta_{ij} dX^i dX^j$ and $ds'^2_{12} = e_{ij} dX^i dX^j$, where $i, j = 1,2$. Taking into account that it is now considered as only full differentials of functions (1) such as

$$dX'^1 = \frac{\partial u}{\partial X^1} dX^1 + \frac{\partial u}{\partial X^2} dX^2 ;$$

$$dX'^2 = \frac{\partial v}{\partial X^1} dX^1 + \frac{\partial v}{\partial X^2} dX^2 ,$$

algorithm (5) simplifies and metric coefficients e_{ij} will be disclosed as follows:

$$e_{11} = \left(\frac{\partial u}{\partial X^1}\right)^2 + \left(\frac{\partial v}{\partial X^1}\right)^2 ;$$

$$e_{22} = \left(\frac{\partial u}{\partial X^2}\right)^2 + \left(\frac{\partial v}{\partial X^2}\right)^2 ;$$

$$e_{12} = \frac{\partial u}{\partial X^1}\frac{\partial u}{\partial X^2} + \frac{\partial v}{\partial X^1}\frac{\partial v}{\partial X^2} . \quad (9)$$

Assume that due to deformation the coordinate axis X^1 is reflected on the plane X^1OX^2 by projection X'^1. The direction of the projection X'^1 relative to X^1 defines the angle $\psi_{12}^{(1)}$. Similarly, the direction of the projection X'^2 of axis X^2 defines the angle $\chi_{12}^{(1)}$. These directions are shown in a scheme (Fig. 2, a). Explicit expression of directions $\psi_{12}^{(1)}$ and $\chi_{12}^{(1)}$ are the result of ratio of the differentials of the coordinate axes projections on a mapping plane in the final state, which are presented in the coordinate system of the initial plane as:

$$\left.\frac{dX'^2}{dX'^1}\right|_{\substack{dX^3=0\\X^2=const}}=\left(\frac{\partial v}{\partial X^1}\right)\Big/\left(\frac{\partial u}{\partial X^1}\right)=tg\psi_{12}^{(1)}; \quad (10)$$

$$\left.\frac{dX'^2}{dX'^1}\right|_{\substack{dX^3=0\\X^1=const}}=\left(\frac{\partial v}{\partial X^2}\right)\Big/\left(\frac{\partial u}{\partial X^2}\right)=tg\chi_{12}^{(1)}. \quad (11)$$

Considering formulas (10) and (11), from the difference $\chi_{12}^{(1)}-\psi_{12}^{(1)}=\eta_{12}^{(1)}$ we obtain:

$$tg\eta_{12}^{(1)}=\frac{\frac{\partial u}{\partial X^1}\frac{\partial v}{\partial X^2}-\frac{\partial v}{\partial X^1}\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}\frac{\partial u}{\partial X^2}+\frac{\partial v}{\partial X^1}\frac{\partial v}{\partial X^2}}.$$

The expression in the numerator is associated with an absolute indicator of changes in the area of the domain Δ in its projection on a plane X^1OX^2. This is a determinant of the tensor formed by the coefficients (9): $e_{11}e_{22}-e_{12}^2=\det e_{ij}$. Therefore

$$tg\eta_{12}^{(1)}=\frac{\sqrt{\det e_{ij}}}{e_{12}}. \quad (12)$$

And now for the indicator of angular distortions $\varepsilon_{12}^{(1)}=90^\circ-\eta_{12}^{(1)}$

$$ctg\varepsilon_{12}^{(1)}=\frac{\sqrt{\det e_{ij}}}{e_{12}}. \quad (13)$$

Expressions of the angle $\eta_{12}^{(1)}$ by formulas (7) and (12) and equally the angle $\varepsilon_{12}^{(1)}$ by formulas (8) and (13) are respectively identical. This you can easily verify by simple transformations of formulas (12) or (13) taking into account the measure of deformation $ds_{12}'^2-ds_{12}^2$.

On the same basis implemented solutions and obtained explicit expressions of directions $\psi^{(1)}$, $\chi^{(1)}$ and indicators of angular distortion $\eta^{(1)}$, are $\varepsilon^{(1)}$ in projections in other coordinate planes of the system X^i ($i=\overrightarrow{1,3}$). In particular, in projection on a plane X^1OX^3

$$\left.\frac{dX'^3}{dX'^1}\right|_{\substack{dX^2=0\\X^3=const}}=\left(\frac{\partial w}{\partial X^1}\right)\Big/\left(\frac{\partial u}{\partial X^1}\right)=tg\psi_{13}^{(1)}; \quad (14)$$

$$\left.\frac{dX'^3}{dX'^1}\right|_{\substack{dX^2=0\\X^1=const}}=\left(\frac{\partial w}{\partial X^3}\right)\Big/\left(\frac{\partial u}{\partial X^3}\right)=tg\chi_{13}^{(1)}. \quad (15)$$

Directions that are expressed by formulas (14) and (15), shown in a scheme (Fig. 2, b). A scheme (Fig. 2, c) shows the axe's directions of the deformed system in projection on a plane X^2OX^3. They are expressed by formulas

$$\left.\frac{dX'^3}{dX'^2}\right|_{\substack{dX^1=0\\X^3=const}}=\left(\frac{\partial w}{\partial X^2}\right)\Big/\left(\frac{\partial v}{\partial X^2}\right)=tg\psi_{23}^{(1)}; \quad (16)$$

$$\left.\frac{dX'^3}{dX'^2}\right|_{\substack{dX^1=0\\X^2=const}}=\left(\frac{\partial w}{\partial X^3}\right)\Big/\left(\frac{\partial v}{\partial X^3}\right)=tg\chi_{23}^{(1)}. \quad (17)$$

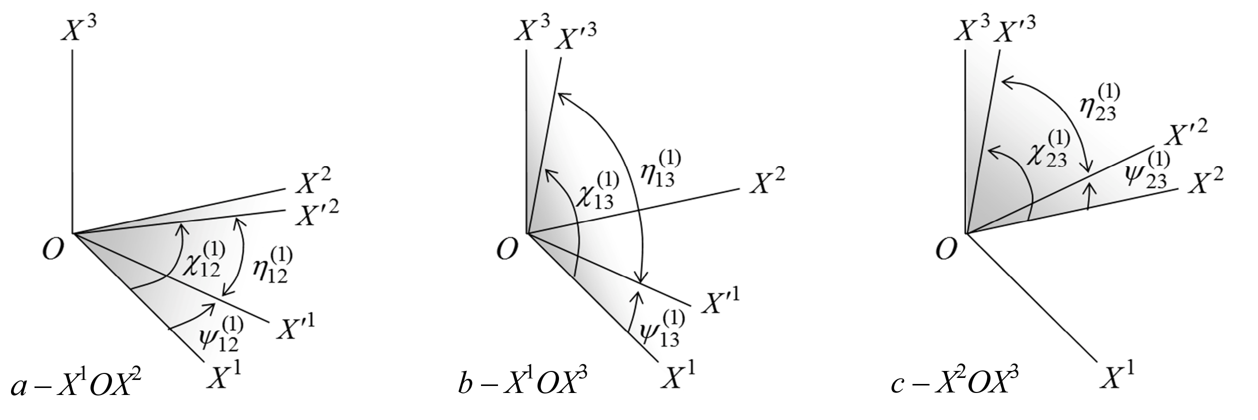

Fig. 2. Axes directions of the deformed system in projections on coordinate planes

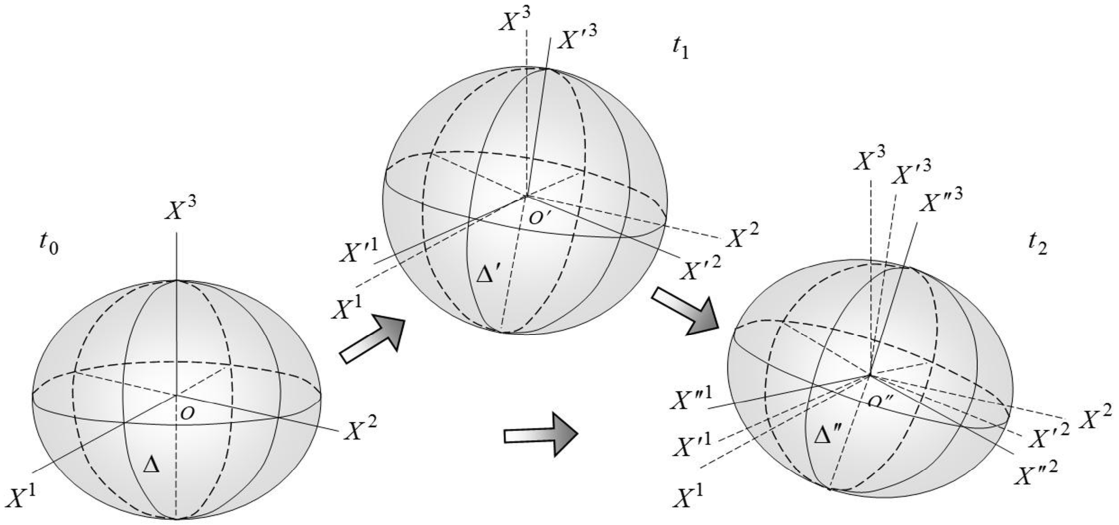

Fig. 3. The transformations scheme of the Δ *domain*

If angles $\eta_{ij}^{(1)}$ and $\varepsilon_{ij}^{(1)}$ are only measures of the loss of the coordinate system orthogonality, the values $\psi_{ij}^{(1)}$ and $\chi_{ij}^{(1)}$ are able to transmit the directions of deformed system axes relative to its initial state. From this point of view the results of task solutions in projections on coordinate planes have obvious advantages. Analysis of the geometric nature of values $\psi_{ij}^{(1)}$ and $\chi_{ij}^{(1)}$ show their direct analogy with geocentric polar coordinates which are traditionally used in geodesy. In particular, the angles $\psi_{12}^{(1)}$ and $\chi_{12}^{(1)}$ have a geocentric longitude λ in directions on the equatorial plane relative to the zero meridians plane. The angles $\psi_{13}^{(1)}$, $\chi_{13}^{(1)}$ and $\psi_{23}^{(1)}$, $\chi_{23}^{(1)}$ with a geocentric latitude φ with directions relative to the equatorial plane onto longitudes $\lambda = 0^\circ$ and $\lambda = 90^\circ$ respectively. If necessary, without any complications from formula calculations, the latitude φ can be replaced by its complement to 90° which referred to as polar distance. Then we can evaluate the position of axes X'^1, X'^2, X'^3 about the axis of rotation in its initial state $X^3 = z$.

The only real obstacle to the practical implementation of the described methods can be an insufficient density coverage of the Earth by GNSS-stations, particularly within the oceans. It is this circumstance that determines the adequacy of constructing a functional model (1) with planetary scale and reliability for the above estimates, that are its consequence. Theoretically, this problem could be solved by a global grid construction, for example in the form of spherical or ellipsoidal quadrangles or triangles, if one involves the Delaunay triangulation method, followed by extrapolation of displacements in grid nodes relative to stations in mainland and insular parts of the Earth. Such a mathematical tool was used in the research practice (see, e.g., [Marchenko et al., 2012]). But the accuracy of extrapolation to great distances will be low and for this reason will probably lose its sense in the idea of expressing the nonlinear deformation on a planetary scale. In this regard, to find the optimal solution, the problem of constructing an adequate functional model requires a detailed study and to date remains open for a comprehensive discussion.

Finally, consider the overall logic in making the decisions regarding task statements in the deformation analysis according to the transformations scheme in Fig. 3.

Let us assume that the hypothesis of the coordinate system deformation at the time moment t_1 is confirmed as such, that does not contradict the empirical data. The empirical values of indicators of angular distortions can be compared to their analogues that are presented in the Table #?.

Consider the studies necessary to evaluate the deformed state of the Earth at the time moment

$t_2 = t_1 + dt_2 = t_0 + dt_1 + dt_2$. If you want to solve the problem relative to datum at the moment t_0 during the period $dt_1 + dt_2$, without taking into account the state of the Earth at the moment t_1, the method does not require any changes. To solve the problem during the period dt_2 relative to t_1 a solution is presented in the above form, but is disclosed in [Tadyeyev, 2015], as unacceptable. Such a formulation of the problem leads to the need of constructing a functional model

$$\left.\begin{aligned} X''^1 &= u'(X'^1, X'^2, X'^3) \\ X''^2 &= v'(X'^1, X'^2, X'^3) \\ X''^3 &= w'(X'^1, X'^2, X'^3) \end{aligned}\right\}, \qquad (18)$$

where $X''^1 = x'', X''^2 = y'', X''^3 = z''$ are diffeomorphic coordinates of the domain Δ'' at the moment t_2. The domain Δ'' is a mapping of Δ' and as the domain of definition of the base functions of the model (18). The metrics of Δ'' define a tensor e'_{ij} and the corresponding quadratic form in coordinates of the domain Δ' take the appearance $ds''^2 = e'_{ij} dX'^i dX'^j$. A tensor e'_{ij} as a geometric image of Δ'' according to transformations (18), also expresses the change of metric properties of the domain Δ'. Since such a change is taken into account relative to the state at the moment t_1, the next deformation analysis must be based on the difference $e'_{ij} - e_{ij}$ and on the measure

$$ds''^2 - ds'^2 = (e'_{ij} - e_{ij}) dX'^i dX'^j .$$

Such a modification of methods is able to provide the deformation fields estimates that are not burdened with the likely influence of the effects of coordinate system distortion and the loss of invariance in relation to the datum.

Scientific novelty and practical significance

The task solutions have been achieved by projective differential geometry methods based on the differential representation of the transformations of Riemannian space images in the form of its complicated diffeomorphic manifolds, in particular, the tangent Euclidean space. Such an approach to solving problems of the deformation analysis in geodynamics is generalized relative to the traditional methods used. On this basis, prospects for filing of transformations by nonlinear functional models are substantiated. The identity of the Earth's parameterization by ITRS and of the tangent Euclidean space parameterization by the Cartesian coordinate system has been proven. Based on the concept of ITRS, her possible deformations are substantiated. The analytical expressions of ITRS angular distortions are obtained. They are considered in the context of the rigid rotation of the Earth, as a component of its global spatial deformation, in projections on coordinate planes. Methods of the deformation analysis is adapted to use the results of GNSS monitoring of coordinates, taking into account the likely deformation of the ITRS. As efficient tools of definition, the current position of the axes of the deformed coordinate system, the obtained results also have practical significance in solving problems of modern geodesy in its relationship to geodynamics in the context of reference frame research.

Conclusions

At this stage of research the formulated perspectives of using the results of GNSS-station's coordinate monitoring in the ITRS for modeling the global Earth's deformation field in the part of the expression of the angular distortion parameters. The task solutions achieved by projective differential geometry methods based on the differential representation of the transformations of Riemannian space images in the form of its diffeomorphic manifolds. It was also applied to the methods of description to the change of the Riemannian metric in the tangent Euclidean space, which is parameterized by the Cartesian coordinate system. Such a theoretical basis enabled us to get the result, which in terms of its content and the practical application has a dual interpretation.

1. In the part an expression of angular distortion parameters are derived the formulas for the angles of the rigid Earth's rotation as a component of its global spatial deformation into projections on ITRS coordinate planes. This result has a direct application in the analysis of global deformation fields for the needs of geodynamics.

2. Based on the geophysical content in the concept of creating the ITRS system, her probable deformations are substantiated. The truthfulness of this hypothesis can be verified by the same angles as are defined in the previous rubric. If to consider them as indicators of angular distortions of the ITRS expressed by deviations from axes orthogonality in ITRF solution epoch (as a datum), the latter should recognize the measures of the oblique system in any epoch of observations that follows . The analytical expressions for coordinate axe's directions of the deformed system are obtained. In terms of this rubric the obtained results could apply in solving problems of modern geodesy in its relationship with geodynamics in the context of reference frame research.

Recommendations for the formulation and solution of deformation analysis problems, in the case of empirical confirmation of the hypothesis of angular distortions of the coordinate system, are presented.

Analytical expressions of angular distortions are given in the general form, which is able to transmit the deformation of a nonlinear character as far it can be expressed by base functions of the model (1). The efficiency of nonlinear models of the Earth's physical fields today is obvious even if we take into account the consequences of their use at the creation of the newest version of the ITRS: the generalization of ITRF2014 solution with an extended modeling of nonlinear movements of stations (seasonal signals and post-seismic deformations) provided a significant increase in its accuracy compared to ITRF2008 [Altamini et al., 2016].

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О. А. ТАДЄЄВ

Кафедра геодезії та картографії, Національний університет водного господарства та природокористування, вул. Соборна, 11, Рівне 33028, Україна, тел. +38(096)7488449, ел. пошта: oleksandrtad@gmail.com

ОЦІНЮВАННЯ ТРИВИМІРНИХ ДЕФОРМАЦІЙНИХ ПОЛІВ ЗЕМЛІ МЕТОДАМИ ПРОЕКТИВНО-ДИФЕРЕНЦІАЛЬНОЇ ГЕОМЕТРІЇ. ЖОРСТКІ ОБЕРТАННЯ ЗЕМЛІ

Мета. Оцінювання жорсткого обертання Землі як складової глобальних деформаційних полів у взаємозв'язку з кутовими спотвореннями геоцентричної просторової системи координат. **Методика.** Розв'язки досягнуто методами проективно-диференціальної (метричної) геометрії на основі диференціального подання перетворень образів ріманового простору у формі його складних диффеоморфних многовидів. Враховуючи сутність системи ITRS, у якій задано вхідні дані, та глобальний масштаб деформаційних полів, рімановим многовидом визначено дотичний евклідовий простір. Щоб виконати завдання, використано прийоми описування змін ріманової метрики у дотичному евклідовому просторі, який параметризований декартовою системою координат. **Результати.** Використовувана основа дала змогу одержати результати, які з погляду їх змісту і практичного застосування мають двояку інтерпретацію. У частині вираження групи параметрів кутових спотворень для потреб деформаційного аналізу встановлено співвідношення для кутів жорсткого обертання Землі в проекціях на координатні площини системи ITRS. Водночас доведено, що ці кути є показниками спотворень системи координат. Гіпотеза ймовірних деформацій геоцентричної просторової системи обґрунтована геофізичним змістом концепції створення ITRS. Аргументовано тотожність умов параметризації Землі системою ITRS і параметризації дотичного евклідового простору декартовою системою координат. На цій основі істинність гіпотези можна перевірити за емпіричними значеннями кутів, які визначені з результатів GNSS-спостережень. За умови достатньої значущості, вони є показниками кутових спотворень системи ITRS чи, у вираженні відхиленнями від ортогональності осей у ITRF-реалізації, мірами косокутної декартової системи на будь-яку епоху спостережень після реалізації. Методами проективно-диференціальної геометрії одержано аналітичні вираження напрямів координатних осей деформованої системи. **Наукова новизна.** Доведено, що підхід до розв'язання задач деформаційного аналізу в геодинаміці на засадах ріманової геометрії є узагальнювальним відносно використовуваного. На такій основі обґрунтовано перспективи подання деформаційних полів нелінійними функціональними моделями. **Практична значущість.** Одержані результати спрямовані на їх використання під час оцінювання глобальних деформаційних полів Землі та вирішення питань сучасної геодезії в її взаємозв'язку з геодинамікою на основі досліджень референцних систем координат. Усі аналітичні вираження показників кутових спотворень подано у загальному вигляді, який здатний передати нелінійні закономірності деформації. Методика деформаційного аналізу адаптована до використання вхідними даними результатів GNSS-моніторингу координат станцій з урахуванням деформації системи ITRS.

Ключові слова: ріманові диффеоморфні многовиди; метричний тензор простору; деформаційний аналіз; жорсткі обертання Землі; деформація системи координат.

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