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STABILITY OF SOLUTIONS OF OPTIMIZATION PROBLEM ARCHITECTURE OF SOFTWARE SYSTEMS

The stability of solutions in the problem of architecture design for software system is examined in the paper with taking to the account the quality attributes for software system, obtained with Analytical Hierarchic Process. For this purpose, for different values of elements' inconsistencies in the matrix of pairwise comparisons, given as random numbers, were compared inconsistency criteria of decision for optimization problem obtained with a classical and with a modified Analytical Hierarchic Process. We also examined the sensitivity of solutions for selection of optimal architecture to the inconsistencies of matrix of pairwise comparisons, and to the errors in determination of priority indices for quality criteria. Obtained results showed the advantages of modified Analytical Hierarchic Process, and they showed the influence of solution stability to the inconsistency of pairwise comparisons on the correctness of results of architecture optimization for software system.

Key words – software system architecture, Analytical Hierarchic Process, optimization.

Introduction.

In connection with the increasing complexity of software systems increasing requirements for their architecture, which conceptually integral should be merged all decisions on the design of the system. The complexity of the problems solved by system makes it impossible for architecture development "from scratch", and the use of existing solutions is unacceptable in connection with constant increase of requirements to the quality of the PS and the rapid improvement of hardware and software platforms. Significant progress in addressing these problems was the development of the technologies of designing an architecture based on the use of architectural patterns. The essence of the technology consists in the presentation of architecture in the form of some structure (carcass), elements of which are selected from a variety of standard components. This technology is presented in [1], in the future, received a major development, and now is widely used by leading corporations software development [2]. But since for a given subject area substation such architectural solutions can be designed in several, the task of choosing the optimal, multi-criteria of quality, solution.

In [3] for the problem of optimal choice of the architecture of a distributed software system is used method of analysis of hierarchies (MAI). But it is known that the use of a standard MAI, with a substantial number of alternatives ($n \geq 9$), leads to significant inconsistencies between the elements of the matrix of pairwise comparisons that generates the error in determining the weight multipliers alternatives. To solve this problem, in[4] a modification of the MAI, which weight multipliers alternatives are determined from the conditions of minimizing inconsistencies matrix of pairwise comparisons, which causes an initial problem into the problem of mathematical programming. In [5] considered the application of the modified MAI to the problem of choosing the optimal architecture of software systems.

In this paper investigation of the influence of inconsistency matrix of pairwise comparisons on the solution of the problem, and whether the modification MAI can improve the solution of the original problem. Compared the solutions obtained modified and standard MAI, for different number of alternatives and magnitudes of the errors of the matrix of pairwise comparisons. Studied well as the influence of errors in determining the weight multipliers architectures and corrections of the priorities of the criteria to change the procedure for ranking alternatives. This is particularly important when evaluating alternative architectural solutions close on certain criteria.

Sustainability MAI to expert data errors.

The structural scheme of the selection process, architectural decisions based on indicators of quality is depicted in Fig. 1.

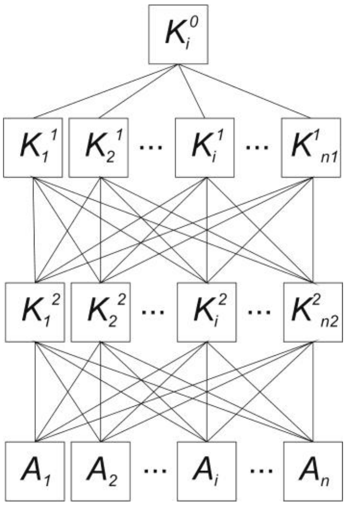

Fig. 1. A hierarchical view of the task of selection of architecture.

Here are such level of quality criteria:

- $K_i^1,\ i=\overline{1,m1}$ – quality criteria the PS in accordance with the standard ISO/IEC 25010;
- $K_i^2,\ i=\overline{1,m2}$ – quality criteria architecture;
- $A_i,\ i=\overline{1,n}$ – alternative architectural decisions.

List of quality criteria in use is determined by the developer PS together with the customer, and quality criteria architecture $\{K_i^2\}$ can be determined by criteria communication $\{K_i^1\}$ on the basis of technology QFD [6] with the recommendations of ISO/IEC 25010. Because we focus on object design technology, the alternative architecture A_i might be linked against standard architectural patterns on the basis of functional requirements [1].

You must choose this architectural solution which would optimized set of criteria $\{K_i^1\},\{K_i^2\}$. It is the task of multicriteria hierarchical optimization for solution of such tasks most commonly used method of analysis of hierarchies Sahati [7].

When you use MAI for solution of such tasks weight multipliers alternatives (criteria) $\{w_i\}$ on each level are using a matrix of pairwise comparisons $B\{b_{ij}\}$, that fill the experts (here b_{ij} defines advantage *i*-s alternatives over the *j-th*).

The coefficients of the matrices should be harmonized, i.e. $b_{ij}=w_i/w_j \quad \forall b_{ij}\in B$. The weighting factors in this case are as components of the eigenvector of the matrix of pairwise comparisons, which correspond to the maximum характеристичному number of the matrix. But with a substantial number of alternatives by operation of experts of various factors matrix $B\{b_{ij}\}$ is uncoordinated and its rank will be different from the unit, that is, the matrix will have multiple eigenvalues. Coherence assessment in case of minor violations are invited to use the index of consistency and relations coherence:

$$I_u = \frac{\lambda_{\max} - n}{n-1}$$

$$I_0 = \frac{I_u}{M(I_u)} \qquad (1)$$

where $\lambda_{\max} = \sum_{i=1}^{n}\left(x_i \times \sum_{k=1}^{n} b_{ik} \right)$ – the maximum value of the eigenvector;

$M(I_u)$ – random consistency.

The index of consistency contains information about the violation of numerical (cardinal) and transitive consistency. Limits of application of the MAI determined by the attitude of $I_0 \le 0{,}1$ [7].

To find weight multipliers from uncoordinated matrix of pairwise comparisons several methods have been developed, the main of which are:

- methods of improving consistency on the basis of the clarification of experts;
- methods of obtaining formally agreed matrices;
- methods of transitive closure, the ratio of benefits [8].

These methods are based on a formal correction of matrices of coefficients of pair comparisons. But because the coefficients b_{ij} with a certain degree of reliability contain information about weight multipliers w_{ij}, it is logical that they all were taken into account in calculations. For this weight multipliers alternatives will search out the conditions to minimize the mismatch in the cells of the matrix $B\{b_{ij}\}$.

As a measure of consistency, which is subject to minimise, you can choose one of the following expressions:

$$(w_i - b_{ij} w_j)^2 \text{ або } |w_i - b_{ij} w_j| . \qquad (2)$$

For the opportunity to choose the degree of consistency in the process of computation enter the limit value consistency $\delta_{\partial on}$ and the degree of consistency we write in the form of

$$\left| \frac{w_i}{w_j} - b_{ij} \right| \le \delta_{\partial on} \cdot b_{ij}, \quad \delta_{\partial on} \ge 0 , \qquad (3)$$

де $\delta_{\partial on}$ – specified limit value.

Then weight multipliers w_i, which minimize (3), you can find the solution of the problem

$$\min_{\{w_i\}} \sum_{i=1}^{n} \sum_{j=1}^{n} (w_i - b_{ij} w_j)^2 \qquad (4)$$

$$a_i \le w_i \le b_i, \quad i = \overline{1,n}.$$

$$-\delta_{\partial on} \cdot b_{ij} \cdot w_j \le w_i - b_{ij} \cdot w_j \le \delta_{\partial on} \cdot b_{ij} \cdot w_j; \quad i, j = \overline{1,n} \qquad (5)$$

Problem (4), (5) is the task of nonlinear programming and through the change of variables can be reduced to an equivalent problem of linear programming:

$$\min_{\{w_i\}} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(y_{ij}^{+} - y_{ij}^{-} \right)$$

$$w_i \ge a_i, \quad i = \overline{1,n}, \qquad (6)$$

$$w_i - b_{ij} w_j = y_{ij}^{+} - y_{ij}^{-}.$$

$$-\delta_{\partial on} \cdot b_{ij} \cdot w_j \le w_i - b_{ij} \cdot w_j \le \delta_{\partial on} \cdot b_{ij} \cdot w_j,$$

$$y_{ij}^{+}, y_{ij}^{-} \ge 0; \quad i, j = \overline{1,n}. \qquad (7)$$

The equivalence of the problems (4), (5) and (6), (7) follows from the fact, that vectors restrictions when y_{ij}^{+} and y_{ij}^{-} are linearly dependent, and therefore any solution is part of the y_{ij}^{+} or in y_{ij}^{-} and therefore, at least (6) corresponds to the minimum (4) [9].

To assess the consistency of the obtained solutions will use the following indicators:

- coefficient of consistency

$$K\left(w_i^*\right) = \frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{1}{b_{ij}}\left|\frac{w_i^*}{w_j^*} - b_{ij}\right|, \tag{8}$$

- as well as the degree of coherence

$$M_1 = \sum_{i=1}^{n} K\left(w_i^*\right), \quad M_2 = \max_i K\left(w_i^*\right). \tag{9}$$

Here w_i^* – values are defined.

Were conducted research on the effectiveness of the method of calculation of weight multipliers, solution of the problem (6), (7) when solving the optimization problem architecture PS the inconsistent matrices $B\{b_{ij}\}$. Thus, for given values of threshold inconsistency $\delta_{\partial on}$ simulated random disturbances matrix $B\{b_{ij}\}$ and there were weight multipliers $w_i^*, i = \overline{1,n}$ standard and modified MAI.

After that there were calculated the coefficients and the degree of coherence (8), (9) for the results obtained by two methods Studied also the errors decision on the ranking of the alternatives. Analysis of the obtained results are presented in the next section.

Experimental investigation of the stability of solutions.

Were taken for investigation alternative architectural decisions of the international project GB (Glass Box) [10]. The study was conducted for different number of architectural alternatives that were evaluated against the following quality criteria:

1. The ability to modification.
2. Scalability.
3. Performance.
4. Cost.
5. Development costs.
6. Portability.
7. Ease of installation.

For each of the criteria was formed matrix $B^s\{b_{ij}^s\}$, $i, j = \overline{1,n}, s = \overline{1,7}$, where b_{ij}^s shows how much *i-th* alternative dominates the *j-th* for the implementation of the *s-th* criterion. The matrix asked a perfectly harmonized. Then simulated error experts by the generation of the random variables K_{ij} in the interval $K_{ij} \in [-0{,}5\cdot\delta_{\partial on} \; +0{,}5\cdot\delta_{\partial on}]$ with a certain step $\Delta\delta$, and the elements of the matrix $B^s\{b_{ij}^s\}$ determined by the formula:

$$b_{ij}^{s*} = b_{ij}^s + K_{ij}\cdot b_{ij}^s. \tag{10}$$

For the obtained matrix $B^{s*}\{b_{ij}^{s*}\}$ determined sets of weight multipliers $\{w_i^s\}, i = \overline{1,n}, s = \overline{1,7}$ standard MAI and as a solution (6), (7). After that calculated the degree of coherence M_1 and M_2, that average multi-criteria of quality.

Figure 2 shows the dependence of the criterion M_1 from the amount of space, which was chosen by K_{ij} for both methods for the case of 15 alternatives.

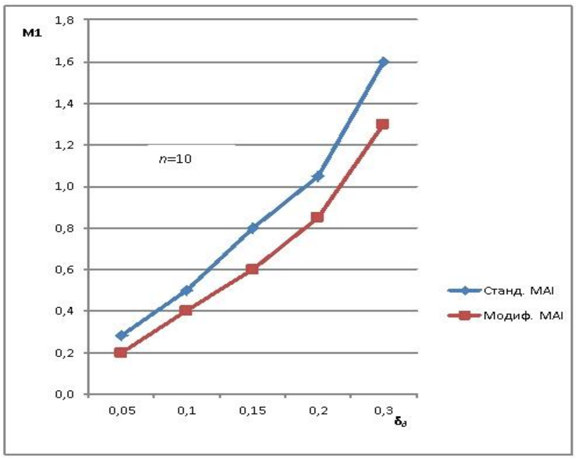

Fig. 2. The dependence of the criterion of the M_1 from the interval errors.

As seen from the graph, modified MAI gives significantly better results according to the criterion M_1, than the standard. Thus, when errors in the matrix $B^{s*}\{b_{ij}^{s*}\}$ within $\delta_{\partial on}$ = 0,15 modified MAI gave 20 percent less than the value of the degree of mismatch solution than the standard.

On Fig. 3 shows a graphic dependence of the degree of coherence M_2 from the interval at which simulated perturbation matrix. From the graph we see that the criterion M_2 with the increase of $\delta_{\partial on}$ advantages modified MAI increase and $\delta_{\partial on}$ = 0,25 the value of the criterion M_2 almost 30 percent less, than for the standard.

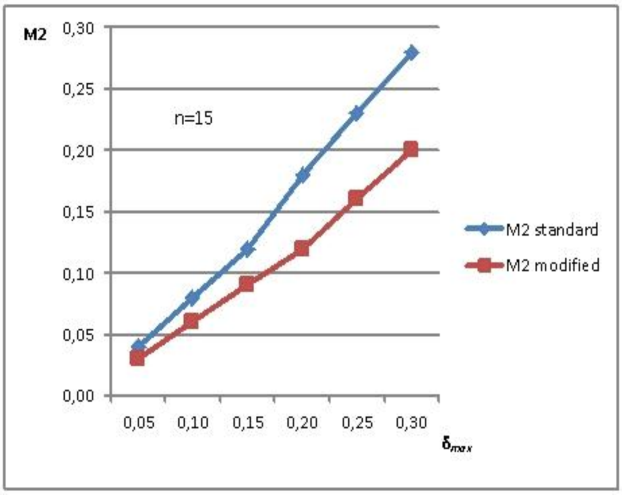

Fig.3 The dependence criteria M_2 interval errors.

Graph also shows that the growth gradient criteria inconsistent decisions for standard MAI higher than that for modified, that testifies to a lesser volatility modified MAI.

Also important is the study of the influence of measurement errors weight multipliers $\{w_i\}$, caused by неувязками in the matrix of pairwise comparisons, the ranking of alternatives $\{A_i\}$ both specific quality criteria, and in totality. This agreed matrix of pairwise comparisons $B^s\{b_{ij}^s\}$; $i, j = \overline{1,n}$; $s = \overline{1,m2}$ there were sets of multipliers $\{w_i^s\}$ and alternative ранжувались for values $\{w_i^s\}$ for each criterion. Thus been ordered sets $\{A_{is}, K_{is}^s\}$, $s = \overline{1,m2}$, $is \in J_s$ – streamlined for values weight multipliers many numbers architectures. After that, according to the above described methodology, are simulated error experts and determined the $\{w_i^*\}$ and was re-ranking $\{A_{is}\}$.

Calculations showed that there was a change of ranking with close values of the w_i already at the border inconsistency $\delta_{\partial on}$ < 0,1. Table 1 shows the results of calculating the weight factors in the four alternative architectures project GB:

1. On the basis of three-level J2EE (THEJ).
2. Three-level use of the platform NET (THTD).
3. Two-level (TWOT).
4. Platform with support for distributed agent (COAB).

Table 1.

Weight alternatives

Quality attributes	Alternatives			
	THTJ	**THTD**	**TWOT**	**COAB**
Modification	0,521	0,172	0,106	0,210
Scalability	0,404	0,402	0,074	0,143
Performance	0,201	0,204	0,347	0,246
Cost	0,166	0,120	0,487	0,227
Development costs	0,152	0,110	0,515	0,223
Portability	0,450	0,050	0,050	0,450
Ease of installation	0,168	0,368	0,256	0,208

The table shows that the evaluation of architectures THTJ and THTD indicators "scalability" and "performance" very close, and so even a slight misalignment of matrix elements $B^s\{b_{ij}\}$ when applying the standard MAI can entail changing the order of ranking of alternatives. Application of the modified algorithm will ensure sustainability of the solution obtained for large values of inconsistencies matrix of pairwise comparisons and thus broaden the scope of the MAI.

For ranking alternatives multi-criteria you must define their priorities. This can be done either direct appointment of priority values experts, or calculation of their matrix of pairwise comparisons. The second option is preferable as it allows to reduce the influence of subjective factors on the result. For this purpose experts of the filled matrix of pairwise comparisons $B^s\{b_{ij}^s\}$, where value b_{ij}^s determines how the impact criterion K_i^2 the prevailing influence of the criterion K_j^2 the implementation of the quality criterion PS K_s^1. Applying the modified MAI, we obtain sets priorities quality criteria architecture $\{P_i^{1s}\}$, $i=\overline{1,m2}$, $s=\overline{1,m1}$. Then the weight of alternative architecture A_i concerning the implementation of the quality criterion PS K_s^1 will be determined by the formula:

$$J_i^{1s} = \sum_{j=1}^{m2} p_j^{1s} \cdot w_i^j, \quad i=\overline{1,n}, \quad s=\overline{1,m1}, \qquad (11)$$

w_i^j – weight multipliers alternatives, defined at the previous stage.

You can now perform a ranking of alternatives $\{A_i\}$ from value $\{J_i^s\}$ for each $s=\overline{1,m1}$.

But as in the development PS nterested several groups of specialists, and their estimation of priorities may significantly differ. In order to take this into account, you must first determine the priorities of each of the groups by forming them matrix of pairwise comparisons, which are priorities criteria $\{P_i^s\}$, $i=\overline{1,k2}$, s – the number of the group of experts. A compromise solution, you can find as average geometric $P_{ij}^{*}=\sqrt[n]{P_{ij}^{1}\cdot P_{ij}^{2}\cdot\ldots\cdot P_{ij}^{n}}$, or as averaged, based on the measure of competence of the expert groups $P_{ij}^{*}=P_{ij}^{\alpha_1}\cdot P_{ij}^{\alpha_2}\cdot\ldots\cdot P_{ij}^{\alpha_n}$ ($\alpha_1,\alpha_2,\ldots,\alpha_n$ – indicators of competence).

Table 2 presents estimates of the priorities of the quality criteria of the different groups of experts received a modified MAI.

Table 2.

Priorities quality criteria

Quality attributes	Experts			The aggregate value
	developers	users	customers	
Modification	0,216	0,294	0,184	0,280
Scalability	0,087	0,092	0,038	0,082
Performance	0,052	0,117	0,087	0,097
Cost	0,245	0,019	0,272	0,135
Development costs	0,245	0,019	0,272	0,135
Portability	0,050	0,155	0,053	0,094
Ease of installation	0,106	0,304	0,093	0,177

These results indicate that the priorities of experts on some indicators are significantly different, and the aggregate value may not be accepted as a compromise. In this case, you can look for a compromise slight adjustment of priorities using the ratio of the

$$D'_{s,i,j}=\frac{\left|J_i-J_j\right|}{\left|w_i^s-w_j^s\right|}\cdot\frac{100}{P_s}. \quad (12)$$

Here $D'_{s,i,j}\left(s=\overline{1,m2};\quad i,j=\overline{1,n},\, i\neq j\right)$ – the minimum change of the value of priority P_K quality criterion K_s, which changes the order of the neighboring alternatives A_i and A_j on the reverse. The smallest value $D'_{s,i,j}$ shows that the priority P_K of attribute K_s is critical changes assessments of pair comparisons.

Using the ratio of (12), for each criterion K_s you can find interval ΔP_s, in which experts can be correction of priorities P_s, directly or through correction values of pairwise comparisons, without changing the ranking of alternatives $D_s^{*}=\min_i D_{s,i,j}=\Delta P_s,\quad i=\overline{1,n},\quad s=\overline{1,k2}$. The relation (12) can also be used in case of change of requirements to the PS in the design process, which can lead to a change of priorities in terms of the criteria.

If you need to conduct the ranking of alternatives in respect of the global quality of PS, it is necessary to define priorities quality criteria PS $\left\{P_i^2\right\},\ i=\overline{1,m1}$, using a modified procedure MAI.

Determine the weight of alternatives in respect of the implementation of the global criterion of quality of PS can be on the values of the indicator:

$$J_i^0=\sum_{s=1}^{m1}J_i^{1s}\cdot P_s^2,\quad i=\overline{1,n} \quad (13)$$

Then the indicator as an alternative A_i multi-criteria will be

$$J_i=\sum_{j=1}^{m2}P_j\cdot w_j^i,\quad i=\overline{1,n}, \quad (14)$$

and the ranking $\{A_i\}$ is held by the values $\{J_i\}$.

From the results of research shows that changing the order of ranking of alternatives can occur as for the error in the determination and through changing priorities $\{P_j\}$. Therefore, the choice of optimal variant of architecture it is necessary to conduct relevant research.

Conclusion

Studies have shown that the use of a standard algorithm for computing the weight factors in the MAI in the optimization problem architecture of a software system can lead to bad decisions in the event of a significant number of alternatives. A modified procedure in MAI can significantly reduce the inconsistency of decisions, even with considerable difficulties matrix of pairwise comparisons. So, for the criteria used, inconsistent application of the modified algorithm provided in some cases, a decrease criteria values from 20 to 30 percent.

Analysis of the results also showed that growth gradient criterion inconsistency $\boldsymbol{M}_1$ increases with errors matrix of pairwise comparisons, that is received in the MAI solution is unstable in these errors. Therefore, it is necessary to conduct additional analysis of the obtained ранжувань alternatives, as the set of criteria and particular criteria as required to build area of compromises [3]. It may be helpful to agreeing on priorities for different categories of specialists in determining the weights of the criteria of quality.

Thus, the use of the modified algorithm in the MAI, and implementation of the above activities will reduce the influence of the errors of pairwise comparisons, as well as instability of the MAI in these errors, and, thus, improve the quality of the solutions of the problem of optimization of architecture PS multi-criteria.

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