**UDK 519.677: 519.17**

[1,2]**Chekurin V.F.**, [3]**Prytula M.H.**, [4]**Khimko O.M.**
[1]Pidstryhach Institute for Applied Problems of Mechanics and Mathematics NAS of Ukraine,

[2]Kujawy and Pomorze University in Bydgoszcz,

[3]Company "Mathematical Center" LTD

[4]Lviv National Polytechnic University

# MATHEMATICAL MODEL FOR STRUCTURE OF GAS TRANSMISSION SYSTEM



**A two-level model for the structure of a natural gas transmission system has been built with the use of methods of set and graph theories. The model is intended for development of software for automation of management by the system.**

**Keywords –gas transmission system, automation of management, software complexes, model of configuration, configuration graph, data structure**

**У статті запропонована дворівнева модель структури газотранспортної системи, побудована із використанням методів теорій множин і графів і зорієнтована на створення програмного комплексу для автоматизації управління магістральними газопроводами.**

**Ключові слова – газотранспортна система, автоматизація управління, програмні комплекси, модель конфігурації, граф конфігурації, структура даних**

## Introduction

Gas transmission systems (GTS) are used to transport large amounts of natural gas across long distances from producing regions to consumption regions. GTS is a global complex system of interconnected trunk pipelines and storage facilities, equipped with compressor stations, junction nodes, valves, pressure regulators and gauges. All these facilities, in the aggregate, form a integral engineering complex the configuration of which can change over its life period, depending on suppliers' and consumers' demands, operation conditions, technical specification etc. Due to that, GTS can be classified as a nonlinear large-scale controlled continuous dynamic system subjected to concentrated and distributed loadings. Essential processes for functioning of this system are: mass, momentum and energy transport by gas mixtures; force and heat exchange between transmitted gas and transmission facilities; mass, momentum and energy exchange between GTS and environment.

Effective control of this system is possible on the basis of computerization of technological processes, administrative management, maintaining services, emergency-repair services and other subdivisions of GTS with the use of hardware-software systems (HSS) [1,2,3]. These HSS should be based on mathematical models. The models form theoretical basis for formulation of direct and inverse problems for simulation of gas flows, optimization of operational modes of compressor station and configuration of GTS. These models can also provide mathematical tools for evaluation of reliability and residual life of GTS, ecological and business risks assessment, etc.

A model of structure of GTS is an important part of mathematical tools of HSS. It is necessary for mathematical formulation of the problem for mass, momentum, and energy transfer in gas mixture transported by GTS, modeling the stress-strained state of elements of gas transport infrastructure, evaluation of their strength and reliability. The model of structure is also required for graphical representations of GTS' configuration correspondingly to the needs of all groups of HSS users, etc.

The model of GTS structure should satisfy the following requirements: a) represent GTS as a heterogeneous system with the thoroughness that is sufficient for implementation of all HSS' functions with given accuracy; b) take into account the variability of GTS' configuration under its operation; c) provide the possibilities for creation an adaptive user HSS interface that can be adjusted to needs of users of different groups.

Graph theory methods are usually used for modeling of GTS' structure [4]. That enables us to take into account the dimensional heterogeneity of GTS, namely — presence of the node and line elements in its structure. This approach is convenient for formation of numerical models of gas-dynamic processes in the system. But such model does not reflect the inherent functional heterogeneity of node and line elements of GTS. The node and line elements can belong to different categories distinguished by their operational functions, technical specifications, parameters etc.

The mentioned approach to modeling of the GTS' structure can considerably complicate implementation of the functions responsible for accounting variability of GTS' configuration, graphical representation of structure of GTS, adaptive user interface etc. This can result in restricted functionality of HSS built on the base of such structure model.

Therewith, the model of GTS' structure is used for mathematical problems of different types: a) modeling of gas-dynamic processes in the gas mixtures, b) calculation of stress-strained state of GTS' elements, c) graphical representation of the structure of GTS and reflection of results of numerical simulation in forms suitable for the users of different groups etc. The problems of different types can require the models that reflect the structure of GTS with different thoroughness. The known approach to modeling of structure of GTS can not provide such facility.

A two-level model of structure of GTS is considered in this paper. The model is built with the use of methods of set and graph theories and is aimed at creation of HSS for automation of management by GTS. The model of configuration is the upper level of the structure model. On this level, GTS is considered as a system integrating heterogeneous physical objects — pipelines (line objects) and node elements of different categories. The lower level just reflects the topology of GTS; it is represented by a configuration graph. Mappings that determine correspondences between objects of models of these two levels are built. Data structures for representation of the physical and topological models in computer programs are considered.

*Remarks regarding denotations used in the paper.* In this paper, to distinguish objects of the mathematical model, the sets and sequences are denoted by upper-case hand-written characters (font **DECOR**); for their elements upper- or lowercase letters of *Times New Roman italic* are used. Mappings between sets are denoted by capital hand-written characters with the use the font FRENCH SCRIPT MT (French Script MT).

## The two-level model of structure of GTS

We consider GTS as a system that joins the node elements of different categories (inputs, outputs, bifurcations, junctions, valves etc.) and the line elements (pipelines) of different categories into a connected network. The line elements of different categories can be distinguished by the pipeline's type (underground or ground-surface pipeline, offshore or onshore pipeline, pipelines mounted on supports or installed with the use of cable structures etc.). They also can differ in their technical features, such as diameter and length of the pipe, roughness of its inner surface etc.

Configuration of GTS can change over its lifetime. The number of node and line elements of various categories can vary as a result of valves opening or closing, installation of new facilities and removing from operation some existing ones etc. To take into consideration the variability of GTS, we consider the two-level model of GTS' structure. The upper level of the model takes into account physical properties and technical features of GTS' elements, whereas the lower level is presented by the configuration graph that is the topological model of GTS (Fig.1).

The collection of node and line elements of various categories forms the set of objects of GTS configuration's model (Fig.2). The configuration chart represents the physical links between the node and line elements, converting the collections of node and line elements into a connected structure.

The graph of configuration represents the topology of the actual configuration of GTS. The set of its vertices is formed from the set of node elements, and the set of edges is formed from the set of the line elements which are presented in the actual configuration of GTS.

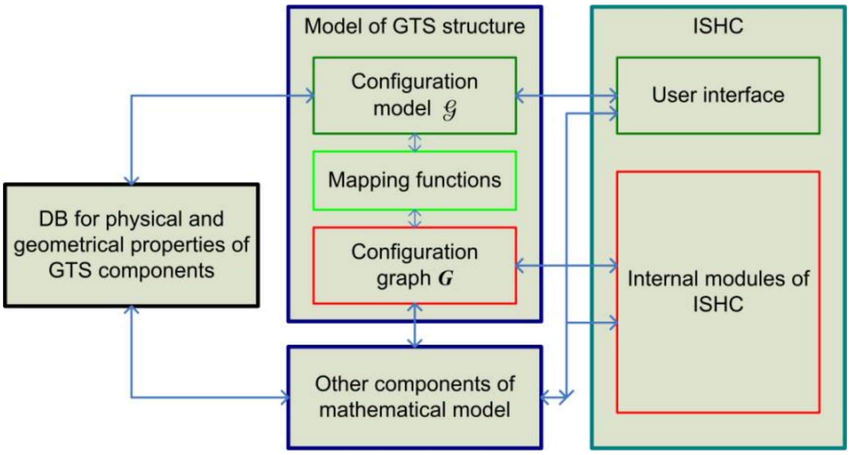


*Fig.1.The two-level model of structure of GTS and other constituents of its mathematical model*

The mappings (see fig.1) are the third constituent of the model of structure. They are destined to establish the correspondences between the objects of configuration model $\mathbf{G}$ on the one hand and the sets of vertices and edges of configuration graph $G$ on the other hand. It enables us to construct the configuration graph on the base of the model of actual configuration and vise versa — to build the configuration model of actual configuration starting from its configuration graph.

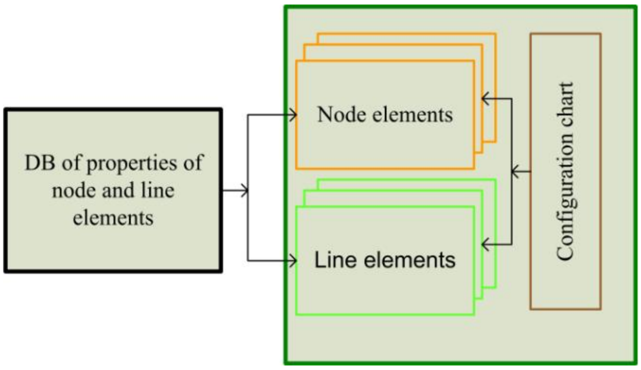


*Fig 2. The model of configuration of GTS*

**Model of configuration of GTS and its graph**

We consider GTS as a 3D structure of interconnected node and line elements, which links the set of inputs $\mathbf{W} = \{W_1, W_2, \ldots, W_{n_\mathbf{W}}\}$ with the set of outputs $\mathbf{O} = \{O_1, O_2, \ldots, O_{n_\mathbf{O}}\}$. The inputs $\mathbf{W}$ and outputs $\mathbf{O}$ are two categories of node elements that make up the set of external node elements. The set of internal node elements also can be divided into several categories, depending on their structure, technical features, operational parameters etc. Let $n_\mathbf{N}$ be the number of the categories of node elements, and $n_K$, where $K \in \{1, 2, \ldots, n_\mathbf{N}\}$, be the number of node elements of the category $K$. Then the set of node elements of the category $K$ is $\mathbf{N}^K = \{N_1^K, N_2^K, \ldots, N_{n_K}^K\}$, where $N_l^K$, $l \in \{1, 2, \ldots, n_K\}$ stands for a node element of category $K$. The set of categories of node elements is $\mathbf{C_N} = \{\mathbf{N}^1, \mathbf{N}^2, \ldots, \mathbf{N}^{n_\mathbf{N}}\}$. For definiteness, we consider the set of inputs $\mathbf{W}$ as $\mathbf{N}^1 \in \mathbf{C_N}$, and the set of outputs $\mathbf{O}$ as $\mathbf{N}^{n_\mathbf{N}} \in \mathbf{C_N}$. Obviously, the set of all node elements is determined as $\mathbf{N} = \bigcup_{K=1}^{n_\mathbf{N}} \mathbf{N}^K$.

Let $\mathbf{C_T} = \{\mathbf{T}^1, \mathbf{T}^2, \ldots, \mathbf{T}^{n_\mathbf{T}}\}$ be the set of categories of pipelines (line elements). Here $n_\mathbf{T}$ stands for the number of pipeline categories; the set of pipelines of category $L \in \{1, 2, \ldots, m_L\}$ is $\mathbf{T}^L = \{T_1^L, T_2^L, \ldots, T_{m_L}^L\}$, where $m_L$ is the number of line elements of category $L$. Then the set of line element is determined as $\mathbf{T} = \bigcup_{L=1}^{n_\mathbf{T}} \mathbf{T}^L$.

The configurtion chart determines connections between the node and line elements. This chart can also reflect the categories of node and line elements, numbering of these elements within categories etc.

Thus, the configuration model reflects the geometrical heterogeneity of GTS (presence of node and line elements in its structure) and its functional heterogeneity (presence of categories of node and line elements). The configuration chart of GTS can be presented by the graph $\mathbf{G} = (\mathbf{N}, \mathfrak{T})$. Its node set is the set $\mathbf{N}$ of node elements structured by the set $\mathbf{C_N}$, and the set of edges is the set $\mathbf{T}$ of line elements structured by the set of pipeline categories $\mathbf{C_T}$: $\mathbf{N} = \bigcup_{K=1}^{n_\mathbf{N}} \mathbf{N}^K$, $\mathbf{T} = \bigcup_{L=1}^{n_\mathfrak{T}} \mathbf{T}^L$. The elements of each of sets $\mathbf{N}^K$ and $\mathbf{T}^L$ are numbered independently: $\mathbf{N}^K = \{N_1^K, N_2^K, \ldots, N_{n_K}^K\}$, $\mathbf{T}^L = \{T_1^L, T_2^L, \ldots, T_{m_L}^L\}$, where $n_K$ is the number of node elements of the category $K$, $m_L$ is the number of line elements of the category $L$, $n_\mathbf{N} = \sum_{K=1}^{n_\mathbf{N}} n_K = |\mathbf{N}|$, $n_\mathbf{T} = \sum_{L=1}^{n_\mathbf{T}} m_L = |\mathbf{T}|$.

The topological model reflects only the dimensional heterogeneity of GTS. It is presented as the graph $G = (\mathbf{X}, \mathbf{E})$, where $\mathbf{X}$ and $\mathbf{E}$ are the sets of vertices and edges. The set $\mathbf{X}$ presents all node elements and the set $\mathbf{E}$ presents all line elements of the configuration model: $|\mathbf{X}| = n_\mathbf{X}$, $|\mathbf{E}| = n_\mathbf{E}$.

To link a model configuration and topological model of the GTS, it is necessary to establish the correspondence $\mathbf{G} \to G$ between the graphs $\mathbf{G}$ and $G$. To do this, correspondences between sets of node elements $\mathbf{N}^K$, $K \in \{1, 2, \ldots, n_\mathbf{N}\}$ and the set of vertices $\mathbf{X} \subset G$ as well as between the sets of line elements $\mathbf{T}^L$, $L \in \{1, 2, \ldots, n_\mathbf{T}\}$ and the set of edges $\mathbf{E} \subset G$ should be established.

**Correspondences between the node elements and vertices of the configuration graph**

Let $\mathbf{I} = \{1, 2, \ldots, n_\mathbf{X}\}$ be a finite sequence of natural numbers, where $n_\mathbf{X} \equiv |\mathbf{X}| = |\mathbf{N}|$. Since the sets $\mathbf{I}$ and $\mathbf{X}$ of the same power, a one-to-one correspondence can be established between them. Any such correspondence $\mathbf{X} \leftrightarrow \mathbf{I}$ introduces a numbering of the vertices. This transforms the set $\mathbf{X}$ into the sequence $\mathbf{X} = \{X_l \ \forall l \in \mathbf{I}\}$, so we can address its elements using indexes: $X_i = \mathbf{X}_i$.

To map the sets of inputs $\mathbf{W}$ and the set of outputs $\mathbf{O}$ onto the set of vertices $\mathbf{X}$, we define the sequences $\mathbf{I}^\mathbf{W}$ and $\mathbf{I}^\mathbf{O}$ that contain the numbers of vertices of the graph $G$ corresponding to the inputs and outputs nodes:

$$\mathbf{I}^\mathbf{W} = \{l_1, l_2, \ldots, l_{n_\mathbf{W}}\}, \ l_i \in \mathbf{I}, \ i \in \{1, 2, \ldots n_\mathbf{W}\}. \quad (1)$$

$$\mathbf{I}^\mathbf{O} = \{l_1, l_2, \ldots, l_{n_\mathbf{O}}\}, \ l_k \in \mathbf{I}, \ k \in \{1, 2, \ldots, n_\mathbf{O}\}. \quad (2)$$

It is obvious that we can address elements of $\mathbf{I}^\mathbf{W}$ and $\mathbf{I}^\mathbf{O}$ by their indexes: $\mathbf{I}_i^\mathbf{W} = l_i \in \mathbf{I}$, $i \in \{1, 2, \ldots n_\mathbf{W}\}$, $\mathbf{I}_k^\mathbf{O} = l_k \in \mathbf{I}$, $k \in \{1, 2, \ldots n_\mathbf{O}\}$.

The sequences (1), (2) determine injections $\mathfrak{N}_{\mathbf{W}}:\mathbf{W}\to\mathbf{X}$ and $\mathfrak{N}_{\mathbf{O}}:\mathbf{O}\to\mathbf{X}$. Mapping $\mathfrak{N}_{\mathbf{W}}$ determines the index (number) $l_i\in\mathbf{I}^{\mathbf{W}}$ of the vertex $X_{l_i}\in\mathbf{X}$, that corresponds to the number $i\in\{1,2,\ldots,n_{\mathbf{W}}\}$ of the input $W_i\in\mathbf{W}$, and mapping $\mathfrak{N}_{\mathbf{O}}$ determines the index (number) $l_k\in\mathbf{I}^{\mathbf{O}}$ of the vertex $X_{l_k}\in\mathbf{X}$ that corresponds to the number $k\in\{1,2,\ldots,n_{\mathbf{O}}\}$ of the output $O_k\in\mathbf{O}$:

$$l_i=\mathfrak{N}_{\mathbf{W}}(i)\equiv\mathbf{I}_i^{\mathbf{W}},\ i\in 1,2,\ldots,n_{\mathbf{W}},\ l_i\in\mathbf{I},\quad l_k=\mathfrak{N}_{\mathbf{O}}(k)\equiv\mathbf{I}_k^{\mathbf{O}},\ k\in 1,2,\ldots,n_{\mathbf{O}},\ l_k\in\mathbf{I} \tag{3}$$

Hence the mapping $\mathfrak{N}_{\mathbf{W}}$ establishes correspondences between inputs $W\in\mathbf{W}$ and vertices $X\in\mathbf{X}$, whereas the mapping $\mathfrak{N}_{\mathbf{O}}$ establishes correspondences between outputs $O\in\mathbf{O}$ and vertices $X\in\mathbf{X}$

$$X=\mathfrak{N}_{\mathbf{W}}(W),\ W\in\mathbf{W},\ X\in\mathbf{X},\quad X=\mathfrak{N}_{\mathbf{O}}(O),\ X\in\mathbf{X},\ O\in\mathbf{O}. \tag{4}$$

So, the mappings $\mathfrak{N}_{\mathbf{W}}$ and $\mathfrak{N}_{\mathbf{O}}$ determine the sets $\mathbf{X}^{\mathbf{W}}\subset\mathbf{X}$ and $\mathbf{X}^{\mathbf{O}}\subset\mathbf{X}$ that correspond to the sets of inputs $\mathbf{W}$ and outputs $\mathbf{O}$ respectively:

$$\mathfrak{N}_{\mathbf{W}}(\mathbf{W})=\mathbf{X}^{\mathbf{W}}\equiv\{X_l\ \forall l\in\mathbf{I}^{\mathbf{W}}\},\qquad \mathfrak{N}_{\mathbf{O}}(\mathbb{O})=\mathbf{X}^{\mathbf{O}}\equiv\{X_l\ \forall l\in\mathbf{I}^{\mathbf{O}}\}. \tag{5}$$

Similar mappings can be introduced for other categories of the node elements. Commonly we will designate the sequence of the numbers of graph $G$ vertices, that correspond to the node elements $\mathbf{N}^K$ of the category $K$ as $\mathbf{I}^K$:

$$\mathbf{I}^K=\{l_1,l_2,\ldots,l_{n_K}\},\ l_i\in\mathbf{I},\ i=1,2,\ldots,n_K,\ K=1,2,\ldots,n_{\mathbf{N}}. \tag{6}$$

The sequence (6) defines the mapping $\mathfrak{N}_K$ that determines index (number) $l_i\in\mathbf{I}$ of the vertex $X_{l_i}\in\mathbf{X}$ corresponding to the index $i\in\{1,2,\ldots,n_K\}$ of the node element $N_i\in\mathbf{N}^K$ of $K$ category:

$$l=\mathfrak{N}_K(i)\equiv\mathbf{I}_i^K,\ i=1,2,\ldots,n_K,\ K=1,2,\ldots,n_{\mathbf{N}},\ l\in\mathbf{I}, \tag{7}$$

hence it determines correspondences between node elements $N_i\in\mathbf{N}^K$ of the configuration model $\mathbf{G}$ and vertices $X_l\in\mathbf{X}$ of graph $G$

$$X=\mathfrak{N}_K(N),\ N\in\mathbf{N}^K,\ X\in\mathbf{X}, \tag{8}$$

When the mapping $\mathfrak{N}_K$ acts on the set $\mathbf{N}^K$ it separates the corresponding subset $\mathbf{X}^K$ in the set $\mathbf{X}$:

$$\mathfrak{N}_K(\mathbf{N}^K)=\mathbf{X}^K\equiv\{X_l\ \forall l\in\mathbf{I}^K\}, \tag{9}$$

The collection $\{\mathfrak{N}_K,K=1,2,\cdots,n_{\mathbf{N}}\}=\mathfrak{N}$ constitutes the mapping $\mathfrak{N}$ that for each given node element $N_i\in\mathbf{N}^K\subset\mathbf{G}$ of a given category puts into correspondence a unique element $X_l\in\mathbf{X}\subset G$:

$$X_{\mathrm{K}}=\mathfrak{N}(N_i,K). \tag{10}$$

Each of the mappings $\mathfrak{N}_K$, $K=1,2,\cdots,n_{\mathbf{N}}$ is an injection from the set $\mathbf{N}_K$ of node elements of the $K$-category into the set $\mathbf{X}$ and a bijection between this category of node elements and corresponding to it subset of vertices $\mathbf{X}^K$:

$$\mathbf{N}^K\xrightarrow{\mathfrak{N}_K}\mathbf{X},\quad \mathbf{N}^K\overset{\mathfrak{N}_K}{\leftrightarrow}\mathbf{X}^K\subset\mathbf{X} \tag{11}$$

Hence for each $\mathfrak{N}_K$, $K=1,2,\ldots n_{\mathbf{N}}$ there exists an inverse mapping $\mathfrak{N}_K^{-1}$. For each given vertex $X\in\mathbf{X}_K$. it determines a unique node elements $N\in\mathbf{N}_K$ of category $K$:

$$N_i=\mathfrak{N}_K^{-1}(X_l),\ X_l\in\mathbf{X}^K,\ N_i\in\mathbf{N}^K,\ \mathbf{N}_K=\mathfrak{N}_K^{-1}(\mathbf{X}^{\mathbf{K}}) \tag{12}$$

To implement the mapping (12), it is necessary to find the index $i$ of the element in the sequence $\mathbf{I}^K$ the value of which equals $l$:

$$\mathfrak{N}_K^{-1}:\ X_l \to N_i \mid\ \mathbf{I}_i^K = l\ . \tag{13}$$

That can be realized by the search algorithm that need time proportional to $n_K$.

Whereas any two different subsets $\mathbf{X}^K$ and $\mathbf{X}^L$ do not intersect: $\mathbf{X}^K \bigcap \mathbf{X}^L = \varnothing\ \ \forall K, L \in \{1,2,\ldots n_{\mathbf{N}}\}, L \neq K$, and all subsets $\mathbf{X}^K\ \ K \in \{1,2,\ldots n_{\mathbf{N}}\}$ in the aggregate form the whole set $\mathbf{X}$ of vertices: $\bigcup_{K=1}^{n_{\mathbf{N}}} \mathbf{X}^K = \mathbf{X} \subset G$, the inverse mapping $\mathfrak{N}^{-1}$ exists, that for any vertex $X_l \in \mathbf{X} \subset G$ determines just one node elements $N_i \in \mathbf{N}^K$ of category $K$, such, that $\mathfrak{N}_K(N_i) = X_l$:

$$\mathfrak{N}^{-1}(X_l) = N_i \in \mathbf{N}^K \tag{14}$$

The mapping $\mathfrak{N}^{-1}$ can be implemented algorithmically through search the sequence $\mathbf{I}^K, K = 1,2,\ldots,n_{\mathbf{N}}$, that contains the element with value $l$, and subsequent determination of its index $i \in [1,2,\ldots,N_K]$ in this sequence:

$$\mathfrak{N}^{-1}:\ l \to (K,i) \mid\ \ K \in \{1,2,\ldots,n_{\mathbf{N}}\}, i \in \{1,2,\ldots,n_K\},\ \mathbf{I}_i^K = l \in \mathbf{I} \tag{15}$$

This algorithm is implemented by simple search, and it takes the time proportional to $n_{\mathbf{X}} n_K$.

The mapping $\mathfrak{N}$, together with its inverse one $\mathfrak{N}^{-1}$, establishes one-to-one correspondence between the set of node elements $\mathbf{N} \in \mathbf{G}$, structured by categories $\mathbf{C_N}$, and the set of vertices $\mathbf{X} \in G$. Due to this, we can consider the set $\mathbf{C_X}$ of vertex categories $\mathbf{C_X} = \left\{\mathbf{X}^K\ \ \forall K \in \{1,2,\ldots,n_{\mathbf{N}}\}\right\}$, where $\mathbf{X}^K = \mathfrak{N}_K\left(\mathbf{N}^K\right)$. The mapping $\mathfrak{N}$ establishes one-to-one correspondence between the sets of categories $\mathbf{C_N}$ and $\mathbf{C_X}$.

**Correspondences between the line elements and the edges of configuration graph**

Any edge $E \in \mathbf{E}$ of the graph $G$ can be specified by a pair of vertices $X_1, X_2 \in \mathbf{X}$ connected by the edge: $E = (X_1, X_2)$. Numeration of the vertices convert the set $\mathbf{X}$ into a sequence, hence, any edge $E \in \mathbf{E}$ can be specified by a pair of natural numbers $(i,\ j) \to E_{ij} \in \mathbf{E}$. We introduce a sequence of pairs of natural numbers:

$$\mathbf{J^E} = \left\{(i_1, j_1),(i_2, j_2),\cdots(i_\kappa, j_\kappa),\ldots,\left(i_{n_{\mathbf{E}}}, j_{n_{\mathbf{E}}}\right)\right\},\ i_k, j_k \in \mathbf{I}\ . \tag{16}$$

The members $i \in \mathbf{I}$ and $j \in \mathbf{I}$ of any pair $(i, j) \in \mathbf{J^E}$ are numbers of corresponding vertices $X_i \in \mathbf{X}$ and $X_j \in \mathbf{X}$.

The sequence (16) introduces a numbering on the set of edges and establishes the correspondence $\mathfrak{E}$ between the pairs $(i_\kappa, j_\kappa) \in \mathbf{J^E}$ and indexes $\kappa \in \mathbf{J} \equiv \{1,2,\ldots,n_{\mathbf{E}}\}$

$$\mathfrak{E}:\ \kappa \to (i, j), \tag{17}$$

If the graph $G$ does not contain parallel edges, the mapping (20) is a one-to-one correspondence. In this case, the inverse mapping $\mathfrak{E}^{-1}$ puts a unique index $\kappa \in \mathbf{J}$ of the edge into correspondence to the pair $(i, j) \in \mathbf{J^E}$ for each pair:

$$\mathfrak{E}^{-1}:\ (i, j) \to \kappa \mid\ J_\kappa^{\mathbf{E}} = (i, j) \tag{18}$$

Mapping (17) can be implemented by access to element of the sequence $\mathbf{J}^{\mathbf{E}}$ on its index: $(i_\kappa, j_\kappa) = \mathbf{J}^{\mathbf{E}}_\kappa$. Mapping (18) can be implemented algorithmically by search in the sequence $\mathbf{J}^{\mathbf{E}}$ the element with value $(i, j)$ and consequent determination of its index $\kappa$.

The sequence (16) specifies the set of edges $\mathbf{E}$ as collections of two-indexed $E_{ij}$ or one-indexed $E_\kappa$ elements and converts it into the sequence:

$$\mathbf{E} = \{E_{ij}\ \forall (i, j) \in \mathbf{J}^{\mathbf{E}}\},\ \ \mathbf{E} = \{E_\kappa\ \forall \kappa \in \mathbf{J}\} \tag{19}$$

When the graph $G$ contains parallel edges, the mapping (17) remains one-to-one correspondence, whereas mapping (18) can posses several values:

$$\mathcal{E}^{-1}:\ (i, j) \to \{\kappa_l, l = 1, 2, \ldots, m_{(i,j)}\} \mid J^{\mathbf{E}}_{\kappa_l} = (i, j),\ l = 1, 2, \ldots, m_{(i,j)} \tag{20}$$

where $m_{(i,j)}$ stands for number of the edges linking the vertices $i$ and $j$.

The mappings $\mathcal{E}$ and $\mathcal{E}^{-1}$ specify correspondences between two- and one-indexed edges. When the graph $G$ does not contain parallel edges, we have:

$$E_k = E_{ij} \mid (i, j) = \mathcal{E}(\kappa),\ \ E_{ij} = E_\kappa \mid \kappa = \mathcal{E}^{-1}(i, j). \tag{21}$$

To map the set of pipelines $\mathbf{T}$ of the configuration model $\mathbf{G}$ with edges $\mathbf{E}$ of the configuration graph $G$, for the each category $\mathbf{T}^K$, $K = 1, 2, \cdots, n_{\mathbf{T}}$ we introduce the sequence $\mathbf{J}^K$ that contains the numbers $\kappa \in \mathbf{J}$ of the edges corresponding to the pipelines of this category $T \in \mathbf{T}^K$:

$$\mathbf{J}^K = \{\kappa_1, \kappa_2, \ldots, \kappa_{m_K}\}\ \ \kappa_i \in \mathbf{J},\ i = 1, 2, \ldots, m_K. \tag{22}$$

The sets of different sequences $\mathbf{J}^K$ and $\mathbf{J}^L$ do not intersect: $\mathbf{J}^K \cap \mathbf{J}^L = \varnothing,\ \forall K, L \in \{1, 2, \ldots n_{\mathbf{T}}\}$, $K \neq L$ and in the aggregate $\mathbf{J}^K$, $K \in \{1, 2, \ldots n_{\mathbf{T}}\}$ form the set the edge indexes $\mathbf{J}$: $\bigcup_{K=1}^{n_{\mathbf{T}}} \mathbf{J}^K = \mathbf{J}$.

The sequence $\mathbf{J}^K$ specifies a mapping $\mathcal{P}_K$ that for any given index $i$ of a pipeline $T_i \in \mathbf{T}^K$ of the category $K$ puts the number $\kappa \in \{1, 2, \ldots, n_{\mathbf{E}}\}$ of the edge $E_\kappa \in \mathbf{E}$ into correspondence to this pipeline: $\kappa = \mathcal{P}_K(i)$. Since any index $i \in \{1, 2, \ldots, m_K\}$ specifies a unique pipeline $T_i \in \mathbf{T}^K$, and the index $\kappa = \mathcal{P}_K(i)$, $\kappa \in \{1, 2, \ldots, n_{\mathbf{E}}\}$ specifies a unique vertex $E_\kappa \in \mathbf{E}$, the mapping $\mathcal{P}_K$ determines the injection

$$\mathcal{P}_K : T_i \to E_\kappa,\ T_i \in \mathbf{T}^K,\ E_\kappa \in \mathbf{E} \tag{23}$$

and specifies the subset $\mathbf{E}^K \subset \mathbf{E}$ corresponding to the set of pipelines $\mathbf{T}^K$ of category $K$:

$$\mathbf{E}^K = \mathcal{P}_K(\mathbf{T}^K) = \{E_\kappa\ \forall \kappa \in \mathbf{J}^K\},\ \ K = 1, 2, \ldots, n_{\mathbf{T}} \tag{24}$$

The mappings $\mathcal{P}_K$, $K \in \{1, 2, \ldots, n_{\mathbf{E}}\}$ are bijections between the sets $\mathbf{T}^K$ and $\mathbf{E}^K$, hence the inverse mappings $\mathcal{P}_K^{-1}$ exist:

$$\mathcal{P}_K^{-1}(E_\kappa) = T_i \Big|\ T_i \in \mathcal{T}^K \wedge \mathcal{P}_K(i) = \kappa,\ \ K = 1, 2, \ldots, n_{\mathbf{T}} \tag{25}$$

The set $\mathcal{P} = \{\mathcal{P}_K\ K = 1, 2, \ldots, n_{\mathbf{T}}\}$ of the mappings $\mathcal{P}_K$ forms the mapping that for any given pipeline $T_i \in \mathbf{T}^K$, $i \in \{1, 2, \ldots, m_K\}$ of category $K = 1, 2, \ldots, n_{\mathbf{T}}$ determines a unique edge $E_\kappa \in \mathbf{E}$: $E_\kappa = \mathcal{P}(T_i, K)$.

The sets $\mathbf{E}^K$ do not mutually intersect: $\mathbf{E}^K \cap \mathbf{E}^L = \varnothing\ \ \forall K, L \in \{1, 2, \ldots, n_{\mathbf{T}}\}, K \neq L$ and they form in the aggregate the set $\mathbf{E}$ of all edges: $\bigcup_{K=1}^{n_{\mathbf{T}}} \mathbf{E}^K = \mathbf{E}$. Hence the inverse mapping $\mathcal{P}^{-1}$ exists. It determines, for any given edge $E_\kappa \in \mathbf{E}$ of the configuration graph $G$ a unique pipeline $T_i \in \mathbf{T}^K$ of the category $K \in \{1,2,\ldots,n_{\mathbf{T}}\}$ and number $i \in \{1,2,\ldots m_K\}$ of the pipe in this category:

$$\mathcal{P}^{-1}(E_\kappa) = (T_i, K) \big| \left(\mathcal{P}_K(T_i) = E_\kappa,\right) \wedge \left(\mathbf{J}_i^K = \kappa\right). \tag{26}$$

Algorithmic implementation of the mapping $\mathcal{P}^{-1}$ includes a search of the sequence $\mathbf{J}^K$ which contains an element containing an element which equals $\kappa$ among the sequences $\mathbf{J}^1$, $\mathbf{J}^2$, …, $\mathbf{J}^{n_{\mathbf{T}}}$. The number $i = \mathcal{P}_K^{-1}(\kappa)$ of the found element $\kappa$ in the sequence $\mathbf{J}^K$ is the number of the pipeline $T_i \in \mathfrak{T}^K$ corresponding to the edge $E_\kappa$.

The mapping $\mathcal{P}$ specifies the one-to-one correspondence between the set of line elements $\mathbf{T} \in \mathbf{G}$, structured by the categories $\mathbf{C_T}$, and the set of the edges $\mathbf{E} \in G$. It also specifies the set $\mathbf{C_E} = \left\{\mathbf{E}^1, \mathbf{E}^2, \ldots, \mathbf{E}^{n_{\mathbf{E}}}\right\}$ of edge categories which correspond to the categories of the pipelines $\mathbf{C_T}$, and establishes one-to-one correspondence: $\mathbf{C_E} \xleftrightarrow{\mathcal{P}} \mathbf{C_T}$ between them.

**Data structures of the topological model**

The configuration graph $G$ is completely determined by the sequence $\mathbf{J^E}$. It can be represented in computer memory by the following structure:

$$\mathbf{J^E}(G):\ \mathbf{array}\left[1..N_{\mathbf{E}}\right] \mathbf{of\ record}\, i, j : \left[1..N_{\mathbf{X}}\right] \mathbf{end\ record}.$$

It needs memory capacity of the order $O(2n_{\mathbf{E}})$

With the use of $\mathbf{J^E}$, some other structures can be also calculated for numeric representation of the configuration graph $G$, in particular, adjacency matrix $\mathbf{A}(G)$ and incidence matrix $\mathbf{B}(G)$.

We will restrict here by the case of non-oriented graph $G$ that does not contain parallel edges.

Then entries $a_{ij}$ of adjacency matrix $\mathbf{A}(G)$ can be determined via the set of edges as it is in[5]:

$$a_{ij} = \begin{cases} 1\ \forall i, j : \left(E_{ij} \in \mathbf{E} \vee E_{ji} \in \mathbf{E}\right) \\ 0\ \forall i, j : \left(E_{ij} \notin \mathbf{E} \wedge E_{ij} \notin \mathbf{E}\right) \end{cases} \tag{27}$$

To represent the matrix $\mathbf{A}(G)$ in computer memory, the 2D array can be used

$$\mathbf{A}(G) : \mathbf{array}\left[1..N_{\mathbf{X}}, 1..N_{\mathbf{X}}\right] \mathbf{of}\ 0..1$$

It requires memory of the order $O\left(n_{\mathbf{X}}{}^2\right)$.

This matrix can be easily calculated with the use of the structure $\mathbf{J^E}$:

$$a_{ij} = \begin{cases} 1\ \forall (i, j) \in \mathbf{J^E} \vee \forall (j, i) \in \mathbf{J^E} \\ 0\ \forall (i, j) \notin \mathbf{J^E} \wedge \forall (j, i) \notin \mathbf{J^E} \end{cases} \tag{28}$$

Calculations should be done for each pair $(i, j) \in \mathbf{J^E}$. That takes the time of the order $O(n_{\mathbf{E}})$.

The incidence matrix $\mathbf{B}(G)$ is a $n_{\mathbf{X}} \times n_{\mathbf{E}}$-matrix $\mathbf{B} = \{b_{i\kappa} \in \{0,1\}, \forall i \in \mathbf{I}, \forall \kappa \in \mathbf{J}\}$. Its $i$-th row $b_{i\kappa}\ \forall \kappa \in \{1, 2, \cdots, \mathbf{J}\}$ specifies the set $\mathbf{E}^{(i)}$ of edges incident to the vertex $X_i$ [5]. To represent the matrix $\mathbf{B}(G)$ in computer memory, 2D array can be used [5]:

$$\mathbf{B}(G):\ \mathbf{array}\left[1..n_{\mathbf{X}}, 1..n_{\mathbf{E}}\right] \mathbf{of}\ 0..1,$$

It requires memory of the order $O\left(n_{\mathbf{X}} n_{\mathbf{E}}\right)$.

To built the matrix $\mathbf{B}(G)$ for non-oriented graph $G$ using the structure $\mathbf{J}^{\mathbf{E}}$, it is necessary to determine the set $\mathbf{E}^{(i)}$ of the edges incident to this vertex for each vertex $X_i \in \mathbf{X}$. The set $\mathbf{J}^{(i)}$ of these edges' indexes can be calculated with the use of the mapping $\mathcal{E}^{-1}$, and the set $\mathbf{E}^{(i)}$ can be calculated as

$$\mathbf{E}^{(i)} = \left\{ E_\kappa \ \forall \kappa \in \mathbf{J}^{(i)} \right\}. \qquad (29)$$

With the use of the sets $\mathbf{J}^{(i)}$ or $\mathbf{E}^{(i)}$ the elements $b_{i\kappa}$ of the matrix $\mathbf{B}(G)$ can be calculated

$$b_{i\kappa} = \begin{cases} 1 \, \forall i, \kappa \mid \left( i \in \mathbf{I} \wedge \kappa \in \mathbf{J}^{(i)} \right) \\ 0 \, \forall i, k \mid \left( i \in \mathbf{I} \wedge \kappa \notin \mathbf{J}^{(i)} \right) \end{cases}, \qquad b_{i\kappa} = \begin{cases} 1 \, \forall i, \kappa \mid \left( i \in \mathbf{I} \wedge E_\kappa \in \mathbf{E}^{(i)} \right) \\ 0 \, \forall i, k \mid \left( i \in \mathbf{I} \wedge E_\kappa \notin \mathbf{E}^{(i)} \right) \end{cases} \qquad (30)$$

To build the sets $\mathbf{J}^{(i)} \ \forall i \in \mathbf{I}$ according to the formula (32), time of the order $O\left(n_{\mathbf{X}} n_{\mathbf{E}}\right)$ is needed. Thus, time needed for calculation of the matrix $\mathbf{B}(G)$ by Formula (34) is of the order $O\left(n_{\mathbf{X}}^2 n_{\mathbf{E}}\right)$.

## Conclusions

A two-level model of GTS structure has been suggested. The model can be used under development of software intended for automation of management by GTS. The configuration model is the upper level of the GTS structure model. It represents the configuration of GTS as a collection of interconnected node and line elements that link the sets of inputs and outputs. Physical properties of the node and line elements are taken into consideration at this level. For this purpose, the sets of node and line elements both are structured according to their categories. Thus, the upper level of the model reflects the structural heterogeneity of GTS (presence of the node and line elements in its structure) and its physical inhomogeneity (the categories of node and line elements are taken into consideration).

At the lower level, the model takes into account just dimensional heterogeneity of GTS. It is presented as a graph reflecting the topology of GTS as a collection of vertices and edges.

The mappings establishing correspondences between the objects of models of different levels have been built. This enables to delimit the scopes of variables corresponding to the object of physical and topological models in the program environment. The variables of the upper level can be made visible just in interface modules of the program. The variables of the lower level can be made visible only in internal modules of the program. Due to that, users receive possibilities for editing of the configuration model: adding and removing objects of the physical model, applying independent numeration for the object of different categories, applying different geometric primitives and colors for their displaying, simple transformation of configuration etc.

The objects of the model can be presented in a computer by means of simple data structures which do not require substantial storage capacity. The mappings which establish the correspondences between objects of different level can be implemented with the use of fast search algorithms.

*1. Химко М.П., Фролов В. А.., Павленко В.А., П'янило Я.Д., Притула Н.М. Розрахунок параметрів газотранспортних систем. // Науково-виробничий журнал України "Нафтова и газова промисловість". - 2006 р. №3. - С.33-37.2. Чекурін В. Ф. До побудови програмної системи для моделювання та оптимізації процесів транспортування природного газу// Фізико-математичне моделювання та інформаційні технології, 2007, №5,- С. 158-169. 3. Herran-Gonzalez A., De La Cruz J. M., De Andres-Toro B, and Risco-Martın J. L. Modeling and simulation of a gas distribution pipeline network// Applied Mathematical Modelling, 33(3):1584–1600,2009. 4. Кристофидес Н. Теория графов. Алгоритмический подход. «Мир», М., 1978. 5. Новиков Ф.А. Дискретная математика для программистов. «Питер», СПб, 2000.*