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MODELS OF HYPOTHETICAL IMAGE FOR IMAGE SEGMENTATION BY CUMULATIVE HISTOGRAM AND PIXEL DENSITY

The algorithmis for image threshold segmentation by cumulative histograms and pixel density of real and hypothetical images is considered. The one and multifragment models of hypothetical image are considered. Testing and experimental results are presented

Keywords: visual pattern, cumulative histogram, segment pixel density, threshold, models of hypothetical image.

Introduction

Indexing is an important tool in the systems of finding images by content. The formation speed and the adequacy of image features are the major criteria for the identification of the quality of these features used by specified systems.

Determining the image features for indexing requires fast algorithms of image segmentation. Nowadays, there is a great variety of publications on the methods of image segmentation. They can be generally divided into two classes: those that are based on finding the intensity threshold and those that divide the image into regions with certain features. The first ones determine the intensity thresholds based on histograms. Among them, the algorithms of determining the minimal intensity [1], convexity [2], moments [3], entropy [4], minimal errors [5,6] etc. can be distinguished. The typical example of the methods from the second class is the graph-based image segmentation [7]. The drawbacks of the abovementioned and some other algorithms are the different thresholds for similar images even within the algorithms of the same class. Most algorithms are fairly bulky, especially those using graph models or those based on statistical calculations. Modern CBIR-systems process millions of images in real time and therefore need extremely fast and quite accurate image feature determination tools. Segmentation algorithms are an important part of these tools.

In this article, the algorithms that meet the requirements of automatic CBIR-systems have been presented, namely the ones that are simple to develop and have a clear physical content.

For the cumulative histogram we have:

$$V_F\ (s) = \sum_{i=1}^{s} V\ (i) \tag{1}$$

where $V\ (i)$ are frequencies by intensity, $V_F\ (s)$ is an accumulative frequency for the given intensity, n is a quantity of the cumulative histogram intervals, s, i are interval numbers (interval values).

1. Segmentation based on cumulative histogram and pixel density in segments

For the hypothetical image we shall build a normalized cumulative histogram according to the formula:

$$V_{FH}(s) = (1/n) * s, \qquad s = 1,..., n, \tag{2}$$

where $V_{FH}(s)$ is the quantity of the pixels (the accumulated frequency) of the hypothetical image in the intensity interval 1-s, this is a set of pixels in which all the intensities are represented in equal quantity.

The number of intensity pixels of each value equals , where N, M are the dimensions (proportions) of the image for which the search of the segmentation threshold is being conducted, n is the quantity of the intervals of the cumulative histogram.

We shall call this model a single-fragment model, since the given histogram is shown with a single segment for the entire intensity interval 0 – 225.

Let us build the function of the difference between the cumulative histogram of the real and the hypothetical images:

$$D(s) = V_F(s) - V_{FH}(s), \qquad s = 1, n. \tag{3}$$

We consider that the coordinates of the extrema indicate the possible segmentation thresholds of the image. The method of determining the extrema is presented in [8,9].

For the hypothetical image, let us construct pixel density in segments according to the formula:

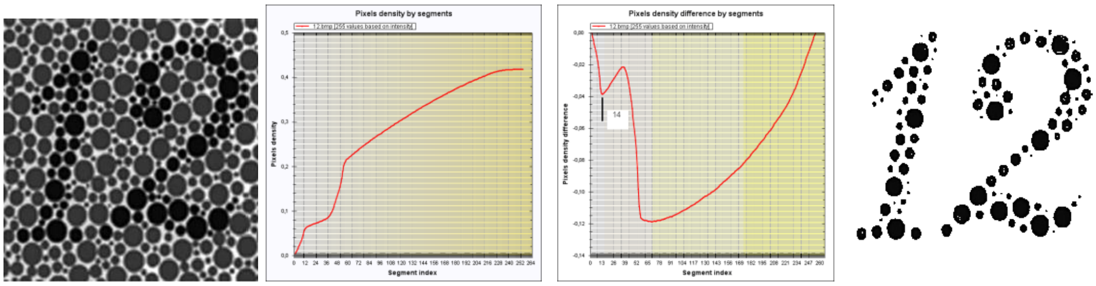
$$G_{CSH}(1 \div s) = (G_{CS}(1 \div 256)/n) \times s, \quad s = 1, n, \tag{4}$$

Let us build the function of the difference between the densities of the real and hypothetical images:

$$D_S(1 \div s) = G_{CS}(1 \div s) - G_{CSH}(1 \div s), \qquad s = 1, n. \tag{5}$$

where $G_{CSH}(1 \div s)$ – is the pixel density of the segment of the hypothetical image in the intensity interval $1 \div s$, $G_{CS}(1 \div 256)$ is the value of the pixel density of the ultimate segment (of the complete image), $G_{CS}(1 \div 256)/n$ is the pixel density of each fragment of the hypothetical image.

The segmentation thresholds are considered to be the extrema coordinates of function. The minimum indicates that, before it, a rapid increase of segment pixel density took place, and after it, the increase speed declines. The maximum, on the contrary, demonstrates the previous slow increase and the further quicker increase. The search for extrema coordinates is conducted in two intervals: from the black colour to the median for the segmentation of the black background, and from the median to the white colour for the segmentation of the grey one. Thus, the coordinate of the extremum indicates the content-based part of the image which can be segmented. In fig. 1c, according to function the segmentation threshold I=14 and the segment have been found.



a b c d

Fig.1. the image (a), the segment pixel density (b), the difference between the densities of the real and hypothetical images (c), the segmented image (d)

It should be mentioned that both the first and the second methods include beside I=14 two more extrema I=37, I=59. However, number 12 in these segments is noised by the elements of the texture (Fig. 2a). Thresholds I=14 and I=59 can be seen in fig. 4a in which the projection of the intensity of image “12” from the side of plane XOZ [10]. According to the results of the comparison of the segmentation methods in [11], the smallest threshold is I=47, and according to the Otsu algorithm, I=116 [12]. The respective segments are shown in fig.2. Thus, it is impossible to find threshold I=14-16 based on the statistical methods.

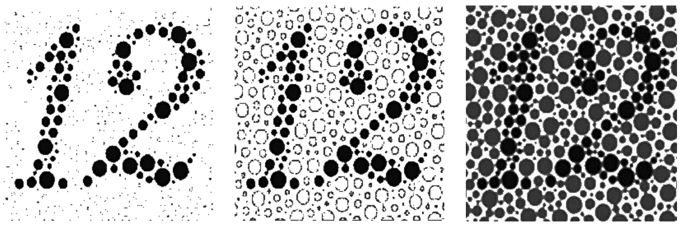
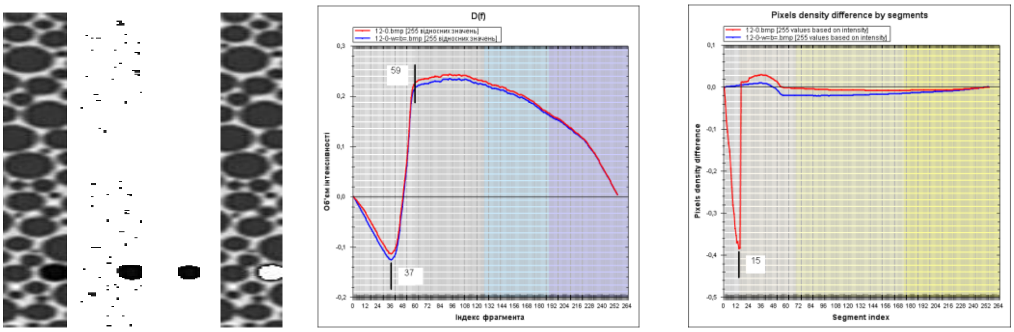


Fig.2. The segments of image “12”: I=37, I=47, I=106

3. The study of the segmentation method

Let us apply the first method to the fragment of image “12” that contains a small part of the black number 12 itself (let us call it “ellipsis”), and the rest is the grey texture (fig. 3a). One of the graphs of function D(s) obtained by means of the cumulative histogram method has a minimum indicating the segmentation threshold I=37. The segment is full of additional pixels (fig.3b). The graph of segment pixel density (one of the two in fig. 3h) for the image in fig. 3a has a minimum I=15 which is considered to be a threshold, and the segment in fig. 3b is obtained. The acquired pure segment is distracted from the original one and thus the image without ellipsis in fig. 3d is obtained. The two developed segmentation methods are applied to it. Function D(s) is similar to the function of the original image (fig. 3e), and the pixel density does not have a minimum in position I=15 (fig. 3h).

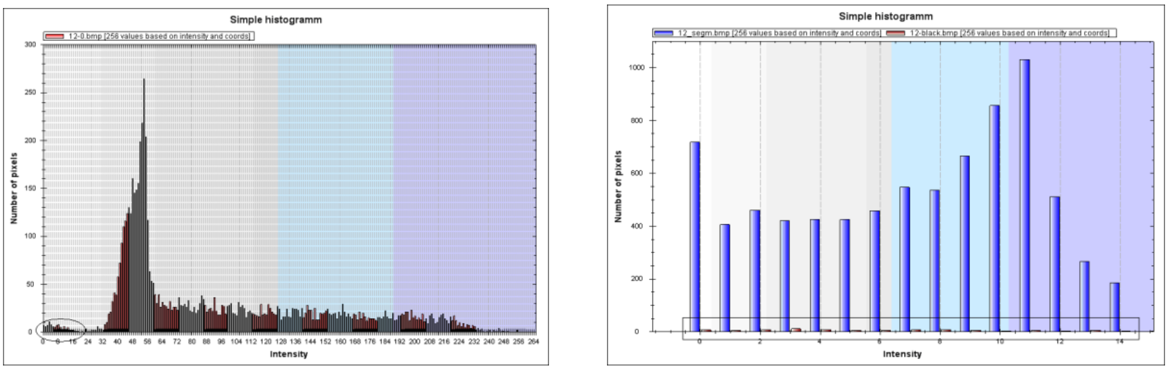
From this experiment we can conclude that the cumulative histogram method is sensitive to the percentage of the segmented part of the background. There is a critical value below which the thresholds can go through the relatively small values of pixel frequencies. The method of pixel density operates with other characteristics with other characteristics, and its extrema indicate the opposite direction of the change of pixel density in a segment.



a b c d e f

Fig.3. The image, its segments, the differences of the cumulative histograms and segment pixel densities

The quantitative and the qualitative content of the fragment of the image in fig.3a can be valued from the histogram in fig.4a. The contribution of the black ellipsis that should be segmented is small in comparison with the complete fragment, and it declines (circled in fig.4a). The contribution of the pixels of this black ellipsis to the image of number 12 is even smaller, which can be observed in the histogram in fig.4b (the columns are very small). The black colour is not homogeneous – it declines evenly. Therefore, in fig.4e there is no extremum responsible for the decline in the number of the black pixels of the ellipsis.



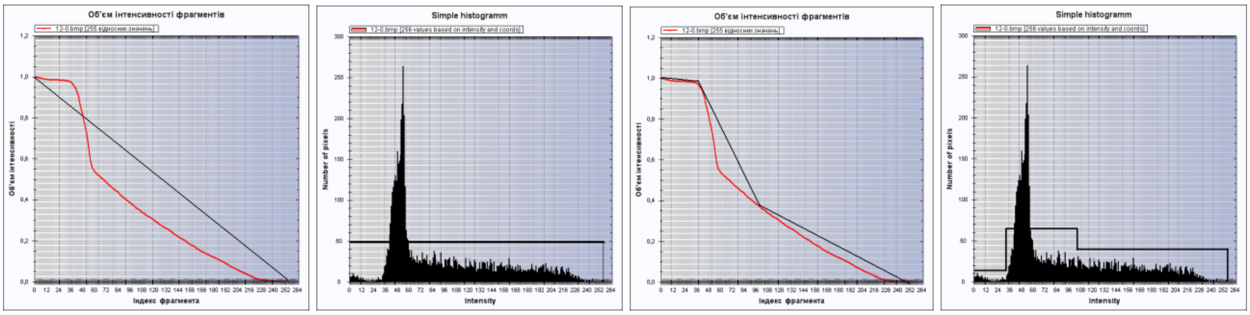
a b

Fig.4. Histograms of: the fragment of the image of “12” (a), number 12 and the ellipsis (b)

The conducted experiments have shown that the cumulative histogram model yields good results if the cumulative histogram of the original image has distinct locations of the speed change of pixel intensity increase and decrease. From the obtained dependencies for the image of the small black ellipsis we can conclude that among the big values of extrema, the extrema, the values of which are much smaller, disappear. With the increase of the value of coordinate s the accuracy of finding the extremum diminishes. Therefore, next we shall consider the multi-fragment model of finding thresholds.

4. Improving the cumulative histogram method

We shall use the hypothetical model not of the complete image, but of its fragments (by intensity). The cumulative histogram will appear as a piecewise linear function. The segment is characterized by the angle of the linear increase (decrease), the initial and final coordinates. In fig.5a,b the cumulative histograms and the histograms of the original and the hypothetical images for the whole intensity interval are presented. In fig. 5c,d the same functions for the separate intervals of the fragments of the images (the values are contingent) are demonstrated.

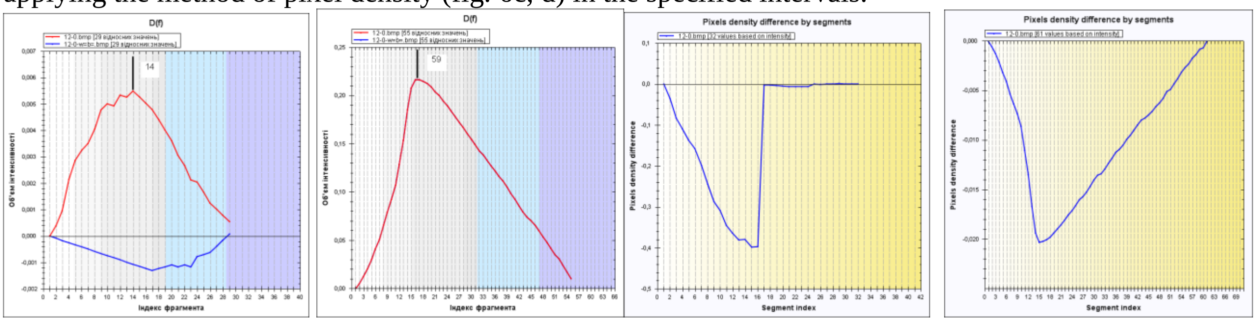


a b c d

Fig.5. The cumulative histograms and the histograms of: the image of "12" and the single-fragment (a, b) and the multi-fragment model (c, d) of the hypothetical image

The beginnings and the endings of the intervals are chosen based on the global extrema coordinates of function D(s) obtained for the uninterrupted segment. In order to level the influence of the extrema coordinates on the new dependencies of function D(s), they are accepted shifted a few units to the left and to the right of the extremum.

According to the graph in fig.3e, we have the following intervals: 0 – 37, 37 - 101, interval 102 - 255 is not considered due to the absence of both the extrema and the necessity to segment the gray color. Function *D(s)* is defined in smaller intervals: 0-32, 42-97 (the cumulative histogram of the hypothetical image is formed automatically on the basis of the cumulative histogram of the image in the specific interval). For the fragments of the image of the black ellipsis (fig.3a) and without the black ellipsis (fig. 3d) the respective dependencies of function *D(s)* are obtained (fig. 6a, b). In the second interval functions *D(s)* of both images coincide. In the first interval for the fragment with the black ellipsis, function *D(s)* has a distinct maximum with coordinate 14 to which the segment with the black ellipsis in fig.3c corresponds. The function value at the maximum point is 0,005, whereas the global maximum for the complete interval is 0,25, that is I is 50 times bigger than the local maximum. Thus, the method allows us to find a threshold in a fragment with a avery small contribution of the pixels of the desired segment. The minimum of the second curve (technological) is caused by the lack of increase or decrease of the cumulative histogram. Coordinate 96 of maximum D= in the second graph corresponds with the marked bending point of the cumulative histogram in fig. 4e. In other words, the improved model of the hypothetical image specified the coordinates of the extremum unnecessary for us in this case. The same coordinates of the thresholds are obtained by applying the method of pixel density (fig. 6c, d) in the specified intervals.



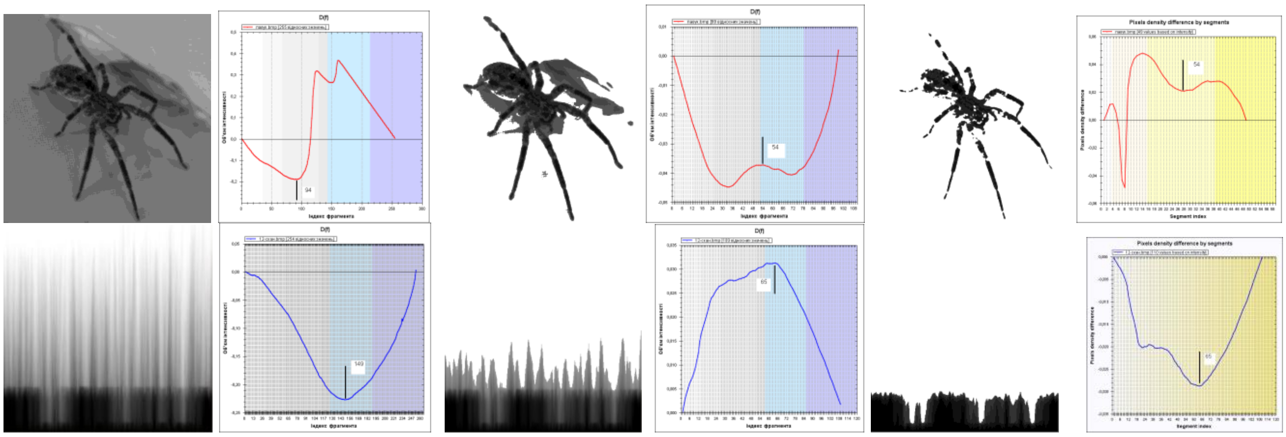
a b c d

Fig. 6. Function D(s) (a, b) and pixel density (c, d) for the two intervals

The segmentation algorithm by the cumulative histogram has a linear complexity from the number of pixels *N*M* (the formation of the histogram) and the intensity levels n (the linear search of extrema), that is, $O(N*M)+O(n)$. The segmentation method by pixel density has an algorithmic complexity $O(N*M*n)$ from the number of levels and from the number of pixels since for each segment, the dispersions of pixel intensity values of the pixels from the level that make up each segment are found.

5. The experiments

In testing the developed algorithms the images for which it is visually difficult to determine the regions for segmenting and obtain the desired components have been used.



a b c d e f

Fig.7. Test images and their segments obtained by different methods

Fig 7 shows an image (a), the thresholds by the single-fragment model of the hypothetical image and the respective segments (b, c), the thresholds by the multi-fragment model and the respective segments (d, e), the thresholds by pixel density (f) which coincide with the ones found in fig. 7d.

Conclusions

The methods of finding the thresholds of image segmenting that are based on the cumulative histograms of the given and the hypothetical images have been suggested. Single- and multi-fragment models that make it possible to determine the thresholds that correspond to different extrema values of the function of the difference between the cumulative histograms of the real and hypothetical images have been presented. The algorithm is characterized by simplicity and the lack of the calculation of any statistical characteristics, also by the linear algorithmic complexity with respect to the dimension of the image and the intensity interval. It is designed for multiple use in determining image features in CBIR.

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