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RESEARCH OF DECONVOLUTION OPERATION USE IN RECONSTRUCTION METHODS OF LOCAL DISTORTION ELIMINATION

Physical process of blurring emergence has been analyzed. Through conducted experiments it has been proved that image blurring formation is adequately described by the model based on convolution, i. e. wrapping. It is shown that blurring center or discrete function of point scattering comprises information about trajectory and uniformity of motion, which has caused an image distortion. It is determined that extremum number of averaged normalized column values of Fourier image distorted by artificial blurring correlates with parameters of blurring.

Key words: distortion, deconvolution, function of point spread (PSF), distortion area, image reconstruction.

Introduction

Blurring is such a distortion type that emerges as a result of dynamic changes of objects of attention or background during frame exposure.

While researching the image, blurred artificially or in a natural way, the biggest attention should be paid to the spectral image features. This determines construction and use of metrics, obtained on the base of the spectral image-signal features. Such an approach allows better understanding of nature and mechanism of natural blur formation, enabling the development of reconstructive technologies.

Formalization of the image distortion process as a result of motion comes to the construction of mathematical blur model that in its turn similar to the distortion case (result of refocusing) is determined by the operation of wrapping (convolution). Formal determination of the operation is the following. Let some discrete distorted image $f(x, y)$ of the dimension $M \times N$ with the blur center $h(x, y)$ and its dimension $m \times n$ is given This center is titled Point Spread Function (PSF) [4]. In addition, light sensitive matrix imposes randomly distributive additive noise $n(x, y)$. Then resulting image $g(x, y)$ with the distortion, which emerged as a result of a camera motion, will be determined by the following formula:

$$g(x,y) = h(x,y) \otimes f(x,y) + n(x,y), \quad (1)$$

where the sign $\otimes$ means the wrapping operation.

The noise $n(x, y)$ is an integral part of up-to-date matrices, what is revealed via random deviations of intensity function values of the image point from real color value.

Noise reasons in digital sensors can be different, but the main are tempo streams in cells of matrix and heat oscillations. The other reasons are charge spreading, photon ricochet and their flight of light sensitive matrix area. There are factors that influence upon the noise quantity. In compliance with ISO they are: matrix type, pixel size, temperaure, and magnetic pickup etc.

In most cases the noise is Gaussian, which parameters are average value and dispersion that characterize the most frequent value and other value deviations. Noise is additive and does not correlate with the image and does not depend on the pixel location. This stochastics complicates the procedures of reconstructive algorithms development for images, as far as despite of the distortion type, noise to a greater or lesser extent is always present.

Problem setting

The main purpose of the work is determination of digital image characteristics for the use of deconvolution operations in reconstructive methods of local distortion elimination. The local distortion type is determined by the object motion or registration device.

Point Spread Function

The image distortion is determined by the value of PSF $h(x, y)$. The function determines the distortion type of every image pixel. During the process of distortion every pixel of an initial image changes into a spot in case of refocusing and in case of a certain blur into a segment. The same process can be represented in the other way: the intensity function value of every distorted image pixel is an integral characteristic of pixel values in not distorted image. As a result of these representations a distorted image is formed. The law, by which every pixel is distorted, is determined by the Point Spread Function.

In a discrete case the PSF can be represented by the matrix operator. As a rule, its dimension is less that the dimension of a whole image and is an important distortion characteristic. The intensity function value depends on the PSF, which determines the wrapping (convolution) operation and in (1) is depicted as the symbol$\otimes$. Hereby some part of the initial image is wrapped into one pixel.

In a discrete case the convolution operation is shown in (1), and its explicit form is [3]:

$$g(x,y) = h(x,y) \otimes f(x,y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} h(i,j)\, f(x+i, y+j), \qquad (2)$$

where $a = (m - 1) / 2$, $b = (n - 1) / 2$.

From the wrapping operation and from (2) it follows that the sum of all PSF matrix values is equal to one. Physical content of the PSF matrix values lies in such their definition when every of them stands for the light proportion of the given point, which is observed in the other coordinates. Obviously, in any convolutional image distortion the intensity function value of every point is distributed over different coordinates without exactly.

There are typical distortion functions. The very title "Point Spread Function" means that the very function is represented by the matrix, which with its structure repeats the distortion type.

If in case of refocusing of an image, every point is transformed into a spot, then all not null elements of the PSF matrix are located in the form of spherical spot with a radius of every point dispersion.

In case of motion distortion, i.e. blurring, all not null elements of the mentioned matrix are located along the trajectory of distortion motion. This is predetermined by the fact that every point is blurred along the trajectory during exposuring.

PSF is represented as a binary matrix and determined as Heaviside function of two variables or as a discrete function, which consists of two Heaviside step functions [2]:

$$h = u(0) - u(a), \qquad (3)$$

where a – length of blur area, i.e. the length of trajectory of distorting motion during the period of a frame exposure in pixels.

PSF can be represented as the sum of Dirac delta functions as far as the Dirac delta function is a primary Heaviside [2]:

$$h = \sum_{k=0}^{a} \delta(k). \qquad (4)$$

The wrapping operation of any function by the Dirac function is a shift of the processed function by the value of Dirac function parameter. Taking this into account, the clear wrapping operation (1) can be written as follows [1]:

$$h = \sum_{k=0}^{a} \delta(k) ¤ f = \frac{1}{a+1}\left(f(x,y) + f(x+1,y) + ... + f(x+a,y) \right) = \frac{1}{a+1} \sum_{k=0}^{a} f(x+k,y). \qquad (5)$$

In the formula (5) the noise n from (1) is ignored, and only the clear operation of convolution is reflected.

The necessity to multiply the sum of the formula (5) by the coefficient $\frac{1}{a+1}$ is substantiated by the fact that in case of convolution the intensity function values of an image are mixed. Nevertheless, totally they should not grow. Hereby, having been summed by (5), they should be normed, i.e. brought back to their order via division into several items.

Frequency characteristic of blurring

Frequency characteristics are the most interesting for the research of blurred images, as far as they reveal the character and parameters of distortion. To research the frequency characteristics of signals the Fourier transform is used. [1].

The Fourier transform is an integral transform of one complex-valued function of one valid variable into the other. The transform expands the given function into oscillator functions.

The Fourier transform of a function is mathematically determined as a complex function, which is set as an integral [1]:

$$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt\,. \tag{6}$$

The Fourier transform is applied to obtain the frequency spectrum of non periodic function, e.g. electric signal, i.e. for the representation of a signal as a sum of harmonic oscillations. The transform result is a set of periodic functions. That is a fundamental tool to represent some signal with indefinite periodicity and often non-understood nature as a set of functions with exactly defined periodicity. This helps to realize the phenomenon sense, which generates a researching non-periodic signal. That is why the area of Fourier transform use is wide enough from searching radio signals of extraterrestrial civilization to researching of language signals.

So far as a discrete Fourier transform is a linear transform, the discrete Fourier transform of the sum of elements is equal to the sum of discrete Fourier transforms of these elements. Then, the Fourier transform of a convoluted image will be as follows [1]:

$$\mathrm{F}\,\{g\} \sim \mathrm{F}\left\{\sum_{k=0}^{a} f(x+k,y)\right\}=\sum_{k=0}^{a}\mathrm{F}\,\{f(x+k,y)\}. \tag{7}$$

During the use of (5) and (7) the set of images is used for the summing operation. In this set all images are equal, as far as it is considered the full blur, which is caused by the camera motion, and the very images differ only in some shift *k*. If take out *k* from the very image *f*, then with the use of the circular shift rule it is possible to obtain the discrete Fourier transform of solitary primary image, which is a part of the blurred image [1]:

$$\mathrm{F}\,\{f(x+k,y)\}=e^{-j\frac{2\pi k}{N}}\,\mathrm{F}\,\{f(x,y)\}, \tag{8}$$

where N – the full length (product of dimensions) of image.

Having substituted (8) with (7) and neglecting the correction coefficient $\frac{1}{a+1}$, we can present the final spectrum of the image distorted by the camera motion via the discrete Fourier transform [1]:

$$F(g)=\sum_{k=0}^{a} e^{-j\frac{2\pi k}{N}} F\{f(x,t)\}. \tag{9}$$

So far as the noise is of random nature, in the formula (9), that represents the frequency characteristics of the image, it has been ignored.

With the help of (9) we can show the connection between the Fourier transform that is not distorted by the image *f* and the Fourier transform of the image *g*, distorted by the blur. In the resulting representation the difference is equal to the sum of exponential functions $e^{-j\frac{2\pi k}{N}}$. In the Fig. 1 left [1] the graph of the sum of exponential functions is shown, and the right part of the image is the visual representation built on the base of the graph. The graph and the images reflect the frequency characteristics, which impose on the not distorted image, if conduct the wrapping operation, i.e this is the separated convolution.

The blurring model adequacy verification on the base of wrapping (convolution) operation should be conducted with the help of two types of images: the images distorted in a natural way, i.e. by the camera motion, and images artificially distorted by the wrapping operation. In the ideal verification case the distortions should be realized with the help of blurring centers, which are maximally similar with one another.

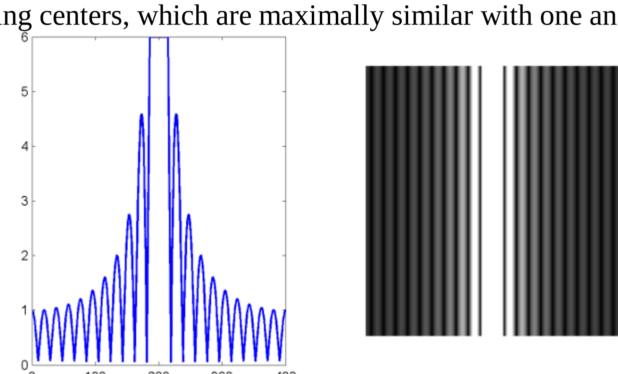

Fig. 1. Sum of exponential functions. Graph and synthesized image on its base

First, the convolution operation with the known blurring center is conducted. The blurring center is chosen to imitate the camera motion. This means that in case of a full blurring along the image plane the blurring center will represent one of the simplest distorting motions. If the motion is uniform and strictly horizontal with the length of 10 points, then the blurring center will be a vector-row with the dimension of 10 elements; value of every element will be 0,1. Such an approach to the blurring center construction is accepted only because of the implementation simplicity, as far as the motion character is not principal while determining the blurring nature.

In the Fig. 2 the example of not distorted image and the image distorted artificially with the use of described-above blurring center, which the horizontal blur imitates.

The research of these images were based on the use of the Fourier transform.

The formula (6) operates with continuous functions. Accordingly, the Fourier transform result is uninterrupted oscillator functions. It is impossible to use them for a digital image that is a matrix of discrete intensity function values. For this case it is recommended to use the Fourier transform variant that is titled as the discrete Fourier transform (DFT) [1]:


Fig. 2 Original image and the result of its artificial distortion

By its sense the discrete Fourier transform is a certain Fourier transform, but for the case of discrete values. If at the input the continuous Fourier transform takes a continuous function, and at the output returns a complex continuous function set, then at the input the discrete Fourier transform takes the sequence of simple or complex numbers, and at the output returns a set of complex values of the same length N. *The sequence is calculated by the formula* (10) [5].

$$X_k = \sum_{n=0}^{N-1} x_n e^{-2i\pi k \frac{n}{N}} . \qquad (10)$$

In practice, the other type of the transform is used, namely the discrete fast Fourier transform. The fast Fourier transform can achieve the same results with the use of significantly less computational resources. If the complexity of the discrete Fourier transform calculation is estimated as $O(N^2)$, then the fast Fourier transform as $O(N \log N)$.

That is why the fast discrete Fourier transform was used to research the difference between the output and artificially distorted images.

Comparative results are presented in Fig. 3

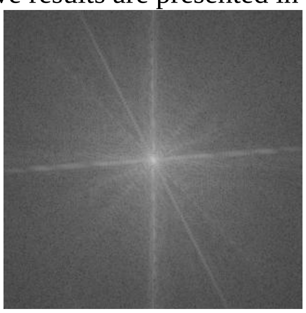
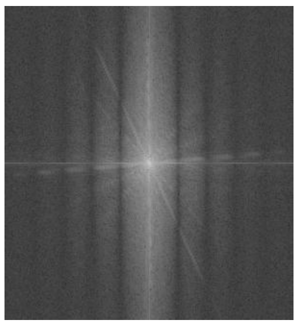

Fig. 3. Vizualization of discrete Fourier transform results before and after artificial distortion by horizontal blurring center

The right part of Fig. 3 refers to the output image, and the left – to the result of artificial distortion by the horizontal motion center.

The amplitude spectrum research in logarithmic scale has revealed that a natural image has no precise regularity of repetitions in its spectrum. All its constituents, both real and imaginary parts have chaotic character similar to the noise, what is characteristic for natural images.

Instead in the distorted image some systematicness that is shown in vertical periodical lines appeared. These lines are present in both real and imaginary parts of Fourier transform of images. If sum the real and imaginary parts and calculate their average values in every column of the image, then the sequence will be depicted with a graph in Fig. 5 [3]

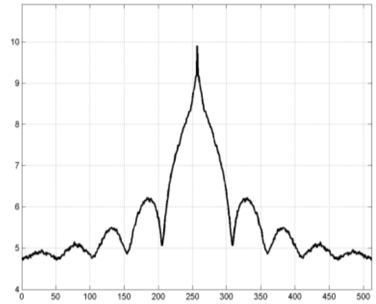

Fig. 5. Averaged normalized quantities of column values of Fourier transfer of the image, distorted by the artificial horizontal blur

To compare the same characteristic but for the case of not distorted image (the right part of Fig. 3) consider the Fig. 6.

The main difference between graphs in Fig. 5 and Fig. 6 lies in the presence of the distorted image (8) of periodical extremums in the graph. A natural not distorted image has no such extremums, what leads to formulation of two statements.

Statement 1.1.

The extremum number of averaged normalized values of Fourier transform columns of an image that is distorted with an artificial blur correlates with the parameters of this blur.

To confirm the statement 1.1 let's consider numeral parameters of the researched image and artificial horizontal blur. The results represented in the Fig. 5 concern the artificial horizontal blur of 10 pixels. The extremum number is equal to 9. The central extremum is of doubled thickness, what is equal to the sum of 10 extremums. The regularity can be explained by repetitions of the intensity function values by imposing color onto the neighboring points while blurring. That is why the extremum number is very close to the blur length. In the ideal case they are equal because every point is imposed onto the other ten points.

The similar experiment has been repeated with other 20 images. The case of vertical blur was considered separately. The results of all experiments were similar to those described. In such a way basing on the results of practical experiments the thesis on extremum number correlation with the characteristics of artificial blur has been confirmed.

The damping at the ends of the graph shown in the Fig. 5 emerges resulting from sorting results of the Fourier transform.

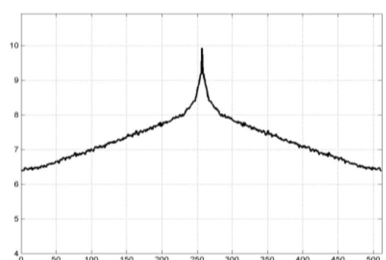

Fig. 6. Averaged normalized quantities of column values of Fourier transfer of natural not-distorted image

Statement 1.2.

The number and frequency of extremums of averaged normalized values of Fourier transform image, distorted by an artificial blur, is an invariant for the change of this blur parameters.

The formulated statement determines that there are typical blurring parameters, which are independent from the change of blurring operator parameters. To verify the conclusion several more experiments with the same initial image but different blur parameters have been conducted. Fig. 7 and 8 illustrate the results of the experiments for the case of artificial horizontal blur.

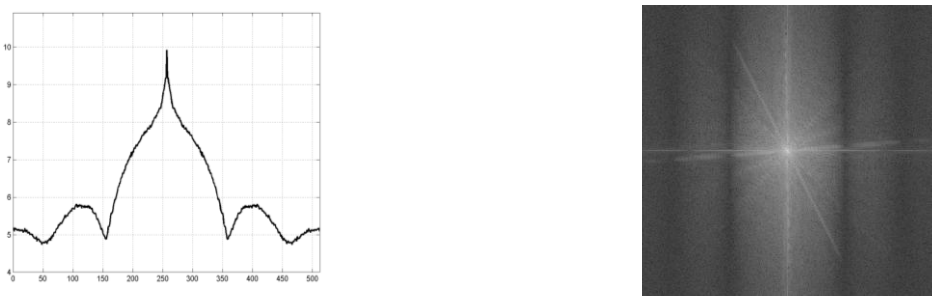

Fig. 7. Vizualization of discrete Fourier transform results for the image
with artificial horizontal blur of 5 pixels

Right part of every figure is an image of sorted spectrum, calculated similar to the result depicted in the Fig. 3. Appropriately, the left part is a graph formed with the averaged values of columns of the right part image. That is a peculiar section of this image, determined similar to the exemplified in the Fig. 5.

The given graphs of experimental research confirm the correctness of statement on invariance.

The tendency of interchange of positive and negative local extremums with the central global extremum is typical and similar to the case of exponential model which graph in Fig. 1. This means that this image spectrum feature can be taken as one of the criteria for image blur recognition.

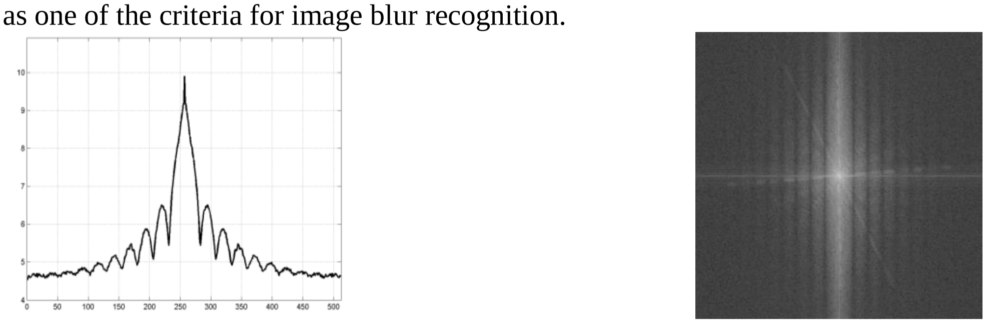

Fig. 8. Vizualization of discrete Fourier transform results
for the image with artificial horizontal blur of 20 pixels

The final verification of the blur model adequacy lies in comparison of the data obtained during studies of the artificially distorted images with the corresponding data for images with natural blurring.

The library of images and the methodology used are presented in [3]. To obtain natural blurring with known parameters and pre-defined blurring character, the authors have designed for this a special device, which is a camera on a moving rail. The camera moves uniformly and rectilinearly and records a chessboard with a sufficiently considerable exposure time. Thus, the obtained image is distorted by a full blurring under the influence of uniform rectilinear motion, i.e. the blurring with the same formal parameters as in the above experiments is obtained. The obtained in such a way image is shown in Fig. 9 [3].

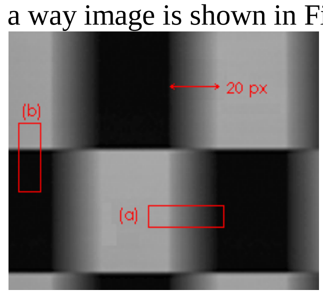

Fig. 9. Image example with natural horizontal blur

The fact that the blurring character meets the expected one you can see clearly from the image itself. Because of the peculiarity of the chessboard image, which lies in the use of only two colors (black – the minimum brightness and white – the maximum brightness), located in the opposite positions on the scale of brightness, shift directions, their length and uniformity of distorting motion are visible for brightness fluctuations in the distorted picture. The character of the image brightness drop indicates the horizontal movement within the areas *a* and *b*. Within the vertically oriented area *b* the changing brightness is very sharp like on the original image, what indicates a lack of movement in the horizontal direction. At the same time, the brightness drop in the horizontally oriented area *a* is clearly visible. Moreover, if you build a graph of the average brightness by the image columns in this area, it can be seen that the brightness is changed almost by the linear law, what proves that the distorting motion was not only linear in the horizontal direction but uniform as well. The motion length, i.e. the distance passed by every image point during the exposure time of the given frame, can be measured by the width of the image (band) area, on which the image brightness drop occurred. That is where the intensity function is not purely white or black it is a mixture of these colors. In this example the width of the area marked with a red arrow is 50 points. This is the trajectory length of the distorting motion.

All of the studies described above and their results with the artificially distorted images were performed with the natural image with the clear blurring that is shown in the Fig. 9. The results of these studies, in particular a frequency graph and its images obtained like in Fig. 7, 8, shown in Fig. 10.

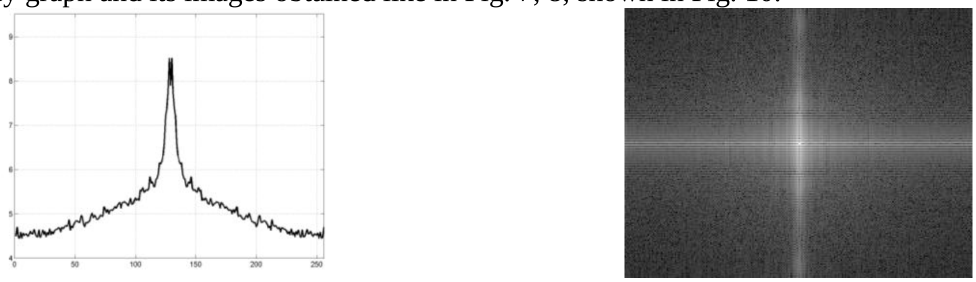

Fig. 10. Vizualization of discrete Fourier transform results for the image with natural blur

The results suggest that natural images contain much more noise than that blur which was obtained artificially. This is proved by a greater number of small extremums on the graph of Fig. 10. Obviously, a natural image pattern is not as clear and unambiguous as the image with the artificial blur. However, it is possible to separate features specific for the horizontal blur case. With the pixel by pixel analysis of the image it has been determined that the blur length is 20 pixels. On the graph shown in Fig. 10 the 18 regular extremums can be separated what is sufficiently close to the real blur parameter. These peaks are similarly visible on the synthesized image shown in Fig. 10.

Taking into consideration the fact that in cases of natural and artificial distortions the identical spectral characteristics are proved, we can conclude that the representation of the blur model based on the convolution operation is adequate. Hence, the reconstructive algorithms based on the deconvolution operation can be implemented to eliminate blur on natural images.

Some difference between artificial and natural blurs is explained with the noise of lightsensitive matrix. Such mixing closes maximally by the final result to the natural case. As the practical research proves, even small admixture of noise worsens significantly the results of the image reconstruction. Due to its random nature the noise is not perceived by a human eye while perception of a whole picture. But while implementing reconstructive algorithms based on the inverse wrapping operation the noise makes additional essential distortions to the renewed image.

The blur nature research of the section was conducted with the use of uniform rectilinear horizontal distortion for the simplicity of presentation. The distortion of real natural images occurs with much more complex trajectories. These trajectories are not so clearly reflected in the image spectrum. Therefore, the described above method of detecting the blur presence and parameters can not be used for all cases of distortion without further modifications.

Conclusions

Physical process of blur emergence has been analyzed. The thesis that image blur formation is adequately described by the model based on convolution, i.e. wrapping, has been proved via experiments.

It has been shown that the blur center or discrete point spread function comprise information about trajectory and motion uniformity, which caused the image distortion.

It has been found that the difference between the Fourier images distorted by the full image blur and not-distorted images is equal to the sum of exponential functions. Based on this formulated statement the extremum number of the averaged normalized values of the Fourier transform columns (in case an image is distorted by the artificial blur) correlates with the blur parameters.

It has been proved that the extremums number and frequency of averaged and normalized values of the Fourier transform of the image distorted by the artificial blur is an invariant to the changes of the blur parameters.

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