ГЕОДЕЗІЯ

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REGIONAL QUASIGEOID DETERMINATION: AN APPLICATION TO ARCTIC GRAVITY PROJECT

Purpose. Investigation to study quasigeoid computations based on the regional gravimetric data and different types of nonorthogonal basis functions was assessed to be important. When measurements from only restricted regions of the Earth surface are available, global spherical harmonics loose their orthogonality in a limited region, so the determination of the coefficients of the model, usually by using the least squares method, is numerically unstable. In spite of this fact, there is a specific solution for Laplace equation for the situation of a spherical cap when the boundary conditions are appropriate. **Methods.** Our solution uses the gravity anomalies in the Arctic area taken from the Arctic Gravity Project (AGP). The method applied on this data set is adjusted spherical harmonic analysis (ASHA). Computation of the quasigeoid heights was performed by the "Remove - Restore" procedure in three steps. On the first step the free air gravity anomalies of the EGM 2008 model up to degree/order 360 were substracted from the initial gravity anomalies of the AGP to get rid of the low frequency gravity field content. On the second step the approximation of the residual gravity anomalies was based on the ASHA method. The construction of the normal equations matrix may lead to the time consuming procedure. For this reason the discrete orthogonality property in longitude for the chosen basis system was taken into account and led to the significant decrease of the computational time of the residual coefficients $\overline{a}_{km},\overline{b}_{km}$. On the last step the residual quasigeoid heights (high frequency components of the gravity field) were computed via the residual harmonic coefficients $\overline{a}_{km},\overline{b}_{km}$ and added to the global contribution of quasigeoid heights taken from the EGM2008 model up to degree/order 360 (low frequency components of the gravity field). **Results.** Hence the gravity field model was constructed and compared with AGP gravity anomalies. Also the obtained model of quasigeoid heights was compared with quasigeoid heights from 49 GNSS/leveling points. **Scientific novelty and practical significance.** In this paper the modification of ASHA method was developed, which makes it possible to significantly accelerate the process of computing the unknown coefficients in the construction of local gravitational fields. This allows to compute local gravitational fields of higher orders. It is well known that quasigeoid accuracy depends on the order of model.

Keywords: gravity anomalies, quasigeoid heights, adjusted spherical harmonic analysis, spherical cap harmonic analysis.

Introduction

The construction of high-precision quasigeoid heights usually can be carried out using the model or operational approaches of physical geodesy. The operational approach corresponds to the method of least-squares collocation and requires a prior study of additional information about the Earth gravitational field (Moritz, 1980; Sideris, 2005). Such approach leads to the optimal linear estimates and allows to get a stable solution. The disadvantage of this method is the large order of the inverted matrix, which is equal to the number of initial data (observations). In the model approach an order of inverted matrix is much smaller and equal to the number of parameters. In this case different sets of basis functions are usually used, which are preferred in local gravity field modeling due to the large number of data to be processed. For example, the sequential multipole analysis was developed for the approximation of disturbing potential and implemented using radial potential multipoles (Marchenko, 1998; Marchenko et al., 2001), which are connected with the radial basis functions. Another type of model approach represents the so-called spherical cap harmonic analysis (SCHA) that involves the associated Legendre functions of the integer degree and noninteger order (Haines, 1985). These functions form two orthogonal subsets. In every set corresponding functions are mutually orthogonal over the spherical cap. However in general these functions are not orthogonal and it is quite difficult to compute eigenvalues and norms for their high orders. For that reason it is possible to use a special model approach of the adjusted spherical harmonic analysis (ASHA) (de Santis, 1992) for the approximation of the local gravity field (Jiancheng et al., 1995; De Santis & Torta, 1997). The ASHA technique provides the projection of initial data from a segment of sphere to hemisphere and leads to the spherical functions of integer degree and integer order. This paper focuses on this alternative ASHA method to the gravity field approximation (as the addition to the SCHA approach) within the procedure of "Remove-Restore".

The traditional gravimetric quasigeoid determination is based on the gravity anomalies Δg, which are given with respect to the quasigeoid (wich is unknown at this stage and has to be determined using just this gravity measurements). Nevertheless, the consideration of Δg in Molodensky sense will lead to the quasigeoid heights in the Arctic area within latitudes [65°,90°].

The National Imagery and Mapping Agency (NGA) collects the Δg sets in the frame of the Arctic gravity project (AGP) (NGA, 2008) in order to build a high-precision quasigeoid heights in the Arctic area including the construction of gravity anomalies grid with resolution of (5' × 5') using data of airborne gravimetry, satellite altimetry and gravimetric data from nuclear submarines (SCICEX). Fig/ 1 illustrates the gravity anomalies from the AGP within latitudes [65°, 90°].

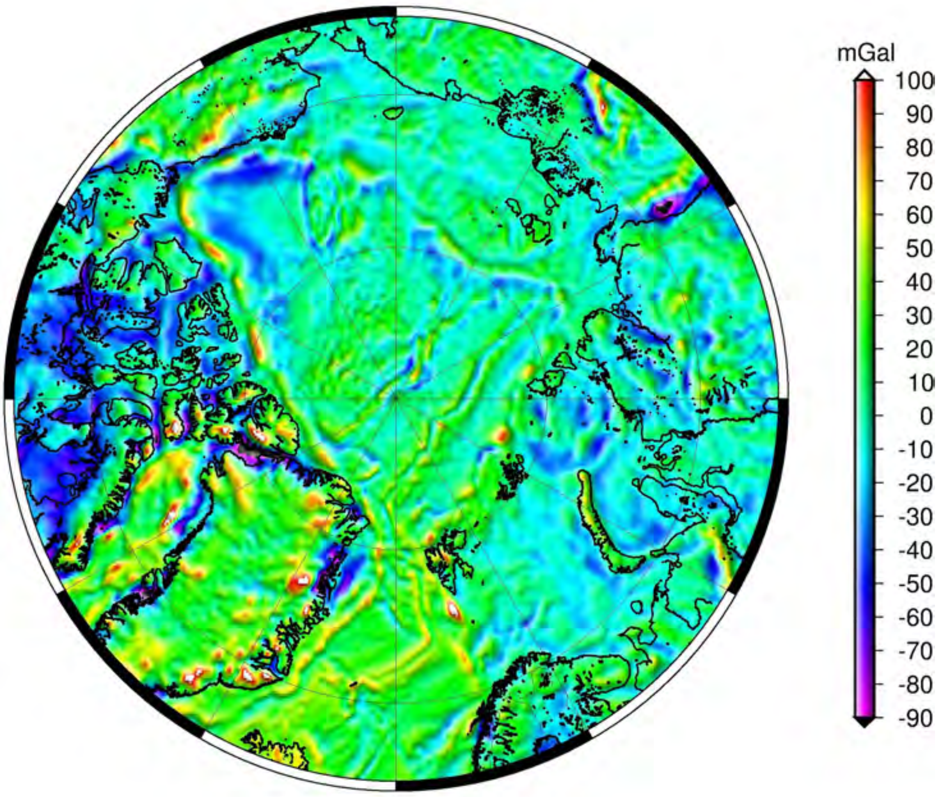

Fig. 1. Gravity anomalies from AGP Δg_{AGP} [mGal]

Methods
Spherical cap harmonic analysis

As well known, the approximation of the geopotential function V and its functionals in some part of sphere has a better results in the case of the suitable base functions system. According to Sturm-Liouville theorem (Churchill, 1963; de Santis & Falcone, 1995) values m and n are nonnegative integers for the whole sphere. However, if some function is defined on a segment of a sphere (fig. 2) the boundary conditions depending on latitude are (Haines, 1985):

$$\frac{dP_{n_k(m)m}(\cos\theta_0)}{d\theta} = 0 \; for \; k - m = even \qquad (1)$$

$$P_{n_k(m)m}(\cos\theta_0) = 0 \; for \; k - m = odd \qquad (2)$$

where θ_0 is the half-angle of segment (polar distance), k is the index, which regulates real (noninteger) n for some m ($n_k \geq k$).

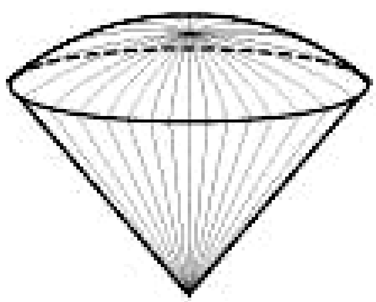

Fig. 2. Segment of sphere

Indeed the system of functions (1) is orthogonal with weight $\sqrt{\sin\theta}$ in the interval [0; θ_0]. The system (2) is orthogonal with the same weight as the system (1) in the interval [0; θ_0], but, in general, functions (1) and (2) are not mutually orthogonal.

For computation of the functions (1) and (2) it is appropriate to express them via a hypergeometric series (Hobson, 1931)

$$P_{n_k m}(\cos\theta) = \sin^m\theta \cdot F(m - n_k, m + n_k + 1, m + 1, \frac{1-\cos\theta}{2}) \quad (3)$$

For computation the n_k it is possible to expand the equations (1) and (2) as a following hypergeometric series (Hwang & Chen, 1997):

$$\bar{F}(n_k, m, \frac{1-\cos\theta_0}{2}) = 0, \quad (4)$$

$$n_k \frac{1-\cos\theta_0}{2} \bar{F}(n_k, m, \frac{1-\cos\theta_0}{2}) - (n_k - m)\bar{F}(n_k + 1, m, \frac{1-\cos\theta_0}{2}) = 0, \quad (5)$$

where

$$\bar{F}(n_k, m, \frac{1-\cos\theta}{2}) = F(m - n_k, m + n_k + 1, m + 1, \frac{1-\cos\theta}{2}). \quad (6)$$

Table 1

Values n_k for segment of sphere $\theta_0 = 25^\circ$

k/m	0	1	2	3	4	5	6	7
0	0.000							
1	5.004	3.806						
2	8.296	8.296	6.632					
3	12.148	11.743	11.324	9.318				
4	15.587	15.586	14.923	14.223	11.938			
5	19.331	19.079	18.824	17.961	17.044	14.518		
6	22.821	22.821	22.384	21.937	20.909	19.811	17.072	
7	26.523	26.339	26.154	25.567	24.963	23.792	22.538	19.606

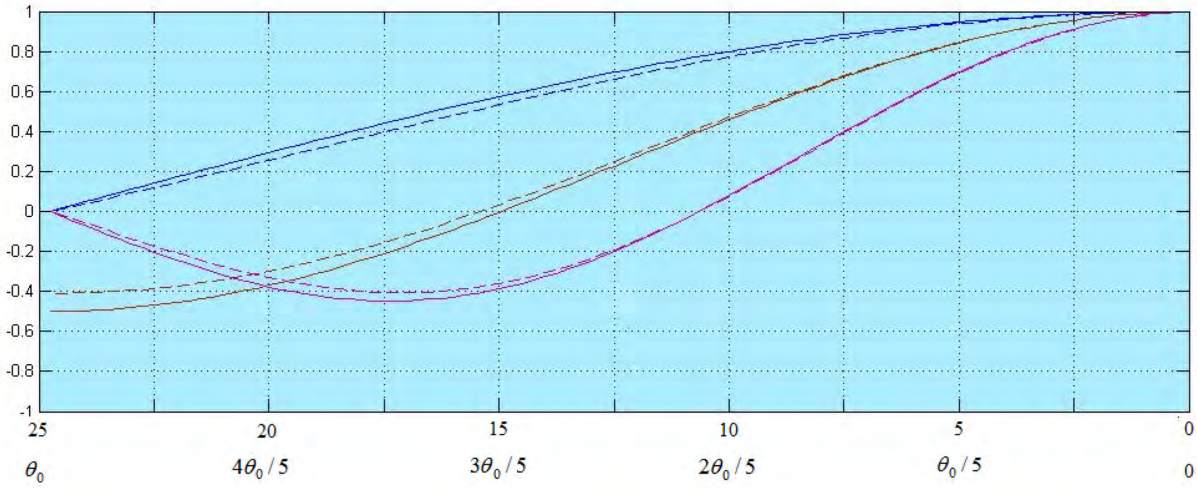

Fig. 3. Functions $P_{10}(\cos s\theta), P_{20}(\cos s\theta), P_{30}(\cos s\theta)$ on segment of sphere $\theta_0 = 90^\circ$ (solid lines) and $P_{n_1 0}(\cos\theta), P_{n_2 0}(\cos\theta), P_{n_3 0}(\cos\theta)$ on segment of sphere $\theta_0 = 25^\circ$ (dashed lines), where $\theta \in [0; 25^\circ]$

For example, values n_k for the segment of sphere $\theta_0 = 25°$ are shown in table 1.

Adjusted spherical harmonic analysis

The computation of zeros of the functions (4), (5) and their norms (Hwang & Chen, 1997) leads to time consuming procedure. It is much easier and more efficient to work with functions, wich can be represented by a finite series.

Let us consider the transformation from the coordinate system on the segment of the sphere (r, θ, λ) to the new coordinate system on the hemisphere (r', θ', λ') (de Santis,1992):

$$r' = r,\ \lambda' = \lambda,\ \theta' = s \cdot \theta \qquad (7)$$

where $s = \dfrac{\pi}{2\theta_0}$.

After the transformation (7) eigenvalues of these functions become integer and nonnegative on the hemisphere, and, therefore, these functions can be expanded into a finite hypergeometric series. It should be noted that functions on different segments of the sphere are similar (fig. 3).

Sketch of computations

The procedure "Remove-Restore" (Hofmann-Wellenhof & Moritz, 2005) is traditionally used for computation of high-precision quasigeoid heights. According to this procedure, let us separate the quasigeoid height ζ in the two parts:

$$\zeta = \delta\zeta + \zeta_M \qquad (8)$$

where $\delta\zeta$ and ζ_M represent the contributions to quasigeoid height corresponding to high frequency and low frequency components of the gravity field respectively. Generally a priori model up to degree/order 360 is subtracted for constructing gravity model within procedure "Remove-Restore". In our experiment, the contribution ζ_M was computed using the global gravitational model EGM 2008 (Pavlis et al., 2008) up to degree/order 360 (Fig. 4):

$$\zeta_M = \frac{GM}{\gamma R} \sum_{n=2}^{360} \left(\frac{a}{R}\right)^n \sum_{m=0}^{n} \left\{ \bar{C}_{nm} \cos(m\lambda) + \bar{S}_{nm} \sin(m\lambda) \right\} \bar{P}_{nm}(\cos\theta) \qquad (9)$$

$$\Delta g_M = \frac{GM}{R^2} \sum_{n=2}^{360} \sum_{m=0}^{n} (n-1) \left\{ \bar{C}_{nm} \cos(m\lambda) + \bar{S}_{nm} \sin(m\lambda) \right\} \bar{P}_{nm}(\cos\theta) \qquad (10)$$

The Δg were computed from the EGM 2008 model up to degree/order 360 (Fig. 5):

After operation "Remove" the residual values of gravity anomalies were obtained (Fig. 6):

$$\delta\Delta g = \Delta g_{AGP} - \Delta g_M \qquad (11)$$

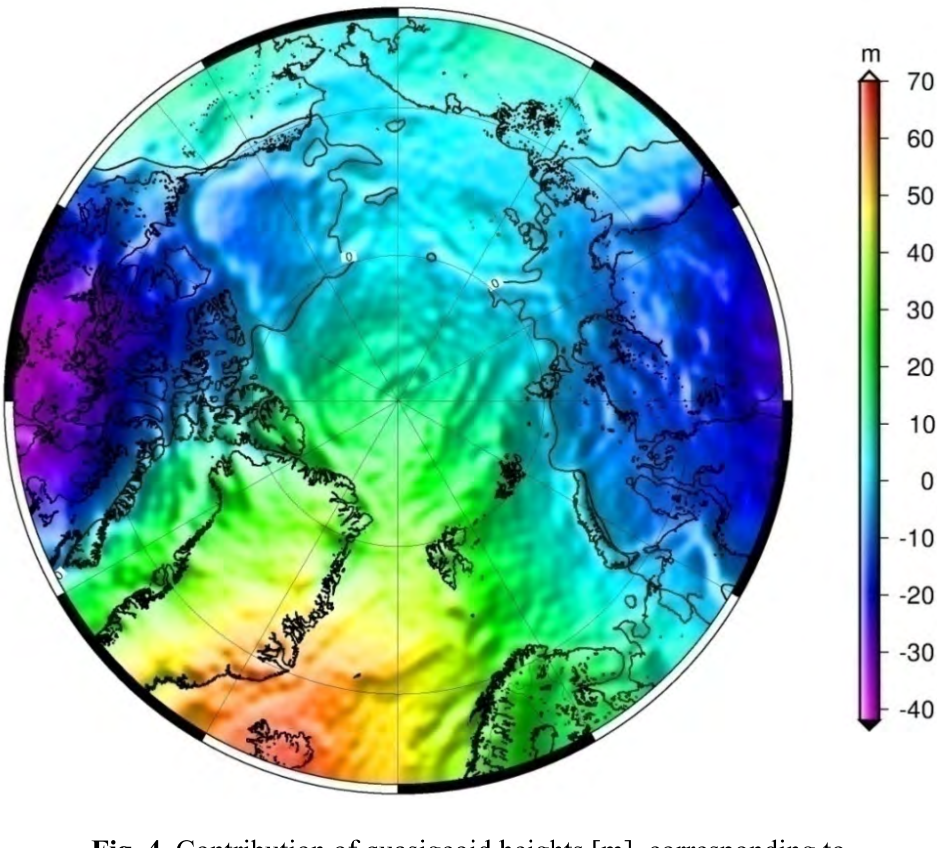

Fig. 4. Contribution of quasigeoid heights [m], corresponding to the long-wave features (up to degree/order 360) of the EGM 2008 gravitational field

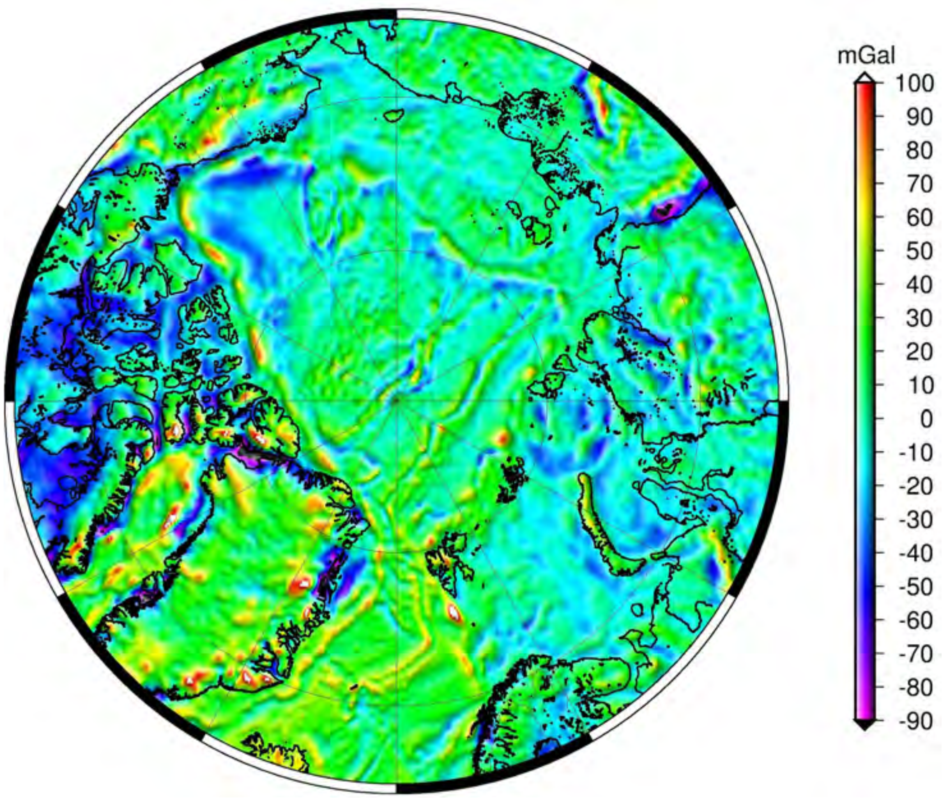

Fig. 5. Gravity anomalies [mGal], computed from model EGM 2008 up to degree/order 360

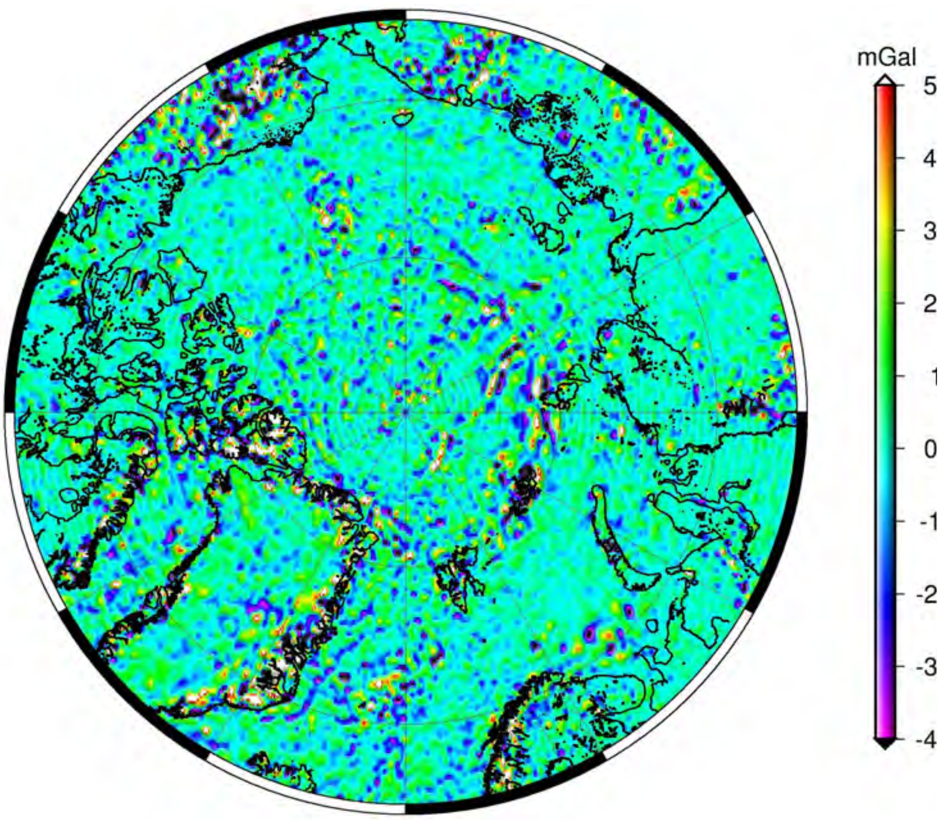

Fig. 6. Residual values of gravity anomalies $\delta\Delta g$ [mGal]

Results

The construction of the regional gravitational field model by the modified ASHA approach

Because in the ASHA method we came to integer eigenvalues the approximation of the residual values of gravity anomalies will be based on the ASHA approach in the following form

$$\delta\Delta g_{\mathrm{mod}} = \frac{GM}{R^2}\sum_{n=2}^{K_m^*}\sum_{m=0}^{k}(n-1)\left\{\bar{a}_{km}\cos(m\lambda) + \bar{b}_{km}\sin(m\lambda)\right\}\bar{P}_{nm}(\cos s\theta) \quad (12)$$

where K_m^* is maximum order of model, s = 3.6, $\bar{a}_{km},\bar{b}_{km}$ are fully normalized ASHA coefficients of model.

To determine the optimal order K_m^* let us consider resolutions of different models. As well known, resolution of global model (Seeber, 2003) is

$$\lambda = \frac{2\pi r}{K_m} \; or \; \frac{\lambda^o}{2} = \frac{180}{K_m}. \quad (13)$$

In turn, resolution of the model, constructed using ASHA, according to (de Santis, 1992) is

$$\lambda = \frac{4\theta_0 r}{K_m^*} \; or \; \frac{\lambda^o}{2} = \frac{2\theta_0^0}{K_m^*}. \quad (14)$$

Combining equations (13) and (14) for $\theta_0 = 25^\circ$ we get (Dzhuman, 2014)

$$K_m^* = \frac{50}{180}K_m. \quad (15)$$

Thus, the model (10) is equivalent to model ASHA up to degree/order 100. Evidently it is necessary to construct the ASHA model with degree/order more than 100 for representing the residual gravity anomalies. For construction of high-precision quasigeoid heights it is enough to adopt $K_m^* = 150$.

We decided to use the least squares method for computation of the unknown coefficients $\bar{a}_{km},\bar{b}_{km}$. If K_m^* is large it is inconvenient to inverse the corresponding normal equations matrix. Therefore, it is necessary to locate the initial data on the regular grid. Then we will be able to use the discrete orthogonality relationships in longitude. The distance between parallels in this grid can be arbitrary, and the distance between meridians must be constant. In this case we get (Marchenko & Dzhuman, 2014; Sneeuw, 1994)

$$\left.\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{4}\sin m\lambda_i^j = 0;\\
&\sum_{i=1}^{r}\sum_{j=1}^{4}\cos m\lambda_i^j = 0,\; m \neq 0;\\
&\sum_{i=1}^{r}\sum_{j=1}^{4}\sin m_1\lambda_i^j \cdot \sin m_2\lambda_i^j = 0,\; m_1 \neq m_2;\\
&\sum_{i=1}^{r}\sum_{j=1}^{4}\cos m_1\lambda_i^j \cdot \cos m_2\lambda_i^j = 0,\; m_1 \neq m_2;\\
&\sum_{i=1}^{r}\sum_{j=1}^{4}\sin m_1\lambda_i^j \cdot \cos m_2\lambda_i^j = 0,
\end{aligned}\right\} \quad (16)$$

where r is number of points in the first octant ($0;\frac{\pi}{2}$).

We gridded residual gravity anomalies $\delta\Delta g$ on such grid using the cubic spline interpolation.

Let us introduce the abbreviations

$$\left.\begin{aligned}
\bar{R}_{nm}(\vartheta_k,\lambda_i^j) &= \bar{P}_{nm}(\cos\vartheta_k)\cos m\lambda_i^j;\\
\bar{S}_{nm}(\vartheta_k,\lambda_i^j) &= \bar{P}_{nm}(\cos\vartheta_k)\sin m\lambda_i^j.
\end{aligned}\right\} \quad (17)$$

and

$$\Sigma = \sum_{k=1}^{s}\sum_{i=1}^{r}\sum_{j=1}^{4} \quad (18)$$

where s is a number of parallels.

In such case according to (Marchenko & Dzhuman, 2014) the unknown coefficients can be easily computed by the formula

$$\left.\begin{aligned}
&\sum_{i=m}^{Km}\sum \bar{R}_{im}\bar{R}_{jm}\cdot x_\sigma = q_\sigma,\; j = m, K_m;\; m = 0, K_m\\
&\sum_{i=m}^{Km}\sum \bar{S}_{im}\bar{S}_{jm}\cdot x_\delta = q_\delta,\; j = m, K_m;\; m = 1, K_m
\end{aligned}\right\} \quad (19)$$

where $\sigma = \sigma(i,m)$, $\delta = \delta(i,m)$, x_σ and x_δ are the unknown coefficients, q_σ and q_δ are components of residual vector.

Evidently, for such a grid the maximum order of matrix, which should be inverted, coincides with the maximum order of the constructed model taking into account equations (16).

Thus, we got unknown coefficients $\bar{a}_{km},\bar{b}_{km}$ up to degree/order 150. The model of residual values of gravity anomalies is shown in Fig. 7.

The main characteristics of gravity anomalies are given in table 2.

Table 2

The main characteristics of gravity anomalies

	Min., mGal	Max., mGal	Mean, mGal	St. dev., mGal
Δg_{AGP}	-167.5	222.7	3.50	27.32
$(\Delta g_M + \delta\Delta g_{\mathrm{mod}})$	-154.3	198.4	3.10	26.62

The relationship between gravity anomalies and disturbing potential can be presented as a solution of Molodensky's boundary-value problem (Hofmann-Wellenhof & Moritz, 2005; de Santis & Torta, 1997):

$$\Delta g = -\left[\frac{\partial T}{\partial r}\right]_{Q_0} - \frac{2T_{Q_0}}{r_{Q_0}} \quad (20)$$

where r is the spherical coordinate, Q_0 is the point on the telluroid obtained from the corresponding point of the physical surface of the Earth. The residual quasigeoid undulation $\delta\zeta$ is found by means of the Bruns formula

$$\delta\zeta = \frac{T}{\gamma}. \quad (21)$$

Thus, taking into account (20) and (21), we get the contribution of the residual quasigeoid heights (Fig. 8) by means of the formula

$$\delta\zeta = \frac{GM}{\gamma R}\sum_{n=2}^{150}\sum_{m=0}^{k}(n-1)\left\{\bar{a}_{km}\cos(m\lambda) + \bar{b}_{km}\sin(m\lambda)\right\}\bar{P}_{nm}(\cos s\theta) \quad (22)$$

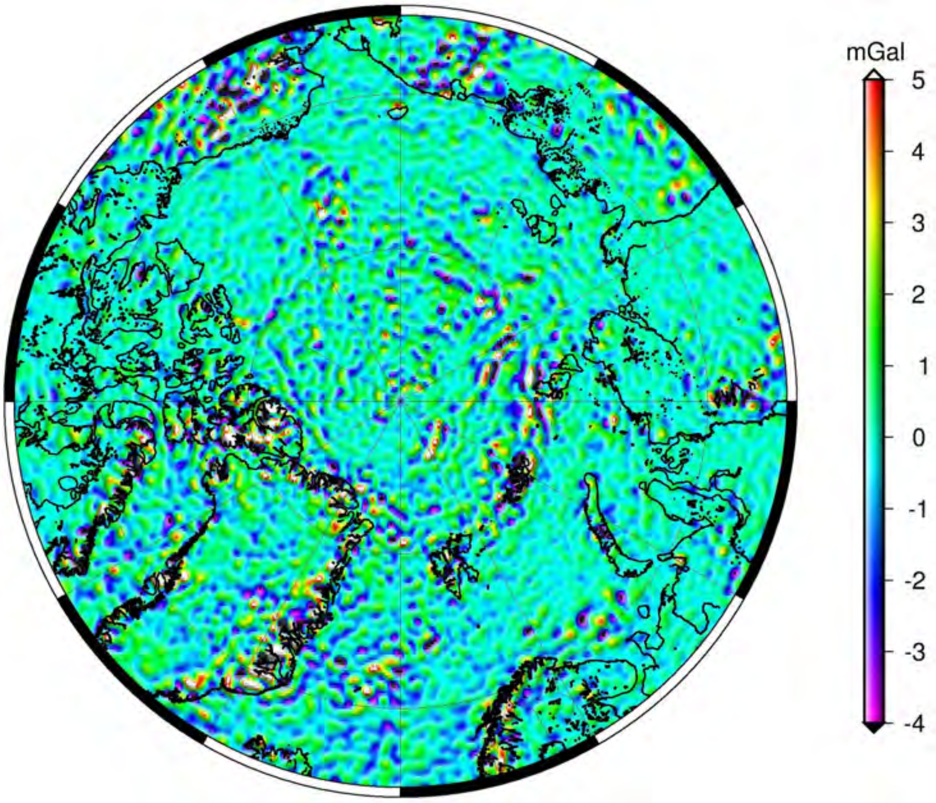

Fig. 7. Model of residual gravity anomalies based
on the modified ASHA method up to degree/order 150 [mGal]

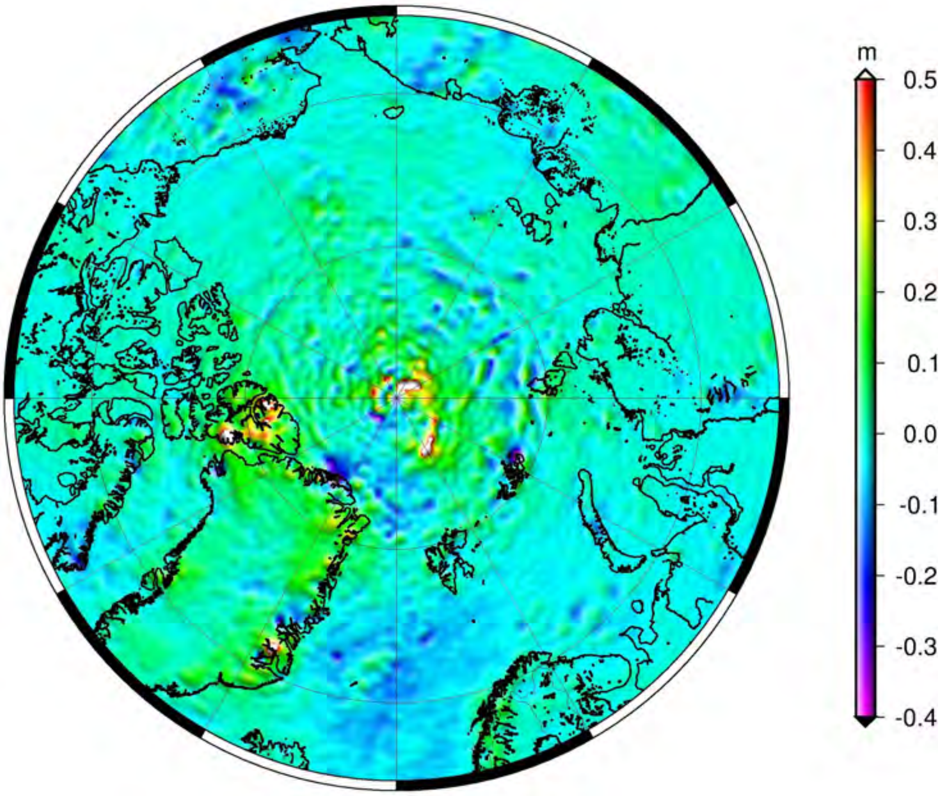

Fig. 8. Contribution of quasigeoid heights $\delta\zeta$ [m]

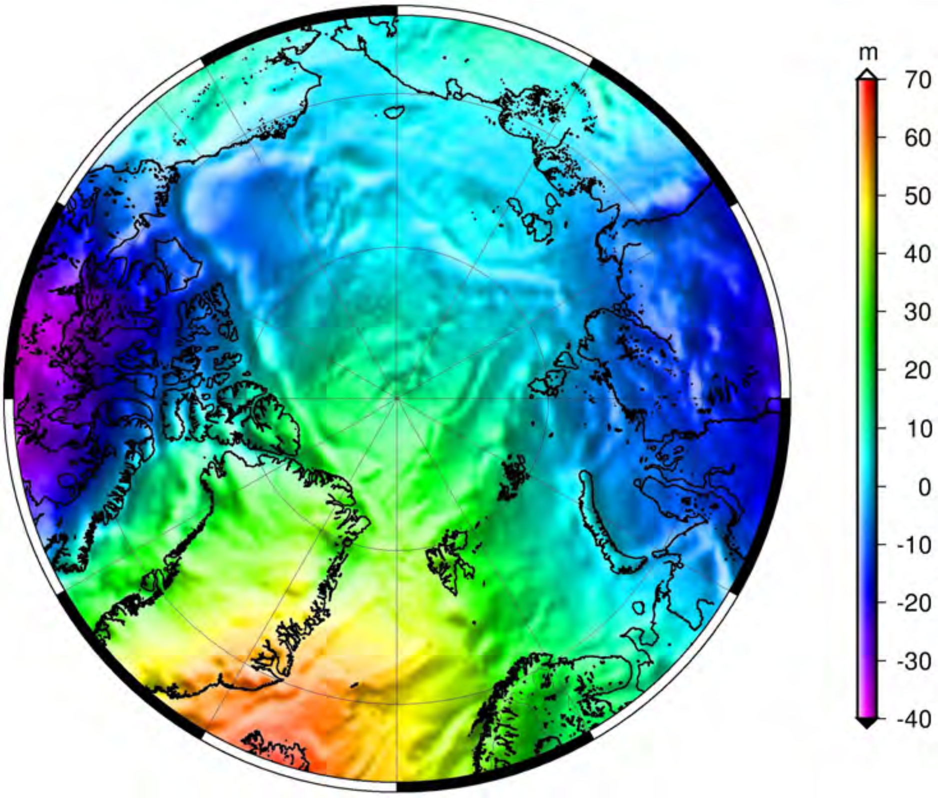

Fig. 9. Map of quasigeoid heights [m] in the Arctic area

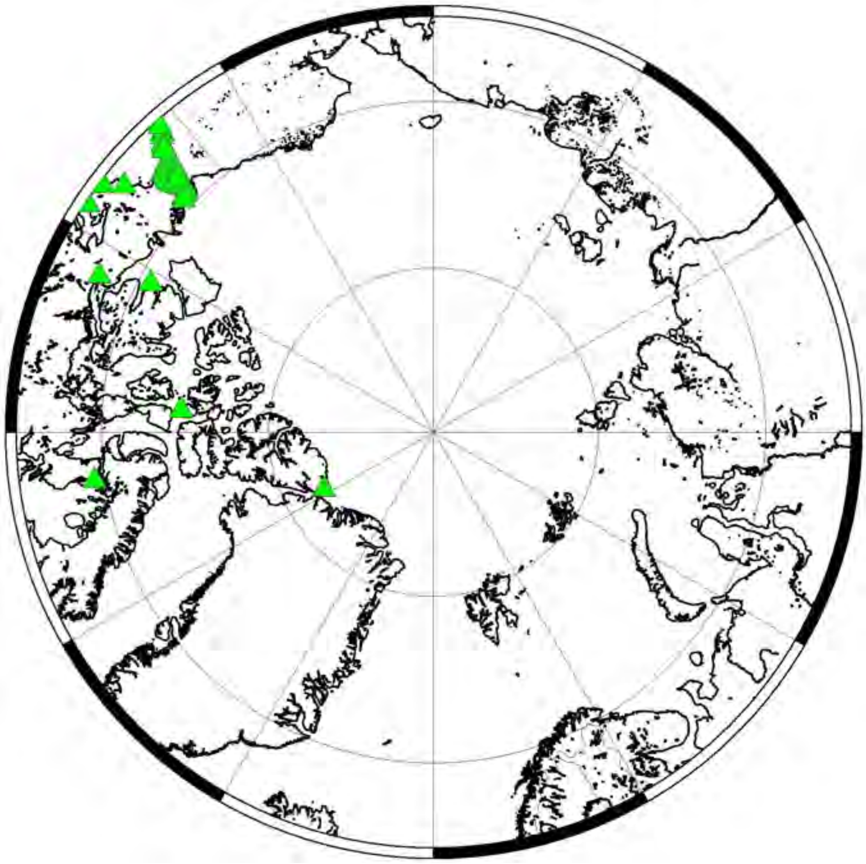

Fig. 10. Placement of points with determined quasigeoid heightsDifferences *d* between the model and measured quasigeoid heights are shown in table 4

Table 4

Differences *d* between the model and measured quasigeoid heights

B, °	L, °	*d*, [m]	B, °	L, °	*d*, [m]	B, °	L, °	*d*, [m]
68,302	226,489	0,153	67,663	226,151	-0,015	65,193	236,566	0,104
67,437	226,228	0,109	67,908	226,425	0,124	68,250	225,204	0,071
67,457	226,246	0,188	68,192	226,563	0,123	65,361	221,700	0,113
67,098	223,874	-0,108	68,562	226,024	0,037	67,792	226,222	0,116
67,245	224,793	-0,156	68,690	225,864	0,076	68,037	226,512	0,137
67,429	225,134	0,059	68,754	225,651	0,084	68,898	225,470	0,116
67,416	225,126	0,070	69,061	225,408	0,095	68,373	225,902	0,094
65,068	221,751	-0,103	69,226	225,757	0,138	68,306	226,473	0,149
65,772	222,154	-0,063	69,437	226,986	0,257	69,438	227,006	0,261
65,899	222,470	-0,002	69,437	227,011	0,291	68,203	224,885	0,025
66,261	223,191	-0,063	68,217	224,998	-0,041	68,292	225,570	0,194
66,449	223,367	-0,021	68,148	224,564	0,059	82,491	297,676	-0,551
66,565	223,692	-0,264	65,275	233,214	0,260	74,691	265,106	-0,463
69,288	226,096	0,119	65,281	233,157	0,248	67,818	244,868	-0,439
65,193	236,575	0,097	66,253	231,356	0,265	69,377	278,190	-0,439
67,543	226,212	0,111	66,257	231,370	0,312	70,736	242,239	-0,393
82,494	297,660	-0,561						

Map of the full quasigeoid surface (8) is shown in the Fig. 9.

The main characteristics of quasigeoid heights are given in table 3.

Table 3

The main characteristics of quasigeoid heights fields

	Min., m	Max., m	Mean, m	St. dev., m
ζ_M	-40.5	68.1	10.8	17.85
ζ	-41.0	68.2	10.9	17.89

Scientific novelty and practical significance

We used 49 points with determined quasigeoid heights using GNSS-leveling to compare our model. GNSS/leveling points with known quasigeoid heights were obtained from International Center for Global Gravity Field Models <http://icgem.gfz-potsdam.de/> (ICGEM). Placement of these points is shown in Fig. 10.

We can see from the table 4, that standard deviation of differences between the model and measured quasigeoid heights is equal to 0.22 m.

Conclusions

Finally we can conclude:

- The approximation of the regional gravity field in the frame of the Arctic Gravity Project was considered and based on the nonorthogonal functions of the SCHA and ASHA methods;
- Among these approaches we prefer the ASHA method that has a certaine advantage caused by the possibility of the representation of the basis functions in the form of a finite hypergeometric series in contrast to the SCHA technique. It is evident that ASHA technique gives the opportunity of the construction of ASHA-model in the analytical or/and gridded forms. The combination of different approaches for the determination of optimal degree/order of model is also discussed;
- The modified ASHA technique provides a good accordance in terms of standart deviation between initial and model gravity anomalies (table 1 and table 2);
- ASHA approach allows to avoid the time consuming procedure in the computations of geodetic functionals;
- The approximation by ASHA technique can be reccomended especially for fast computations of regional gravimetric fields with high orders.

Acknowledgement

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ВИЗНАЧЕННЯ РЕГІОНАЛЬНОГО КВАЗІГЕОЇДА З АРКТИЧНОГО ГРАВІТАЦІЙНОГО ПРОЕКТУ

Мета. В роботі побудовано поле висот квазігеоїда на територію регіону Арктики. Коли в наявності є дані з певного регіону Землі, глобальні сферичні функції втрачають свою ортогональність на даному регіоні, і визначення коефіцієнтів моделі, яке зазвичай проводиться за способом найменших квадратів, стає чисельно нестабільним. Проте є спеціальне рішення рівняння Лапласа для сферичного сегменту. **Метод.** В якості вихідних даних прийнято поле аномалії сили ваги на даний регіон з Арктичного проекту. Побудова квазігеоїда здійснювалася за допомогою процедури "Видалення - Відновлення" в три етапи. На першому етапі від поля аномалій сили ваги з Арктичного проекту віднімалися модельні значення аномалій сили ваги, обчислені за моделлю EGM2008 до 360-го порядку. На другому етапі виконувалося моделювання отриманих залишків аномалій сили ваги за допомогою методу adjusted spherical harmonic analysis (ASHA). Даний метод передбачає редукцію вихідних даних на півсферу і їх моделювання за допомогою системи неортогональних функцій, які задовільняють рівнянню Лапласа. При цьому під час побудови матриці нормальних рівнянь було використано дискретну ортогональність базової системи функцій по довготі, що призвело до значного скорочення часу обчислень невідомих коефіцієнтів. На третьому етапі, використовуючи попередньо знайдені коефіцієнти моделі, було побудовано залишки висот квазігеоїда (короткохвильові ефекти поля), також побудовано внесок квазігеоїда із моделі EGM2008 (довгохвильові ефекти поля), і відновлено повне поле квазігеоїда. **Результати.** Побудовано модель регіонального гравітаційного поля і порівняно її з аномаліями сили тяжіння з AGP. Також отримано модель висот квазігеоїда, яку порівняно з висотами квазігеоїда, взятими з 49 точок GNSS/нівелювання. **Наукова новизна і практична значущість.** В даній роботі розроблено модифікацію методу ASHA, яка дає можливість значно пришвидшити процес знаходження невідомих коефіцієнтів при побудові локальних гравітаційних полів. Це дає можливість будувати локальні гравітаційні поля вищих порядків. Добре відомо, що точність квазігеоїда залежить від порядку моделі.

Ключові слова: аномалії сили тяжіння, висоти квазігеоїда, adjusted spherical harmonic analysis, spherical cap harmonic analysis

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ОПРЕДЕЛЕНИЕ РЕГИОНАЛЬНОГО КВАЗИГЕОИДА ИЗ АРКТИЧЕСКОГО ГРАВИТАЦИОННОГО ПРОЕКТА

Цель. В работе построено поле высот квазигеоида на территорию региона Арктики. Когда в наличии данные из определенного региона Земли, глобальные сферические функции теряют свою ортогональность на данном регионе, и определение коэффициентов модели, которое обычно проводится по способу наименьших квадратов, становится численно нестабильным. Однако есть специальное решение уравнения Лапласа для сферического сегмента. **Метод.** В качестве исходных данных принято поле аномалии силы тяжести на данный регион с Арктического проекта. Построение квазигеоида осуществлялась с помощью процедуры " Удаление – Восстановление" в три этапа. На первом этапе от поля аномалий силы тяжести с Арктического проекта отнимались модельные значения аномалий силы тяжести, вычисленные по модели EGM2008 до 360-го порядка. На втором этапе выполнялось моделирование полученных остатков аномалий силы тяжести с помощью метода adjusted spherical harmonic analysis (ASHA). Данный метод предусматривает редукцию исходных данных на полусферу и их моделирование с помощью системы неортогональных функций, которые удовлетворяют уравнению Лапласа. При этом при построении матрицы нормальных уравнений было использовано дискретную ортогональность базовой системы функций по долготе, что привело к значительному сокращению времени вычислений неизвестных коэффициентов. На третьем этапе, используя предварительно найдены коэффициенты модели, было построено остатки высот квазигеоида (коротковолновые эффекты поля), также построено вклад квазигеоида с модели EGM2008 (длинноволновые эффекты поля), и восстановлено полное поле квазигеоида. **Результаты.** Построена модель регионального гравитационного поля и сравнение ее с аномалиями силы тяжести с AGP. Также получена модель высот квазигеоида, которую по сравнению с высотами квазигеоида, взятыми 3 49 точек GNSS / нивелирования. **Научная новизна и практическая значимость.** В данной работе разработана модификация метода ASHA, которая позволяет значительно ускорить процесс нахождения неизвестных коэффициентов при построении локальных гравитационных полей. Это дает возможность строить локальные гравитационные поля высших порядков. Хорошо известно, что точность квазигеоида зависит от порядка модели.

Ключевые слова: аномалии силы тяжести, высоты квазигеоида, adjusted spherical harmonic analysis, spherical cap harmonic analysis

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