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PREDICTION OF THE PROPER OPERATION AND FAILURE PROBABILITIES IF THE COMPLETENESS OF THE SYSTEM IS SPECIFIED FOR THE HIERARCHICAL BRANCHED TILL THE 4-TH LEVEL SYSTEMS USING THE ARTIFICIAL NEURAL NETWORKS

The software module is developed. By the specified readiness parameters it calculates probabilities of the proper operation and failure-ability for the isotropic symmetric and hierarchical branched systems (HBS).The module is tested/restricted against the systems of the n-level branching whose elements obey the exponential rules. The non-iterative artificial neural network (ANN) has been deployed to the prediction of those characteristics. The reduced to the mean value range errors of the ANN learning and forecasting are calculated as well as the time estimations for the ANN learning and forecasting.

Keywords - hierarchical branched system, artificial neural network forecasting non-iterational training ANN, ANN with “bottles neck”

Introduction

The reliability improvement of complicate systems - is one of the most important engineering problems of any modern industry and the Ukrainian one is not exception as well. It's impossible to construct and deploy any system without estimating the reliability. Because of that there is a necessity in the methods development for prediction the system reliability parameters based on the system's architecture specifics.

The majority of systems might be described as a hierarchical tree-like branched structure (HBS). The computers networks structures are also could be described as HBS. The server - is at the zero-level of such systems, at the intermediate levels - concentrators of the information, and the workstations are on the exits of the system. Networks like that may have symmetric, asymmetric, non-isotropic, with transitions over layers structure with branching up to n-th level. Although they could be with simple as well as with complicate subordination

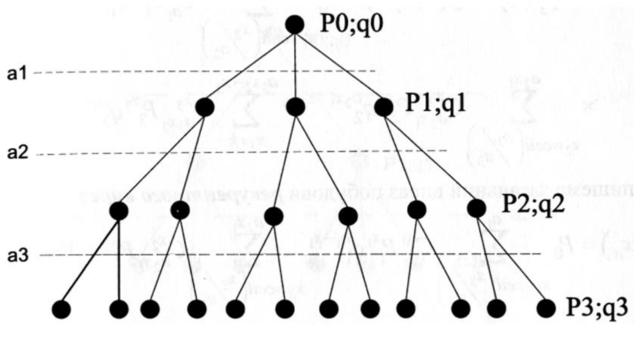

Fig.1. Symmetric HBS is branched to 3-rd level

Algorithm description.

The simplest kind of HBS is symmetric system. For example, let us review the symmetric HBS is branched to 3-rd level (see Fig.1). Where $P_0, q_0, P_1, q_1, P_2, q_2, P_3, q_3$ appropriately are probabilities of proper operation- and failure- abilities of the system elements of the zero, first, second, and third levels. Let a_1, a_2, a_3- are the branching coefficients for the first, second and third levels.

The generating functions and the recursive expression [1] could be used for investigation of the unrecoverable symmetric HBS systems. The generating function $S_m(Z)$ reflects the expansion in powers of the free Z -parameters that selects probabilities P_m^n as coefficients at Z . The generating function of the system is shown on the Fig.1 could be expressed in the next way

$$S_3(Z) = P_0(P_1(P_2(P_3 Z + q_3)^{a_3} + q_2)^{a_2} + q_1)^{a_1} + q_0 \qquad (1)$$

The generating function could be expanded by formula of binomial theorem. If we mark as X the working elements at first, second and third levels.

$$S_3(Z) = P_0 \sum_{x_1=0}^{a_1} C_{a_1}^{x_1} P_1^{x_1} q_1^{a_1-x_1} \sum_{x_2=0}^{a_2 x_1} C_{a_2 x_1}^{x_2} P_2^{x_2} q_2^{a_2 x_1 - x_2} \sum_{x_3=0}^{a_3 x_2} C_{a_3 x_2}^{x_3} P_3^{x_3} q_3^{a_3 x_2 - x_3} Z^{x_3} + q_0 \qquad (2)$$

With (2) the expression (3) could be rewriten in the next way

$$S_3(Z) = P_0 \sum_{x_1=ceil\left(\frac{x_2}{a_2}\right)}^{a_1} C_{a_1}^{x_1} P_1^{x_1} q_1^{a_1-x_1} \sum_{x_2=ceil\left(\frac{x_3}{a_3}\right)}^{a_2 x_1} C_{a_2 x_1}^{x_2} P_2^{x_2} q_2^{a_2 x_1 - x_2} C_{a_3 x_2}^{x_3} P_3^{x_3} q_3^{a_3 x_2 - x_3} \quad (3)$$

where the $ceil\left(\frac{x_i}{a_i}\right), i=\overline{1,3}$ - is function that rounds the result of dividing X per A

Let designate as $P_3(x_3)$ - the likelihood of proper operation of the x_3 system elements at the exit. If $0 < x_3 \le a_1 a_2 a_3$ than, as it is shown in [2,3], we are getting formula (4). If $x_3 = 0$ than we have to add q_0 to the resulting expression (4).

$$P_3(x_3) = P_0 \sum_{x_1=ceil\left(\frac{x_2}{a_2}\right)}^{a_1} C_{a_1}^{x_1} P_1^{x_1} q_1^{a_1-x_1} \sum_{x_2=ceil\left(\frac{x_3}{a_3}\right)}^{a_2 x_1} C_{a_2 x_1}^{x_2} P_2^{x_2} q_2^{a_2 x_1 - x_2} \sum_{x_3=0}^{a_3 x_2} C_{a_3 x_2}^{x_3} P_3^{x_3} q_3^{a_3 x_2 - x_3} \qquad (4)$$

Let all system's elements have an exponential dependency. In this case the $P_3(x_3,t)$ as the probability of proper operation of x_3 output system's elements could be expresssed as

$$P_3(x_3,t) = e^{-\lambda_0 t} \sum_{x_1=ceil\left(\frac{x_2}{a_2}\right)}^{a_1} C_{a_1}^{x_1} e^{-\lambda_1 x_1 t} (1-e^{-\lambda_1 t})^{a_1-x_1} \sum_{x_2=ceil\left(\frac{x_3}{a_3}\right)}^{a_2 x_1} C_{a_2 x_1}^{x_2} e^{-\lambda_2 x_2 t} (1-e^{-\lambda_2 t})^{a_2 x_1 - x_2} * \qquad (5)$$
$$* C_{a_3 x_2}^{x_3} e^{-\lambda_3 x_3 t} (1-e^{-\lambda_3 t})^{a_3 x_2 - x_3}$$

Now we review the n-th level ($n \in Z$) symmetric HBS system. The probability of proper operation $P_n(x_n,t)$ of x_n output system's elements

$$P_n(x_n,t) = e^{-\lambda_0 t} \sum_{x_1=ceil\left(\frac{x_2}{a_2}\right)}^{a_1} C_{a_1}^{x_1} e^{-\lambda_1 x_1 t} (1-e^{-\lambda_1 t})^{a_1-x_1} * \ldots * \sum_{x_{n-1}=ceil\left(\frac{x_n}{a_n}\right)}^{a_{n-1} x_{n-2}} C_{a_{n-1} x_{n-2}}^{x_{n-1}} e^{-\lambda_{n-1} x_{n-1} t} (1-e^{-\lambda_{n-1} t})^{a_{n-1} x_{n-2} - x_{n-1}} *$$
$$* C_{a_n x_{n-1}}^{x_n} e^{-\lambda_n x_n t} (1-e^{-\lambda_n t})^{a_n x_{n-1} - x_n} = e^{-\lambda_0 t} \sum_{x_1=ceil\left(\frac{x_2}{a_2}\right)}^{a_1} C_{a_1}^{x_1} e^{-\lambda_1 x_1 t} (1-e^{-\lambda_1 t})^{a_1-x_1} * \qquad (6)$$
$$* \prod_{i=2}^{n-1} \sum_{x_i=ceil\left(\frac{x_i}{a_i}\right)}^{a_{i-1} x_{i-2}} C_{a_{i-1} x_{i-2}}^{x_{i-1}} C_{a_i x_{i-1}}^{x_i} e^{-\lambda_i x_i t} (1-e^{-\lambda_i t})^{a_i x_{i-1} - x_i} * C_{a_n x_{n-1}}^{x_n} e^{-\lambda_n x_n t} (1-e^{-\lambda_n t})^{a_n x_{n-1} - x_n}$$

Than the probability of failure of x_n system output elements could be calculated by the expression

$$Q_n(x_n,t) = 1 - P_n(x_n,t) \qquad (7)$$

We have developed the python script that in case of priory known of the readiness criteria calculates via equations (6) and (7) the probabilities of proper operation and failure for the n-th level symmetric HBS system with exponential elements' dependencies. The features of this application are the possibilities to dynamically change: the amount of HBS's elements; initial values of these parameters; discretization rate for these parameters.

Inputs of the application are:

n - ($n \in Z$) the number of the HBS system's level;

a_i - coefficients of the branching of i-th level HBS, $i = \overline{1,n}$;

λ_j - j-th element failure intensity $j = \overline{0,n}$;

t - the temporal interval of working the HBS;

x_n - redines criteria.

The calculation's results could be saved to file in comma separated values (csv) format to use them for further training and testing ANN, which is proposed to be used as more universal instrument to predict the possible dynamical change of system's reliability with time at earlier stage of design without compiling programing and debugging of complex equations and bulky mathematical calculations.

File with data for training and testing the ANN must be compliant with the following description:

Columns from left to right - are the input parameters, the 3-rd column from the end - are the time samples with 1 hour sample rate. The column before the last one - $P_n(x_n,t)$, and the last column – is $Q_n(x_n,t)$. In our particular case, the incoming values of $P_n(x_n,t)$ and $Q_n(x_n,t)$ are calculated with the next values of input parameters: $n = 4$; $a_i = \overline{1,6}$, $i = \overline{1,n}$; $\lambda_j = 10^{-3}$ [1/год], $j = \overline{0,n}$; $x_n = \prod_{l=1}^{n} x_l$ .The file has 125900 rows and 6.58 Mb file-size. Data calculation on PC with 2-physical cpu cores (i3; 2.1GHz) and 3G of RAM takes about 18 sec.

The non-iterative ANN based on the "functional on the tabulated functions set" paradigm is chosen to make predictions of $P_n(x_n,t)$ and $Q_n(x_n,t)$, e.g. the ANN of radial basis functions (RBF), that is designed by prof. Tkachenko R.O. [4, 5].

The major advance of such ANN - is the quick learning process due to the non-iterative algorithm, which is based on the method of the orthogonal basis building in the vector space – the particular sample of the investigated process. As a consequence, this ANN could be deployed in the real-time mode. The accuracy warranty of the prediction is provided during the ANN setup, e.g. via customizing the ANN parameters [6, 7]. The external criteria of correspondence of the prediction results to the appropriate values in the checking samples set is used for the ANN training.

The calculation results of $P_n(x_n,t)$ and $Q_n(x_n,t)$ aresplittered to two files: ANN training and ANN testing. In particular, we used sequence of 2000 samples for training, and the next 136 values - for testing.

For the prediction, by the experimental selection way the "bottles neck" ANN which consist of 8 neurons at the incoming layer , 7 – in the hidden layer i and 8 neurons at the outcoming layer is selected. As you could notice, the neurons amount at outcoming layer is higher than at the hidden layer and is equal to the amount of neurons at the incoming layer. The incoming and hidden layersperform the data compression, while the hidden and outgoing layers perform data decompression. ANN like that provides deep data compression with a purpose to reduce the dimension-size of the "incoming and outcomings" to build the map in the new double-measured coordinate system. This procedure realization is possible thanks to deployment of ANN with non-linear connections among synapsis, e.g. ANNs of radial basis functions.

ANN training results according to $P_n(x_n,t)$ for each output are shown on fig.2. and prediction results are on the fig.3.

The mean square error is reduced to range of values for the estimation $P_n(x_n,t)$ is done by ANN was about 1.03%, the max value of error – 6.61%. The error for the estimation $Q_n(x_n,t)$ – was 3.36%, and the max value – 8.87%. Run-time of predictions was less than 1 sec. with acceptable accuracy. The time is taken by ANN to calculate predicted values was about the same that had been taken by applicaton for direct calculation of equations (6) and (7) for small sample sets. But if there were more than 500 samples of either od $P_n(x_n,t)$ or $Q_n(x_n,t)$ than the ANN would took less time than is necessary for drect calculations of (6) and (7).

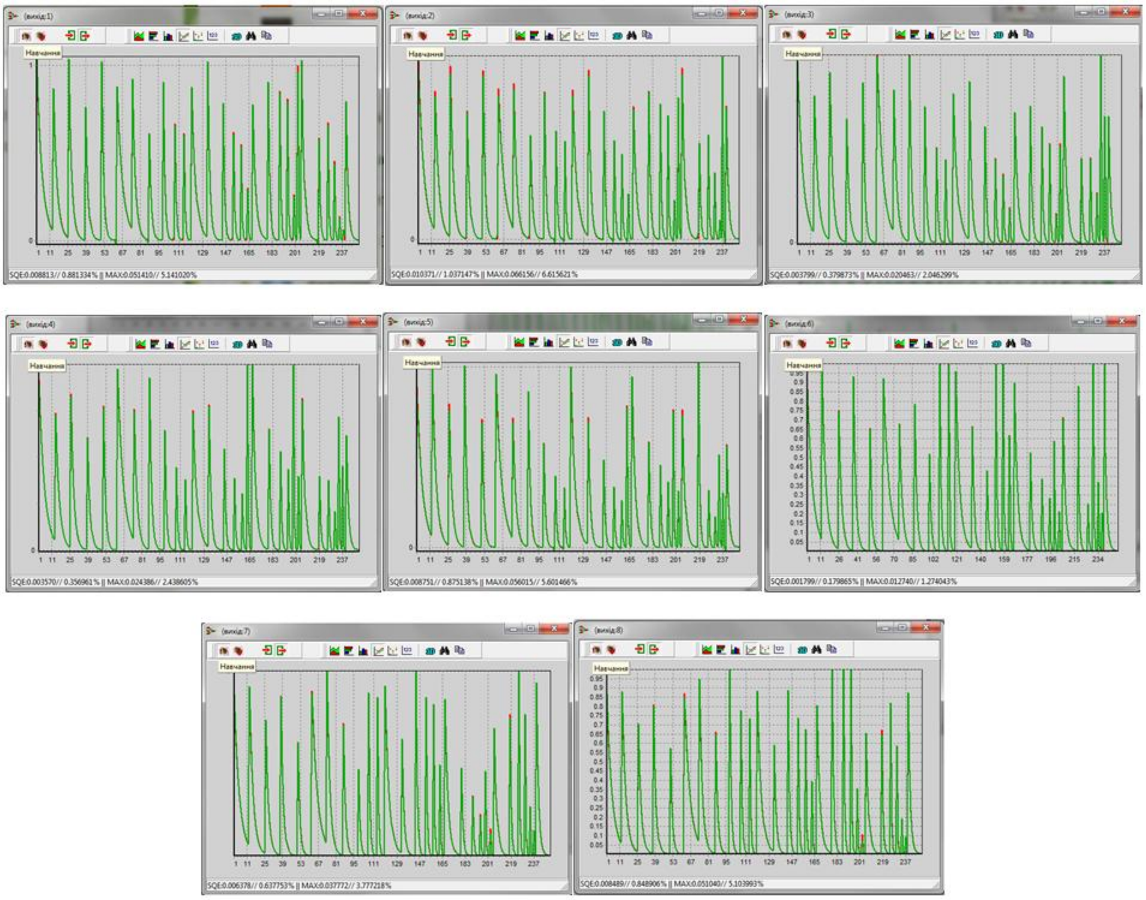

Fig.2. ANN training mode for $P_n(x_n,t)$

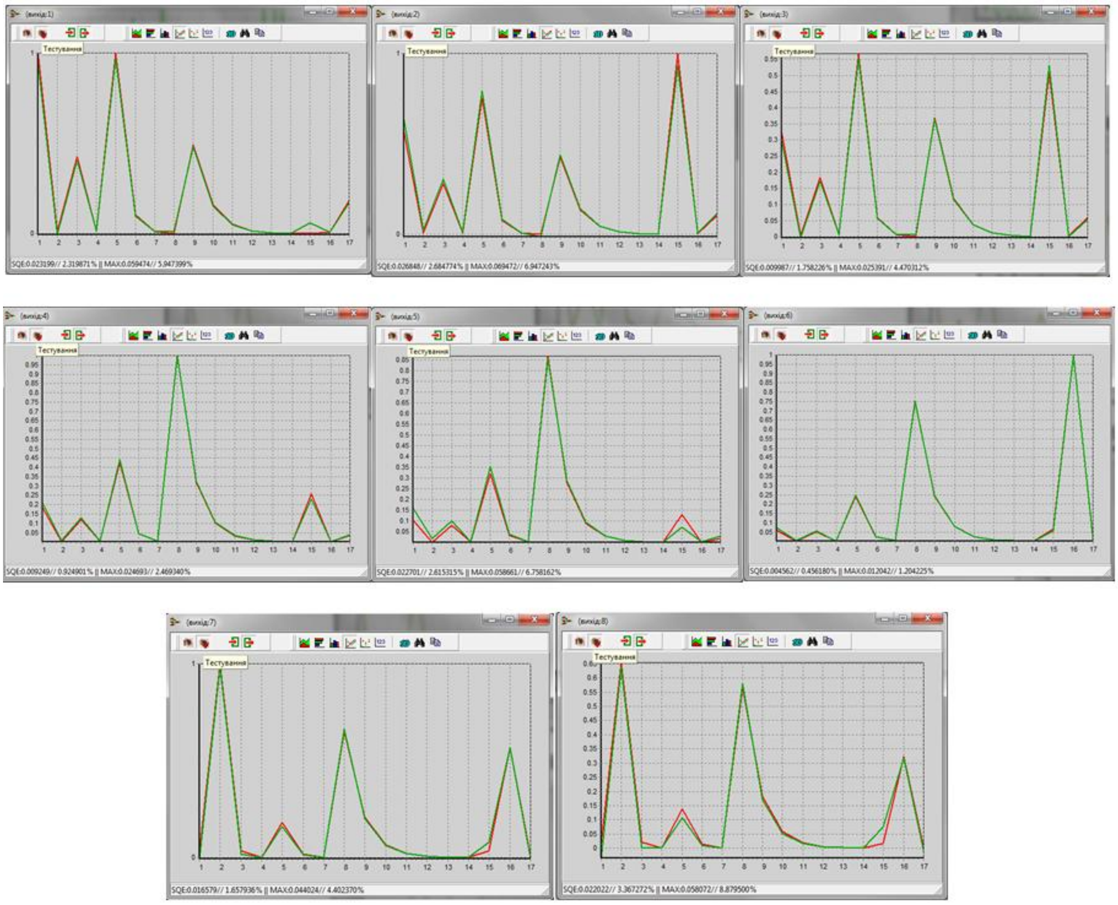

Fig.3. ANN prediction mode of $P_n(x_n,t)$

Table.1. represent the results for comparison that are obtained for 3 modifications of incoming parameters and calculated by $P_n(x_n,t)$ and $Q_n(x_n,t)$ and their predicted by ANN values.

Grid.

The research results

P_1	P_2	$\lvert P_1-P_2\rvert$	Q_1	Q_2	$\lvert Q_1-Q_2\rvert$	P_1	P_2	$\lvert P_1-P_2\rvert$	Q_1	Q_2	$\lvert Q_1-Q_2\rvert$
1	0,942	0,058	0,057	0,057	0,057	0,322	0,352	0,03	0,647	-0,000	0,000
0,568	0,634	0,066	0,365	0,336	0,336	0,243	0,249	0,006	0,750	0,092	0,092
0,323	0,297	0,026	0,702	0,646	0,646	0,183	0,160	0,023	0,839	0,171	0,171
0,184	0,206	0,022	0,793	0,710	0,710	0,138	0,108	0,03	0,891	0,214	0,214
0,104	0,160	0,056	0,839	0,729	0,729	0,104	0,098	0,006	0,901	0,214	0,214
0,059	0,071	0,012	0,928	0,793	0,793	0,078	0,082	0,004	0,917	0,222	0,222
0,034	-0,006	0,04	1,006	0,846	0,846	0,059	0,056	0,003	0,943	0,239	0,239
0,019	-0,034	0,053	1,034	0,850	0,850	0,044	0,045	0,001	0,954	0,241	0,241
0,011	-0,005	0,016	1,005	0,798	0,798	0,034	0,038	0,004	0,961	0,240	0,240
0,006	0,025	0,019	0,974	0,744	0,744	0,025	0,025	0	0,974	0,245	0,245
0,004	-0,003	0,007	1,003	0,751	0,751	0,019	0,015	0,004	0,984	0,247	0,247
0,002	0,008	0,006	0,991	0,718	0,718	0,014	0,009	0,005	0,990	0,246	0,246
0,001	0,017	0,016	0,982	0,688	0,688	0,011	0,010	0,001	0,989	0,238	0,238
0,001	0,004	0,003	0,995	0,681	0,681	0,008	0,008	0	0,991	0,232	0,232
1	0,987	0,013	0,012	-0,321	0,321	0,006	0,005	0,001	0,994	0,229	0,229
0,654	0,637	0,017	0,362	0,009	0,009	0,005	0,005	0	0,994	0,222	0,222
0,428	0,405	0,023	0,594	0,223	0,223	0,003	0,003	0	0,996	0,217	0,217
0,28	0,304	0,024	0,695	0,306	0,306	0,003	0,003	0	0,996	0,211	0,211
0,183	0,172	0,011	0,827	0,420	0,420	0,002	0,001	0,001	0,998	0,207	0,207
0,12	0,128	0,008	0,871	0,447	0,447	0,001	0,000	0,001	0,999	0,202	0,202
0,079	0,100	0,021	0,899	0,459	0,459	0,001	0,013	0,012	0,986	0,183	0,183
0,051	0,055	0,004	0,944	0,488	0,488	0,001	-0,013	0,014	1,013	0,204	0,204
0,034	0,018	0,016	0,981	0,509	0,509	0,001	0,006	0,005	0,993	0,179	0,179
0,022	0,001	0,021	0,998	0,511	0,511	1	0,995	0,005	0,004	-0,814	0,814
0,014	0,013	0,001	0,986	0,485	0,485	0,868	0,856	0,012	0,143	-0,680	0,680
0,009	0,009	0	0,990	0,474	0,474	0,753	0,751	0,002	0,248	-0,580	0,580
0,006	0,005	0,001	0,994	0,464	0,464	0,653	0,662	0,009	0,337	-0,496	0,496
0,004	0,004	0	0,995	0,452	0,452	0,567	0,579	0,012	0,420	-0,418	0,418
0,003	0,003	0	0,996	0,440	0,440	0,492	0,481	0,011	0,518	-0,325	0,325
0,002	0,002	0	0,997	0,428	0,428	0,427	0,439	0,012	0,560	-0,287	0,287
0,001	0,000	0,001	0,999	0,418	0,418	0,37	0,365	0,005	0,634	-0,218	0,218
0,001	0,000	0,001	0,999	0,406	0,406	0,321	0,325	0,004	0,674	-0,182	0,182
1	0,968	0,032	0,031	-0,573	0,573	0,279	0,289	0,01	0,710	-0,150	0,150
0,754	0,791	0,037	0,208	-0,407	0,407	0,242	0,244	0,002	0,755	-0,109	0,109
0,568	0,555	0,013	0,444	-0,182	0,182	0,21	0,202	0,008	0,797	-0,071	0,071
0,428	0,440	0,012	0,559	-0,078	0,078	0,182	0,171	0,011	0,828	-0,043	0,043

In the table we used the following notations: P_1 - values are calculated by (6) - probabilities of proper operation in case of the specified readiness condition anisotropic symmetric branched till 4-th level HBS; P_2 – predicted via ANN - probabilities of proper operation in case of the specified readiness condition anisotropic symmetric branched till 4-th level HBS. $\lvert P_1-P_2\rvert$ - absolute values of difference between P_1 and P_2. Q_1 - calculatedaccording to (7) - probabilities of proper operation in case of the specified readiness condition anisotropic symmetric branched till 4-th level HBS. Q_2 – predicted via ANN - probabilities of proper operation in case of the specified readiness condition anisotropic symmetric branched till 4-th level HBS. $\lvert Q_1-Q_2\rvert$ - absolute values of differences between Q_1 and Q_2.

The results of real (blue line) and predicted (red line)via ANN values of $P_n(x_n,t)$ and $Q_n(x_n,t)$ are on the fig.4.

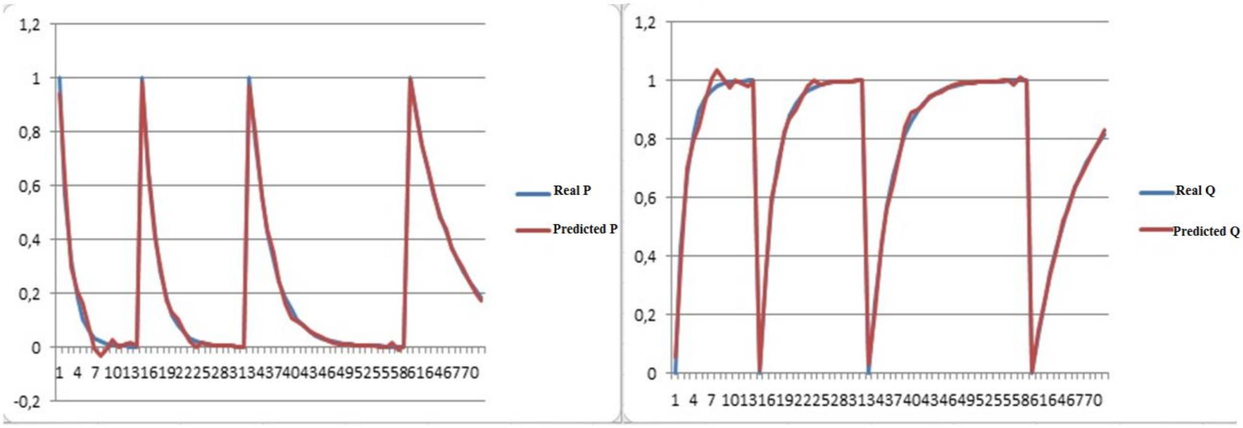

Fig.4. The results of real and predicted via ANN values of $P_n(x_n,t)$ *and* $Q_n(x_n,t)$

Conclusions

The results analysis shows that the ANN deployment to implement the prediction of the reliability characteristics of symmetric HBS gives good enough results. The largest gain in performance when making prediction of the HBS reliability using ANN is similar to the calculation by formulas (6) and (7) in case of large number of branching levels .

Although the performance might be higher, if system consists of more reliable elements, e.g. if necessary to process too big amount of statistic data.

It’s determined that to make prediction of the HBS reliability parameters it’s more suitable touse ANN of the ‘bottles neck’ type, thanks to compression possibility of big amount of data.

ANN could be used to the quick calculations HBS parameters with perfect accuracy, that is especially actual for HBS development at design stage; determining the weakest chain in the system; modelling the HBS work in the real time; and so on.

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