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U. Dzelendzyak[1], **V. Samotyy**[1,2], **I. Dzelendzyak**[3]
[1.] Lviv Polytechnic National University
The Department of computerized automatics' systems
[2.] Cracow University of Technology
The Department of Automation and Information Technology
[3.] Lviv Polytechnic National University
The Department of Marketing and Logistic

SOME DEPENDENCE OF FIBONACCI'S NUMBERS AND GOLDEN CHOPPING

New analytical dependences were established for golden chopping and for polynomials with infinite number of members. It is shown how we can determine analytically infinite polynomial in which the argument is the golden chopping using a Taylor's series. The expressions for calculating these polynomials in which the coefficients are numbers in the Fibonacci's series are displayed.

Key words: Fibonacci's algorithm, golden chopping, power series, difference, divider.

Introduction

In 1202 Italian mathematician Leonardo of Pisa also known as Fibonacci (which means son of Bonacci) wrote a book "Liber abacci" ("Book about abacus") [1]. With this book Europeans first learned of Hindu ("Arabic") numerals, as well as the Fibonacci's sequence.

The Fibonacci's sequence is expressed as as $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$, $F_7 = 13$, $F_8 = 21$, $F_9 = 34$, ... , $F_{77} = 5527939700\,884757$, $F_{78} = 8944394323\,791464$, ... We deal with a game-theoretic framework [2] involving a finite number of infinite populations, members of which have a finite number of available strategies. The payoff of each individual depends on her own action and distributions of actions of individuals in all populations. Fischer's concept, which is presented in [3], is an attempt to use Fibonacci's numbers for constructing the method of market behavior forecast taking into consideration the aspects of price and time. The wide range of Fibonacci's numbers application, especially in statistics, sports, non-Euclidean geometry, RSA codes, coloring of geographical maps, etc. are presented in [4]. Fibonacci's algorithm has also been applied in technical fields. In particular, new algorithms of analog-to-digital conversion are an important outcome of this theory [5], which became the basis for the design of advanced analog-to-digital converters. Fibonacci numbers are used in automatic lines [6], which use a method of determining the state space of automatic lines with storage components based on a Fibonacci number. The obtained formulas give a theoretical basis for constructing mathematical models of automatic lines' complex. Gold sectional are also used in telecommunications theory [7, 8]. The new logical-mathematical tools for modeling systems "man-machine-environment" in telecommunications are shown, and its application in the theories of linear and nonlinear filtering and in solving special problems searching are also shown. In [9] the method of matrix encryption / decryption numerical information using the sequence of Fibonacci numbers are shown, where classical mathematical tools - the theory of matrices is used. A method of detecting and correcting errors in an encrypted matrix, errors which happen in communication channels, is proposed. In this procedure, natural decimals of different sizes are corresponding objects of correcting. It has a principal meaning for the development of theory coding of information.

The general formula of building this sequence is as follows

$$F_n = F_{n-1} + F_{n-2},\ n = 2,\ 3,\ ...,\ F_0 = 0,\ F_1 = 1. \quad (1)$$

Dividing F_{n-1} / F_n golden chopping is obtained.

It turns out that no matter whether this sequence consists of integers or real numbers, F_{n-1} / F_n ratio will always give golden chopping, which is aperiodic irrational number. And the greater the number of steps N the more accurately one can calculate golden chopping. For example, taking a sequence of Fibonacci ratio F_4 / F_5=3/5=0.6 and increasing N to 78 we will have

$$F_{77} / F_{78}=5527939700884757/8944394323791464\approx 0.61803398874989484820458. \quad (2)$$

It is of no import which will be the start value of F_0, F_1. It may be real numbers. Moreover you can pass $F_0 > F_1$. For example, let it be $F_0 = 120.4$, $F_1 = 13.8$. Then according to (1) we can construct the following sequence: 134.2, 148, 282.2, 430.2, 712.4, 1142.6, 1855, 2997.6, 4852.6, 7850.2, 12702.8 20553, 33255.8, 53808.8, 87064.6, 140873.4, 227938, 368811.4, 596749.4, 965560.8, … .

Dividing the last two numbers, we also obtain the golden chopping with a certain approximation, which depends on the number of the sequence members

$$F_{18} / F_{19}=596749.4/965560.8 \approx 0.618\ 033\ 996\ 4. \quad (3)$$

The preliminary version of this paper was presented as the publication [10].

1. Basic relations for golden chopping

Golden chopping is marked as

$$Z \approx 0.618\ 033\ 988\ 749\ 894\ 848\ 204. \quad (4)$$

The notation difference is introduced

$$R = 1 - Z = 0.381\ 966\ 011\ 250\ 105\ 151\ 796. \quad (5)$$

The divider is marked as

$$D = 1/Z = 1.618\ 033\ 988\ 749\ 894\ 848\ 204. \quad (6)$$

The main dependencies for Z, R and D are written down

$$1/Z = Z + 1, \quad (7)$$

$$1/R = 2 + Z. \quad (8)$$

Substituting (7) in (6), we obtain

$$D = Z + 1. \quad (9)$$

If you subtract from (8) (7), we obtain

$$1/R - 1/Z = 1. \quad (10)$$

Substituting (5) into (10) and multiplying the received result by Z

$$\frac{Z}{1-Z} = 1 + Z \quad (11)$$

or

$$Z = (1-Z)(1+Z). \quad (12)$$

Expression (12) gives us a quadratic equation

$$Z^2 + Z - 1 = 0. \quad (13)$$

The solution of this equation is

$$Z_1 = -D, \ Z_2 = Z. \quad (14)$$

The consequence of this solution is $Z_1 Z_2 = -1$, as a consequence of equation (13) is

$$\frac{Z^2}{1-Z}=1. \qquad (15)$$

Substituting (6) into (9)

$$D=1+1/D, \qquad (16)$$

which formulates quadratic equation

$$D^2-D-1=0. \qquad (17)$$

The solution of this equation is

$$D_1=D,\ D_2=-Z. \qquad (18)$$

The consequence of this interpretation is $D_1D_2=-1$, as a consequence of equation (17) is

$$\frac{D^2}{1+D}=1. \qquad (19)$$

2. The basic properties of power series of golden chopping

Try to consider some properties of power series, where the argument is the golden chopping, or difference. To be golden chopping the dependence, which is a consequence of the Taylor's series will be fair

$$\sum_{k=1}^{\infty} Z^k=\frac{Z}{1-Z}. \qquad (20)$$

By comparing (11) and (20), we obtain

$$\sum_{k=1}^{\infty} Z^k=1+Z. \qquad (21)$$

By comparing (7) and (21), we obtain

$$\sum_{k=1}^{\infty} Z^k=\frac{1}{Z}. \qquad (22)$$

The consequence of these equations are the following ratio

$$\frac{Z}{1-Z}=\frac{1}{Z}=1+Z. \qquad (23)$$

Subtracting from (21) Z, that is the first term of the series and taking into account relationship (20), (22), we obtain

$$\sum_{k=2}^{\infty} Z^k=1=\frac{1-Z^2}{Z}=\frac{Z^2}{1-Z}. \qquad (24)$$

Equation (13) gives us the dependence

$$Z^2=1-Z. \qquad (25)$$

Substituting (5) into (25), we obtain

$$Z^2 = R. \qquad (26)$$

Subtracting from (24) Z^2 and taking into account the dependence (26), we have

$$\sum_{k=3}^{\infty} Z^k = 1 - R. \qquad (27)$$

Taking into account that $Z = 1 - R$, we have

$$\sum_{k=3}^{\infty} Z^k = Z. \qquad (28)$$

Multiplying left and right side of (13) on Z

$$Z^3 + Z^2 = Z. \qquad (29)$$

Adding to the left and right side of (29) Z

$$Z + Z^2 + Z^3 = 2Z. \qquad (30)$$

It means that the first three members of the sum (22) can be omitted subtracting from the right side value $2Z$. Then the amount will be started from the fourth element

$$\sum_{k=4}^{\infty} Z^k = \frac{1}{Z} - 2Z = D - 2Z. \qquad (31)$$

Taking into account that $D = Z + 1$ (see (7), (9)) expression (31) can be written as

$$\sum_{k=4}^{\infty} Z^k = 1 - Z = R. \qquad (32)$$

There are several polynomial dependencies golden chopping. One is added to (30) and the received result is multiplied on Z

$$Z + Z^2 + Z^3 + Z^4 = 2Z^2 + Z. \qquad (33)$$

Taking into account the dependence (26) the last expression takes the form

$$Z + Z^2 + Z^3 + Z^4 = 2R + Z. \qquad (34)$$

According to (5) $R + Z = 1$, so (34) can be written differently

$$Z + Z^2 + Z^3 + Z^4 = R + 1. \qquad (35)$$

Subtracting polynomials in (22) according to (35) the first 4 members, and from the right side $R+1$, then the polynomial (22) can be written as

$$\sum_{k=5}^{\infty} Z^k = \frac{1}{Z} - 1 - R. \qquad (36)$$

Substituting (5), (7) into (36) we obtain

$$\sum_{k=5}^{\infty} Z^k = 2Z - 1. \qquad (37)$$

One is added in both parts of equation (35) and the received result is multiplied on Z

$$Z + Z^2 + Z^3 + Z^4 + Z^5 = ZR + 2Z\,, \tag{38}$$

Substituting (5) into (38), we obtain

$$Z + Z^2 + Z^3 + Z^4 + Z^5 = Z(1-Z) + 2Z \tag{39}$$

or

$$Z + Z^2 + Z^3 + Z^4 + Z^5 = 3Z - Z^2\,. \tag{40}$$

Substituting (25) into (40), we obtain

$$Z + Z^2 + Z^3 + Z^4 + Z^5 = 4Z - 1\,. \tag{41}$$

Subtracting polynomials in (22) according to (41) the first 5 members, and from the right side $4Z-1$, then the polynomial (22) can be written as

$$\sum_{k=6}^{\infty} Z^k = \frac{1}{Z} - 4Z + 1\,. \tag{42}$$

Substituting (7) into (42) and obtain the final result

$$\sum_{k=6}^{\infty} Z^k = 3(1-Z)\,. \tag{43}$$

One is added in both parts of equation (41) and the received result is multiplied on Z :

$$Z + Z^2 + Z^3 + Z^4 + Z^5 + Z^6 = 4Z^2\,. \tag{44}$$

Substituting (25) into (44), we have

$$Z + Z^2 + Z^3 + Z^4 + Z^5 + Z^6 = 4(1-Z)\,. \tag{45}$$

Subtracting polynomials in (22) according to (45) the first 6 members, and from the right side $4(1-Z)$, then the polynomial (22) can be written as

$$\sum_{k=7}^{\infty} Z^k = \frac{1}{Z} - 4(1-Z)\,. \tag{46}$$

Substituting (7) into (46) and obtain the final result

$$\sum_{k=7}^{\infty} Z^k = 5Z - 3\,. \tag{47}$$

One is added in both parts of equation (45) and the received result is multiplied on Z

$$Z + Z^2 + Z^3 + Z^4 + Z^5 + Z^6 + Z^7 = 5Z - 4Z^2 \tag{48}$$

or

$$Z + Z^2 + Z^3 + Z^4 + Z^5 + Z^6 + Z^7 = 9Z - 4\,. \tag{49}$$

Subtracting polynomials in (22) according to (49) the first 7 members, and from the right side $9Z-4$, then the polynomial (22) can be written as

$$\sum_{k=8}^{\infty} Z^k = \frac{1}{Z} - 9Z + 4\,. \tag{50}$$

Substituting (6), (7) into (50) and we obtain the final result

$$\sum_{k=8}^{\infty} Z^k = 5 - 8Z \;. \tag{51}$$

One is added in both parts of equation (49) and the received result is multiplied on Z

$$Z + Z^2 + Z^3 + Z^4 + Z^5 + Z^6 + Z^7 + Z^8 = 9Z^2 - 3Z \;. \tag{52}$$

Taking into account (25), we have

$$Z + Z^2 + Z^3 + Z^4 + Z^5 + Z^6 + Z^7 + Z^8 = 9 - 12Z \,. \tag{53}$$

Subtracting polynomials in (22) according to (53) the first 8 members, and from the right side $9-12Z$, then the polynomial (22) can be written as

$$\sum_{k=9}^{\infty} Z^k = \frac{1}{Z} - 9 + 12Z \;. \tag{54}$$

Substituting (7) into (49) and we obtain the final result

$$\sum_{k=9}^{\infty} Z^k = 13Z - 8 \;. \tag{55}$$

These considerations can be continued to identify any partial amount. The grouped results are written

$$\left.\begin{array}{l} \sum_{k=1}^{\infty} Z^k = 1+Z,\ \sum_{k=2}^{\infty} Z^k = 1,\ \sum_{k=3}^{\infty} Z^k = Z,\ \sum_{k=4}^{\infty} Z^k = 1-Z,\ \sum_{k=5}^{\infty} Z^k = 2Z-1, \\ \sum_{k=6}^{\infty} Z^k = 2-3Z,\ \sum_{k=7}^{\infty} Z^k = 5Z-3,\ \sum_{k=8}^{\infty} Z^k = 5-8Z,\ \sum_{k=9}^{\infty} Z^k = 13Z-8,\ldots \end{array}\right\} . \tag{56}$$

We must remember that the Fibonacci's series are defined as $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3 \ldots$. After dependence analysis (56) we can make a deductive conclusion that the coefficients of these expressions is the number of the Fibonacci's series, namely

$$\sum_{k=n}^{\infty} Z^k = (-1)^{n-1}(F_{n-2}Z - F_{n-3}),\ n = 3,\ 4,\ 5,\ \ldots\ . \tag{57}$$

Here we observe a sign change, plus and minus constantly alternating. It is a direct manifestation of the Elliott's wave theory [11], sometimes it is called the rule of alternation. It can be formulated in such a way: complex corrective waves alternate with simple, and strong pulse waves with weak corrective waves.

This output gives another set of expressions, namely (25), (30) (35) (41) (45) (49), (53). We write them as a separate group of formula

$$\left.\begin{array}{l} \sum_{k=1}^{2} Z^k = 1,\ \sum_{k=1}^{3} Z^k = 2Z,\ \sum_{k=1}^{4} Z^k = 2-Z,\ \sum_{k=1}^{5} Z^k = 4Z-1, \\ \sum_{k=1}^{6} Z^k = 4(1-Z),\ \sum_{k=1}^{7} Z^k = 9Z-4,\ \sum_{k=1}^{8} Z^k = 9-12Z,\ldots \end{array}\right\} . \tag{58}$$

The coefficients of these expressions are also subject to Elliott's wave theory. These expressions can be written as recurrent formula

$$\sum_{k=1}^{n} Z^k = S_{n-2} + S_{n-1}Z,\ n = 2,\ 3,\ldots,\ S_0 = 1,\ S_1 = 0. \tag{59}$$

We also calculate another coefficients S_2, S_3, ... using Fibonacci's algorithm, which is based on Elliott's wave theory, namely

$$S_n = (-1)^n (S_{n-1}(-1)^{n-1} + S_{n-2}(-1)^n + (-1)^n) \quad . \tag{60}$$

This expression has no analytical output and is purely heuristic in nature, like expression (57).

If expression (59) substitute the value of golden chopping Z and calculate it for different values n, you can get such a sequence of irrational numbers (confine three digits after the decimal point)

$$1,\ 1.236,\ 1.382,\ 1.472,\ 1.528,\ 1.562,\ 1.584,\ \ldots \quad , \tag{61}$$

which according to (21) asymptotically approaching values of

$$1+Z=1.618\ 033\ 988\ 749\ 894\ 848\ 204\ 58\ .$$

In practice it is very convenient because it saves computer time. There is no need to calculate the amount of power series, if it can be replaced by several arithmetic operations of multiplication, addition and subtraction.

3. The basic properties of power series differences

You can build a similar amount (21) for the differences. According to Taylor's series we have

$$\sum_{k=1}^{\infty} R^k = \frac{R}{1-R}\,. \tag{62}$$

Taking into account the dependence (10), we have

$$\frac{R}{1-R} = Z\ . \tag{63}$$

Substituting (63) into (62), we obtain

$$\sum_{k=1}^{\infty} R^k = Z\ . \tag{64}$$

Subtract from the left and right side of (64) R

$$\sum_{k=2}^{\infty} R^k = Z - R\,. \tag{65}$$

Adding to the left and right sides of (5) the difference R

$$R + R = 1 - Z + R,\ \rightarrow\ Z - R = 1 - 2R. \tag{66}$$

Substituting (5) into (66)

$$Z - R = 2Z - 1\,. \tag{67}$$

Substituting (67) into (65)

$$\sum_{k=2}^{\infty} R^k = 2Z - 1\,. \tag{68}$$

Adding to the left and right side of (5) R^2

$$R + R^2 = 1 - Z + (1-Z)^2\,. \tag{69}$$

Given the dependence (25), we have

$$R + R^2 = 3 - 4Z\ . \tag{70}$$

Subtract from (64) expression (70)

$$\sum_{k=3}^{\infty} R^k = 5Z - 3 . \quad (71)$$

One is added to the left and the right side of (70) and the received result is multiplied on R :

$$R + R^2 + R^3 = 4R(1 - Z) . \quad (72)$$

Taking into account the dependence (5), (26) expression (72) takes the form

$$R + R^2 + R^3 = 4(2 - 3Z) . \quad (73)$$

After subtracting (73) from (64) we obtain:

$$\sum_{k=4}^{\infty} R^k = 13Z - 8 . \quad (74)$$

One is added to the left and the right side of (74) and the received result is multiplied on R :

$$R + R^2 + R^3 + R^4 = 3R(3 - 4Z) . \quad (75)$$

Taking into account the dependence (5), (26) expression (75) takes the form

$$R + R^2 + R^3 + R^4 = 3(7 - 11Z) . \quad (76)$$

After subtracting (76) from (64) we obtain:

$$\sum_{k=5}^{\infty} R^k = 34Z - 21 . \quad (77)$$

These considerations can be continued and any partial amount can be identified. We write the grouped results

$$\sum_{k=1}^{\infty} R^k = Z, \ \sum_{k=2}^{\infty} R^k = 2Z - 1, \ \sum_{k=3}^{\infty} R^k = 5Z - 3, \ \sum_{k=4}^{\infty} R^k = 13Z - 8, \ \sum_{k=5}^{\infty} R^k = 34Z - 21, \ \dots \ . \quad (78)$$

The coefficients of expressions (78) can be written through a series of Fibonacci's numbers

$$\sum_{k=n}^{\infty} R^k = F_{2n-1} Z - F_{2n-2}, \ n = 1, \ 2, \ \dots \ . \quad (79)$$

This output gives another set of expressions, namely (5), (70) (73) (76). We write them a separate group of formula

$$R = 1 - Z, \ R + R^2 = 3 - 4Z, \ R + R^2 + R^3 = 4(2 - 3Z), \ R + R^2 + R^3 + R^4 = 3(7 - 11Z), \dots \quad (80)$$

The coefficients of expressions (80) can be written through a series of Fibonacci's numbers

$$\sum_{k=1}^{n} R^k = F_{2n} - (F_{2n+1} - 1)Z, \ n = 1, \ 2, \ \dots \ . \quad (81)$$

If we compare expressions (56) and (78), we can note the following pattern

$$\left.\begin{aligned} &\sum_{k=3}^{\infty} Z^k = \sum_{k=1}^{\infty} R^k = Z, \quad \sum_{k=5}^{\infty} Z^k = \sum_{k=2}^{\infty} R^k = 2Z - 1, \\ &\sum_{k=7}^{\infty} Z^k = \sum_{k=3}^{\infty} R^k = 5Z - 3, \quad \sum_{k=9}^{\infty} Z^k = \sum_{k=4}^{\infty} R^k = 13Z - 8 \end{aligned}\right\} . \quad (82)$$

The obtained dependences can be summarized

$$\sum_{k=2n+1}^{\infty} Z^k = \sum_{k=n}^{\infty} R^k, \ n = 1, \ 2, \ \dots \ . \quad (83)$$

4. Properties of generalized power series

We must take into account that except the golden chopping Z which according to (19) when adding the power series gives one, other power series also can be built. According to Taylor's series during adding these power series also will asymptotically approach to one, for example

$$\sum_{k=1}^{\infty}\frac{1}{2^k}=1. \qquad 4)$$

The following formula is fair for 1/3, namely

$$2\sum_{k=1}^{\infty}\frac{1}{3^k}=1 \qquad (85)$$

and

$$3\sum_{k=1}^{\infty}\frac{1}{4^k}=1 \qquad (86)$$

and

$$4\sum_{k=1}^{\infty}\frac{1}{5^k}=1. \qquad (87)$$

Analysis of the obtained amount of general formula can be written

$$(N-1)\sum_{k=1}^{\infty}\frac{1}{N^k}=1. \qquad (88)$$

We can make a more general conclusion: the formula (88) is fair for all real (and not only integer) N which satisfy the condition $N \geq 2$.

Try to explore the dependence if $1 < N < 2$. Based on the Taylor's series we again obtain again depending

$$1/2\sum_{k=1}^{\infty}1.5^{-k}=1, \qquad (89)$$

$$1/5\sum_{k=1}^{\infty}1.2^{-k}=1, \qquad (90)$$

$$1/10\sum_{k=1}^{\infty}1.1^{-k}=1. \qquad (91)$$

The received amount have been analysed and then it can be written in general formula

$$(1/N)\sum_{k=1}^{\infty}(1+1/N)^{-k}=1. \qquad (92)$$

This formula also covers written above formulas, namely (84) - (91). Here $N > 0$ is a real number. So substituting in formula (92) N =1 we obtain the expression (84), and at N =0.25 we will obtain the expression (87). Formula (92) can be written in another form, replacing the negative exponent to positive

$$(1/N)\sum_{k=1}^{\infty}\left[\frac{N}{1+N}\right]^k=1. \qquad (93)$$

This formula can be written in a more general form

$$\sum_{k=1}^{\infty}\left[\frac{N}{M+N}\right]^k=\frac{N}{M}, \qquad (94)$$

where M >0 - a real positive number. Formula (93) are special case of (94) when M = 1.

The resulting formula is more general because it covers such cases as partial (22) when N =1, $M = Z$; (64) with $N = Z$, M =1.

Try to prove these two statements. Substituting $N = 1,\ M = Z$ in (94), we have

$$\sum_{k=1}^{\infty}\left[\frac{1}{1+Z}\right]^{k} = \frac{1}{Z}. \qquad (95)$$

Comparing (22) and (95), it remains to prove that

$$\frac{1}{1+Z} = Z. \qquad (96)$$

It is easy to see that this equality is a consequence of equation (7). Thus, formula (22) is a special cases of (94) when $N = 1,\ M = Z$. Substituting $N = Z,\ M = 1$ in (94), we have

$$\sum_{k=1}^{\infty}\left[\frac{Z}{1+Z}\right]^{k} = Z. \qquad (97)$$

Comparing (61) and (94) it remains to prove that

$$\frac{Z}{1+Z} = R. \qquad (98)$$

According to (11) have

$$\frac{Z}{R} = 1 + Z. \qquad (99)$$

Defined in (99) R we will receive expression (98). Thus, formula (64) *is a special* cases of (94) when $N = Z,\ M = 1$.

Conclusion

These new analytical dependences for golden chopping provide analytical basis for their further use in various fields of engineering practice. Expressions for polynomial series make it possible to calculate analytically the sum of infinite series without residual error. It increases the accuracy of calculations, while significantly reducing their number, because instead of calculating the sum of infinite series we can calculate the expression in which there are several operations of addition, subtraction, multiplication and division.

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