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RECOGNITION OF CONDUCTIVE AND HIGH-RESISTIVE INCLUTIONS IN A PIECEWISE HOMOGENEOUSE HALF-SPACE IN PROCESS OF MATHEMATICAL MODELLING OF STEADY OSCILATIONS OF AN ELECTROMAGNETIC FIELD

The numerical-analytic technique for finding electric and magnetic components of electromagnetic field in a piecewise homogeneous conductive half-space is suggested. Electromagnetic field is excited by a horizontal contour with current harmonically changing in time. The problem is formulated and solved by means of the boundary element method.

Keywords: Maxwell's equation system, the Helmholtz equation system, established EMF fluctuations; indirect boundary element method.

Introduction

Nowadays electromagnetic fields (EMF) harmonically varying with time that are generated by artificial sources and propagate within homogeneous and horizontal-stratified models of the Earth crust have been studying by means of well developed spectral analysis theory. It especially refers to a wide range of two dimensional magnetotelluric problems, when an external source is given as a homogeneous magnetic field or a plain wave. Also there are analytic solutions for EMF distribution problems at comparatively simple electrical conditions for foreign local inclusions having a canonical or similar to canonical form in the case of two or three dimensional horizontal-stratified models [1, 2]. Concerning inclusions having complex forms, which better represent a real geoelectric situation, lately numerical and analytical-numerical methods for mathematical modeling of EMF generated by artificial sources have wide been using. Most popular difference methods [3, 4] or finite element methods [5, 6] provide a good accuracy of results but need a digitization of whole space which the heterogeneous body occupies and what in turn need a usage of a bulk memory. A usage of boundary integral equations [7-9] and built on the base of them boundary element methods [10] allows one to discretize only boundary surface of the body what in turn reduce an amount of memory during a realization of the algorithm and provide a good calculation accuracy at inner points.

The paper proposes and implements an analytic-numerical approach based on indirect boundary element method for a solution of direct three dimensional problems of electromagnetic field theory in the case of steady oscillations for a conductive piecewise homogeneous half-space. Full and quasi-stationary models are considered in the paper. An influence of inclusions with higher and lower conductivity (for oil and gas deposits) than within geophysical environment on a EMF distribution is studied.

Problem definition

Let's consider a piecewise homogeneous half-space occupying the region $\Omega = R^{3-} = \{(x_1, x_2, x_3): -\infty < x_1 < \infty, -\infty < x_2 < \infty, -\infty < x_3 < 0\}$ in a three-dimensional Cartesian coordinate system and including the body Ω_2 that has a parallelepiped form and an ideal contact with a geoelectric environment $\Omega_1 = \Omega / \Omega_2$. The geoelectric environment Ω_1 and the body Ω_2 are characterized by constant electrical conductivities σ_1, σ_2, magnetic conductivities μ_1, $\mu_{2,}$, and dielectric conductivities ε_1,

ε_2 (figure 1). On the surface of half-space $\Gamma = \{(x_1, x_2, x_3): -\infty < x_1 < \infty, -\infty < x_2 < \infty, x_3 = 0$ an electrical field (EF) is absent, and in a contour $C \subset \Omega_1$ foreign currents act with an intensity defined as $\vec{\psi}(x,\tau) = (\psi_1(x,\tau), \psi_2(x,\tau), \psi_3(x,\tau))$, where τ – time variable, $x = (x_1, x_2, x_3)$ – point in the space. At the initial moment of time there is null distribution of components of electric field strength $E_i^l(x,\tau)$ (where i=1,2,3, l=1,2) within both geoelectric environment Ω_1 and the body Ω_2 and rates of their variations.

Propagating an EMF within piecewise homogeneous half-space, in which within region Ω_1 foreign currents are acting, is described by Maxwell equations:
for a geoelectric environment ($(x,\tau) \in \Omega_1 \times T$) for an inclusion ($(x,\tau) \in \Omega_2 \times T$)

$$\begin{aligned}
&rot\vec{H}^1(x,\tau) = \sigma_1 \vec{E}^1(x,\tau) + \varepsilon_1 \frac{\partial \vec{E}^1(x,\tau)}{\partial \tau} + \vec{\psi}(x,\tau), && rot\vec{H}^2(x,\tau) = \sigma_2 \vec{E}^2(x,\tau) + \varepsilon_2 \frac{\partial \vec{E}^2(x,\tau)}{\partial \tau},\\
&div\vec{H}^1(x,\tau) = 0, && div\vec{H}^2(x,\tau) = 0,\\
&rot\vec{E}^1(x,\tau) = -\mu_1 \frac{\partial \vec{H}^1(x,\tau)}{\partial \tau}, && rot\vec{E}^2(x,\tau) = -\mu_2 \frac{\partial \vec{H}^2(x,\tau)}{\partial \tau},\\
&div\vec{E}^1(x,\tau) = 0, && div\vec{E}^2(x,\tau) = 0,
\end{aligned} \tag{1}$$

where $\vec{H}^l(x,\tau) = (H_1^l(x,\tau), H_2^l(x,\tau), H_3^l(x,\tau))$, $\vec{E}^l(x,\tau) = (E_1^l(x,\tau), E_2^l(x,\tau), E_3^l(x,\tau))$ – a magnetic field strength vector and an electric field strength vector in Ω_l, $T = \{\tau : 0 < \tau < \infty\}$ respectively.

For many electrodynamics problems it's useful to separate Maxwell equations (1), i.e. to write separately equations for an electric field and a magnetic field. By means of vector differential operation properties we can obtain telegraphy equations system for components of EF $E_j^1(x,\tau)$ (j=1,2,3) in environment Ω_1 and $E_j^2(x,\tau)$ in environment Ω_2:

$$\begin{aligned}
&\Delta E_j^1(x,\tau) - \sigma_1\mu_1 \frac{\partial E_j^1(x,\tau)}{\partial \tau} - \varepsilon_1\mu_1 \frac{\partial^2 E_j^1(x,\tau)}{\partial \tau^2} = \mu_1 \frac{\partial \psi_j(x,\tau)}{\tau}, (x,\tau) \in \Omega_1 \times T,\\
&\Delta E_j^2(x,\tau) - \sigma_2\mu_2 \frac{\partial E_j^2(x,\tau)}{\partial \tau} - \varepsilon_2\mu_2 \frac{\partial^2 E_j^2(x,\tau)}{\partial \tau^2} = 0, (x,\tau) \in \Omega_2 \times T,
\end{aligned} \tag{2}$$

supplemented with boundary

$$E_j^1(x,\tau) = 0,\ (x,\tau) \in \Gamma \times T, \tag{3}$$

and initial conditions

$$E_j^l(x,\tau) = 0, \frac{\partial E_j^l(x,\tau)}{\partial \tau} = 0,\ x \in \Omega_l \text{ при } \tau = 0, \tag{4}$$

and also with ideal electromagnetic contact conditions (continuity of tangential components of electric and magnetic fields and a jump of normal components) on a border of the environment and the body:

$$\begin{aligned}
&E_d^1(x,\tau) = E_d^2(x,\tau), \quad \frac{1}{\mu_1} rotE_d^1(x,\tau) = \frac{1}{\mu_2} rotE_d^2(x,\tau),\\
&\sigma_1 E_n^1(x,\tau) = \sigma_2 E_n^2(x,\tau), \quad rotE_n^1(x,\tau) = rotE_n^2(x,\tau),
\end{aligned} \quad (x,\tau) \in \Gamma_{12} \times T, \tag{5}$$

where Δ – Laplace operator, $\Gamma_{12} = \partial\Omega_1 \cap \partial\Omega_2$, $\partial\Omega_l$ – a border of the region Ω_l.

If we carry out observations in the quite long term after an electric field excitation, physical quantities can be assumed to harmonically vary in time with a circular frequency ω, and it means we deal with a steady oscillations problem. Assuming that $E_j^l(x,\tau) = \tilde{E}_j^l(x,\omega)e^{-i\omega\tau}$,

$\psi_j(x,\tau)=\tilde{\psi}_j(x,\omega)e^{-i\omega\tau}$, our analysis will simplify, because time variable will be excluded with problem (2)-(5):

$$\Delta\tilde{E}_j^1(x,\omega)+\mu_1\omega(\varepsilon_1\omega+i\sigma_1)\tilde{E}_j^1(x,\omega)=-i\omega\mu_1\tilde{\psi}_j(x,\omega),\ x\in\Omega_1,$$
$$\Delta\tilde{E}_j^2(x,\omega)+\mu_2\omega(\varepsilon_2\omega+i\sigma_2)\tilde{E}_j^2(x,\omega)=0,\ x\in\Omega_2, \tag{6}$$

$$\tilde{E}_j^1(x,\omega)=0,\ x\in\Gamma, \tag{7}$$

$$\begin{aligned}&\tilde{E}_d^1(x,\omega)=\tilde{E}_d^2(x,\omega),\quad \frac{1}{\mu_1}rot\tilde{E}_d^1(x,\omega)=\frac{1}{\mu_2}rot\tilde{E}_d^2(x,\omega),\\ &\sigma_1\tilde{E}_n^1(x,\omega)=\sigma_2\tilde{E}_n^2(x,\omega),\quad rot\tilde{E}_n^1(x,\omega)=rot\tilde{E}_n^2(x,\omega),\end{aligned}\quad x\in\Gamma_{12}=\partial\Omega_1\cap\partial\Omega_2. \tag{8}$$

Here $\tilde{E}_j^l(x,\omega)=\tilde{E}_j^{l1}(x,\omega)+i\tilde{E}_j^{l2}(x,\omega)$, $\tilde{\psi}_j(x,\omega)=\tilde{\psi}_j^1(x,\omega)+i\tilde{\psi}_j^2(x,\omega)$ – complex amplitudes of components of an electric field strength vector and strange current sources.

Technique of building the solution. Integral representation of the solution

For building the solution of the problem (6)-(8) the indirect boundary element method was used. According to this method the border between the environment Ω_1 and the body Ω_2 is decomposed into *V arias (boundary elements)* Γ_v^{12}, such that $\Gamma_{12}=\cup_{v=1}^{V}\Gamma_v^{12}$, $\Gamma_v^{12}\cap\Gamma_w^{12}=\varnothing$ when $v\neq w$. In each boundary element Γ_v^{12} we set fictitious current sources $\varphi_{jv}^l(x,\omega)=\varphi_{jv}^{l1}(x,\omega)+i\varphi_{jv}^{l2}(x,\omega)$, ($j$ = 1,2,3). So we will obtain new equations system:

$$\Delta\tilde{E}_j^1(x,\omega)+\mu_1\omega(\varepsilon_1\omega+i\sigma_1)\tilde{E}_j^1(x,\omega)=-\sum_{v=1}^{V}\varphi_{jv}^1(x,\omega)\chi_v-i\omega\mu_1\tilde{\psi}_j(x,\omega),$$
$$\Delta\tilde{E}_j^2(x,\omega)+\mu_2\omega(\varepsilon_2\omega+i\sigma_2)\tilde{E}_j^2(x,\omega)=-\sum_{v=1}^{V}\varphi_{jv}^2(x,\omega)\chi_v, \tag{9}$$

where χ_v – a characteristic function of boundary element Γ_v.

Using a special fundamental solution of the Helmholtz equation $\Phi_h^1(x,\xi,\omega)$, which automatically satisfies a boundary condition (7) and a fundamental solution $\Phi^2(x,\xi,\omega)$ for an inclusion, we can write an integral representation of solution for problems (9),(7),(8) according to components $\tilde{E}_j^1(x,\omega)$ and $\tilde{E}_j^2(x,\omega)$:

$$\tilde{E}_j^1(x,\omega)=\sum_{v=1}^{V}\int_{\Gamma_v}\Phi_h^1(x,\xi,\omega)\varphi_{jv}^1(\xi,\omega)d\Gamma_v(\xi)+I_{cj}(x,\omega,\Phi_h^1),$$
$$\tilde{E}_j^2(x,\omega)=\sum_{v=1}^{V}\int_{\Gamma_v}\Phi^2(x,\xi,\omega)\varphi_{jv}^2(\xi,\omega)d\Gamma_v(\xi), \tag{10}$$

and also obtained on its base an integral representation of derivatives of its components with respect to coordinates x_l:

$$\frac{\partial \tilde{E}_j^1(x,\omega)}{\partial x_k} = \sum_{v=1}^{V} \int_{\Gamma_v} Q_{hk}^1(x,\xi,\omega)\varphi_{jv}^1(\xi,\omega)d\Gamma_v(\xi) + I_{cj}(x,\omega,Q_{hk}^1),$$

$$\frac{\partial \tilde{E}_j^2(x,\omega)}{\partial x_k} = \sum_{v=1}^{V} \int_{\Gamma_v} Q_k^2(x,\xi,\omega)\varphi_{jv}^2(\xi,\omega)d\Gamma_v(\xi), \qquad (11)$$

where $F_h^1(x,\xi,\omega) = F^1(r,\omega) - F^1(r',\omega)$, $F \in \{\Phi, Q_k\}$, $F^l(r,\omega) = F^l(x,\xi,\omega) = F^{l1}(r,\omega) + iF^{l2}(r,\omega)$, $r^2 = \sum_{k=1}^{3}(x_k - \xi_k)^2$, $r'^2 = \sum_{k=1}^{2}(x_k - \xi'_k)^2$, $\xi'_k = \xi_k\,(k = 1,2)$, $\xi'_3 = -\xi_3$, $\xi = (\xi_1,\xi_2,\xi_3) \in R^3$,

$\Phi^l(r,\omega) = \dfrac{\exp(ir\sqrt{\mu_l\omega(\varepsilon_l\omega + i\sigma_l)})}{4\pi r}$, $\Phi^{l1}(r,\omega) = \dfrac{1}{4\pi r}\exp(-rA_l\sin(\dfrac{\theta_l}{2}))\cdot\cos(rA_l\cos(\dfrac{\theta_l}{2}))$,

$\Phi^{l2}(r,\omega) = \dfrac{1}{4\pi r}\exp(-rA_l\sin(\dfrac{\theta_l}{2}))\cdot\sin(rA_l\cos(\dfrac{\theta_l}{2}))$, $Q_k^l(x,\xi,\omega) = \dfrac{\partial\Phi^l(x,\xi,\omega)}{\partial x_k} = (x_k - \xi_k)Q^l(r,\omega)$,

$Q^{l1}(r,\omega) = -\dfrac{1}{r^2}\left(rA_l(\Phi^{l1}\sin(\dfrac{\theta_l}{2}) + \Phi^{l2}\cos(\dfrac{\theta_l}{2})) + \Phi^{l1}\right)$, $A_l = \sqrt{\mu_l\omega}\sqrt[4]{\varepsilon_l^2\omega^2 + \sigma_l^2}$, $\theta_l = arctg(\dfrac{\sigma_l}{\varepsilon_l\omega})$,

$Q^{l2}(r,\omega) = \dfrac{1}{r^2}\left(rA_l(\Phi^{l1}\cos(\dfrac{\theta_l}{2}) - \Phi^{l2}\sin(\dfrac{\theta_l}{2})) - \Phi^{l2}\right)$,

$I_{cj}(x,\omega,F_h^1) = i\omega\mu_1\int_C F_h^1(x,\xi,\omega)\tilde{\psi}_j(\xi,\omega)dC(\xi) = I_{cj}^1(x,\omega,F_h^1) + iI_{cj}^2(x,\omega,F_h^1)$,

$I_{cj}^1(x,\omega,F_h^1) = -\omega\mu_1\int_C (F_h^{11}\tilde{\psi}_j^2 + F_h^{12}\tilde{\psi}_j^1)dC(\xi)$, $I_{cj}^2(x,\omega,F_h^1) = \omega\mu_1\int_C (F_h^{11}\tilde{\psi}_j^1 - F_h^{12}\tilde{\psi}_j^2)dC(\xi)$.

In order to simplify the algorithm let's approximate unknown functions $\varphi_{jv}^{l1}(x,\omega)$, $\varphi_{jv}^{l2}(x,\omega)$ by constants d_{jv}^{l1}, d_{jv}^{l2}. This constants can be found from a system of linear equations built on the base of equations (10), (11) requiring a satisfaction of ideal electromagnetic contact conditions in a collocation sense.

After the system of linear equations is resolved with respect to d_{jv}^{l1}, d_{jv}^{l2}, we can calculate values of an electric field vector by means of formulae (10) at every points x^p of space:

$$\tilde{E}_j^{11}(x^p,\omega) = \sum_{v=1}^{V}\left(d_v^{11}I_{vp}(\Phi_h^{11}) - d_v^{12}I_{vp}(\Phi_h^{12})\right) + I_{cj}^1(x^p,\omega,\Phi_h^1),$$

$$\tilde{E}_j^{12}(x^p,\omega) = \sum_{v=1}^{V}\left(d_v^{11}I_{vp}(\Phi_h^{12}) + d_v^{12}I_{vp}(\Phi_h^{11})\right) + \tilde{I}_{cj}^2(x^p,\omega,\Phi_h^1),$$

$$\tilde{E}_j^{21}(x^p,\omega) = \sum_{v=1}^{V}\left(d_v^{21}I_{vp}(\Phi^{21}) - d_v^{22}I_{vp}(\Phi^{22})\right),\quad \tilde{E}_j^{22}(x^p,\omega) = \sum_{v=1}^{V}\left(d_v^{21}I_{vp}(\Phi^{22}) + d_v^{22}I_{vp}(\Phi^{21})\right),$$

where $I_{vp}(F) = \int_{\Gamma_v} F(x,\xi,\omega)d\Gamma_v(\xi)$, $F \in \{\Phi_h^{11}, \Phi_h^{12}, \Phi^{22}, \Phi^{21}\}$.

Real and imagine parts $\tilde{H}_j^{l1}(x^p,\omega)$, $\tilde{H}_j^{l2}(x^p,\omega)$ of components of a magnetic field vector can be calculated by formulae:

$$\tilde{H}_k^{l1}(x^p,\omega) = \frac{1}{\omega\mu_l}\left(\frac{\partial E_s^{l2}}{\partial x_j} - \frac{\partial E_j^{l2}}{\partial x_s}\right),\ \tilde{H}_k^{l2}(x^p,\omega) = -\frac{1}{\omega\mu_l}\left(\frac{\partial E_s^{l1}}{\partial x_j} - \frac{\partial E_j^{l1}}{\partial x_s}\right). \qquad (12)$$

Here for different k we use different pairs of indexes *s* and *j*, for *k*=1 – *s*=3, *j*=2, for *k*=2 – *s*=1, *j*=3, for *k*=3 – *s*=2, *j*=1.

Quasi-stationary model

In the case of a quasi-stationary model, equations of system (6) obtain new form:

$$\begin{aligned}&\Delta\tilde{E}_j^1(x,\omega)+i\mu_1\omega\sigma_1\tilde{E}_j^1(x,\omega)=-i\omega\mu_1\tilde{\psi}_j(x,\omega),\\&\Delta\tilde{E}_j^2(x,\omega)+i\mu_2\omega\sigma_2\tilde{E}_j^2(x,\omega)=0.\end{aligned} \tag{13}$$

And expressions for fundamental solutions will simplify respectively:

$$\Phi^{l1}(r,\omega)=\frac{1}{4\pi r}\exp(-rA_l/\sqrt{2})\cdot\cos(rA_l/\sqrt{2}),\Phi^{l2}(r,\omega)=\frac{1}{4\pi r}\exp(-rA_l/\sqrt{2})\cdot\sin(rA_l/\sqrt{2}),$$

$$Q^{l1}(r,\omega)=-\frac{1}{r^2}(rA_l(\tilde{\Phi}^{l1}+\tilde{\Phi}^{l2})/\sqrt{2}+\tilde{\Phi}^{l1}),Q^{l2}(r,\omega)=\frac{1}{r^2}(rA_l(\tilde{\Phi}^{l1}-\tilde{\Phi}^{l2})/\sqrt{2}-\tilde{\Phi}^{l1}),$$

here it was taken into account that $\theta_l=\pi/2$, $A_l=\sqrt{\mu_l\omega\sigma_l}$.

Numerical implementation

Task 1. For numerical experiments we considered an inclusion having an parallelepiped form with parameters p_x=100, p_y=100, p_z=30 and lying at a depth of h_0. An external source generating EMF was a square frame with side h=100, and lying at a depth of h_3=-0.001. Dependency on electric current was described by a function $\tilde{\psi}_j(x,\omega)=C_j(x)$, where $C_j(x)$ – projections of an unit vector being collinear to a tangent to a contour C at point $x\in C$. In this particular case integrals from equations (10), (11) had a form:

$$\tilde{I}_{cj}^1(x,\omega,\tilde{F}^1)=-\omega\mu_1\int_C\tilde{F}^{12}C_j(\xi)dC(\xi),\quad \tilde{I}_{cj}^2(x,\omega,\tilde{F}^1)=\omega\mu_2\int_C\tilde{F}^{11}C_j(\xi)dC(\xi).$$

Each side of the parallelepiped was divided into 27 boundary elements (fig. 1).

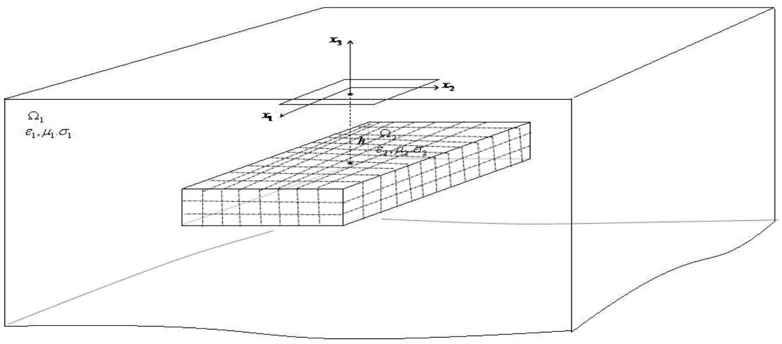

Fig. 1. Geoelectrical model of half-space with an inclusion

The fig. 2 describes dependency vertical components of $H_3^1(0,0,h_3,\omega)$ obtaining at a center of the frame on a angular frequency for full and quasi-stationary models in the case of oil (σ_2=0,25σ_1, μ_2=0.99994μ_1, ε_2=2ε_1, series 3, 6), gas (σ_2=0.1σ_1, μ_2=1.00008μ_1, ε_2=ε_1, series 4, 7) and conductive (σ_2=5σ_1, μ_2=μ_1, ε_2=30ε_1, series 1, 5) inclusions, lying at a depth of h0=40. Also a distribution for homogeneous half-space for full model is described σ_1=1Sm/m, μ_1=4π10^{-7}G/m, ε_1=15ε_0, ε_0=1/36 π 10^{-9} F/m (series 2). As we can see for high-resistively inclusions values $H_3^1(0,0,h_3;\omega)$ in whole frequency range are lesser than for homogeneous half-space, and for conductive inclusions – greater.

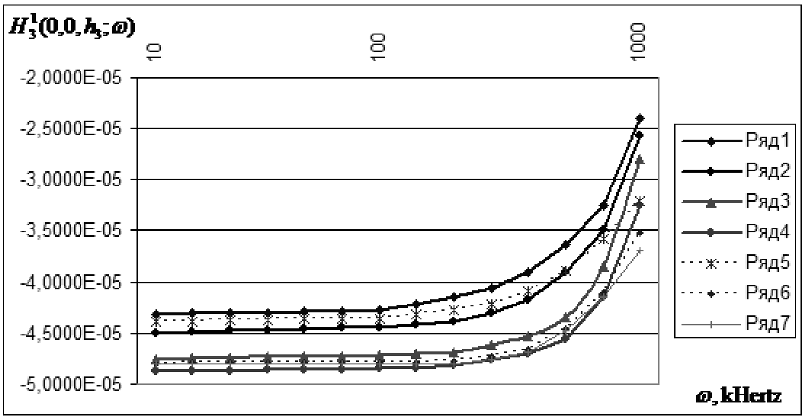

Fig. 2. Influence EM parameters of the inclusion on a vertical component of MF vector

As it's known during an interpretation of results theoretical curves have an important meaning that characterize an amplitude $\left|\rho_{\omega}^{f}\right| / \rho_{1}$ and a phase θ_{ω}^{f} of apparent resistance, where

$$\left|\rho_{\omega}^{f}\right| / \rho_{1} = \frac{\sqrt{(f^{1} f^{01} + f^{2} f^{02})^{2} + (f^{2} f^{01} - f^{1} f^{02})^{2}}}{(f^{01})^{2} + (f^{02})^{2}}, \theta_{\omega}^{f} = arctg \frac{f^{2} f^{01} - f^{1} f^{02}}{f^{1} f^{01} + f^{2} f^{02}},$$

$f^{k} \in \{H_{1}^{1k}, H_{2}^{1k}, H_{3}^{1k}, E_{1}^{1k}, E_{2}^{1k}, E_{3}^{1k}\}$ (k=1,2), $f^{1}, f^{01}, f^{2}, f^{02}$ – real and complex components of a MF vector, index "0" indicate values in homogenous space with a resistance $\rho_1 = 1/\sigma_1$.

Fig. 3, 4 describe a dependence of theoretical curves $\left|\rho_{\omega}^{f}\right| / \rho_{1}$, θ_{ω}^{f} on λ_1 / h_0 built on the base a vertical component of MF at the center of the frame. for a full model.

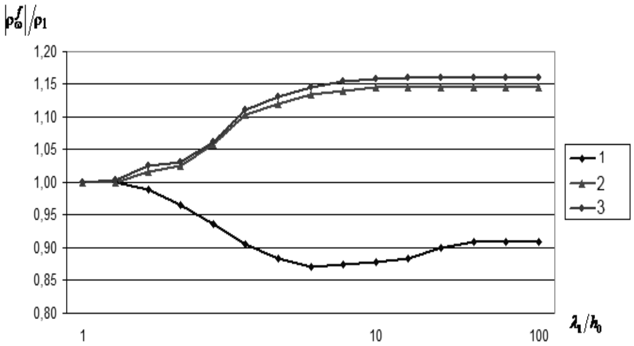

Fig. 3. Dependency amplitude curves on wave lengths

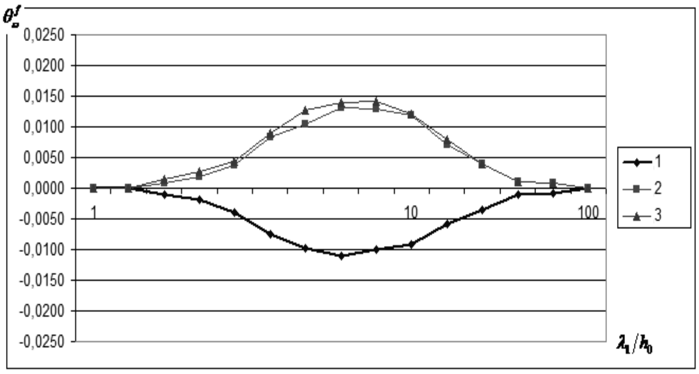

Fig. 4. Dependency phase curves on wave lengths

We also considered a dependency values $\left|\rho_{\omega}^{f}\right| / \rho_{1}$ and θ_{ω}^{f}, built on the base of vertical component of MF, on the depth of lying the conductive inclusion ($\sigma_2 = 5\sigma_1$, $\mu_2 = \mu_1$, $\varepsilon_2 = 30\varepsilon_1$,) – h_0=40 (series 1), h_0=80 (series 2) i h_0=100 (series 3) for frequency rang from 1 kHertz to 100 kHertz. (fig. 5, 6).

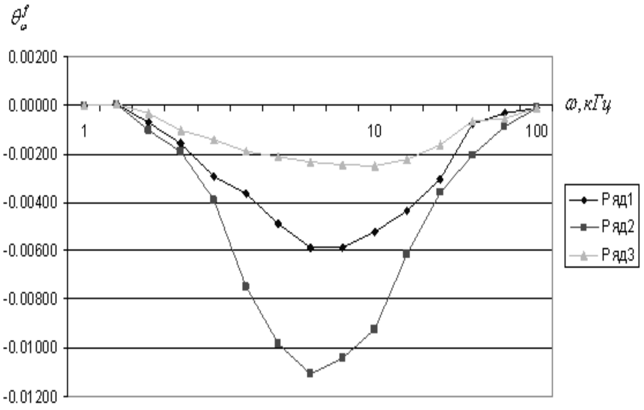

Fig 5. Dependence phase curves on the depth of lying and wave lengths

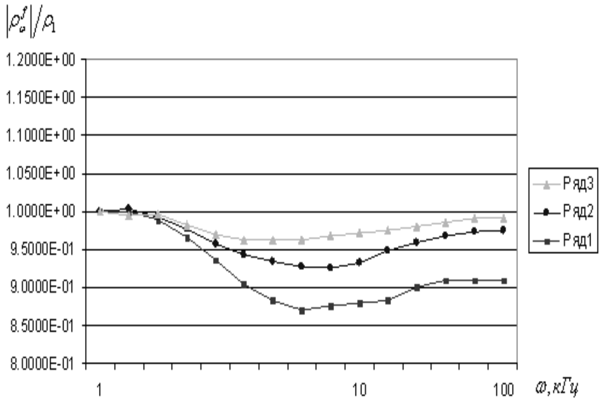

Fig. 6. Dependence amplitude curves on the depth of lying and wave lengths

The case of homogeneous object with any curvilinear boundary

The problem of finding components of EMF for steady oscillations is described by following equations:

$$\Delta\tilde{E}_j(x,\omega)+\mu\omega(\varepsilon\omega+i\sigma)\tilde{E}_j(x,\omega)=-i\omega\mu\tilde{\psi}_j(x,\omega),\ x\in\Omega,$$

and boundary conditions:

$$\tilde{E}_j(x,\omega)=\tilde{E}_{jz}(x,\omega),\ x\in\Gamma\,. \qquad (14)$$

A system of linear equations for finding unknown fictitious sources is built on the base of boundary conditions (14).

Task 2. An introduced approach was also applied to modeling steady oscillations within a three dimensional homogenous object with electromagnetic characteristics conductive (series 1) and oil (series 2) deposits which has an elliptic form with axis *a*=3, *b*=2, *c*=1. A border of object was divided into 27 boundary elements. We also analyzed an influence of a foreign source on components EMF within the object. We considered second-type boundary conditions ($\tilde{E}_1(x,\omega)=\tilde{E}_2(x,\omega)=0$, $\tilde{E}_3(x,\omega)=0.000005\cdot(x_3/c+1)/2$).

The fig. 7 shows a dependency of a tangent component of MF on an angular frequency ω (in a frequency range from 1 to 20 kHertz) at the center of the body. The fig 8. describes a dependency of a normal component EF distributed on Ox_3 axis at the segment [-1, 1].

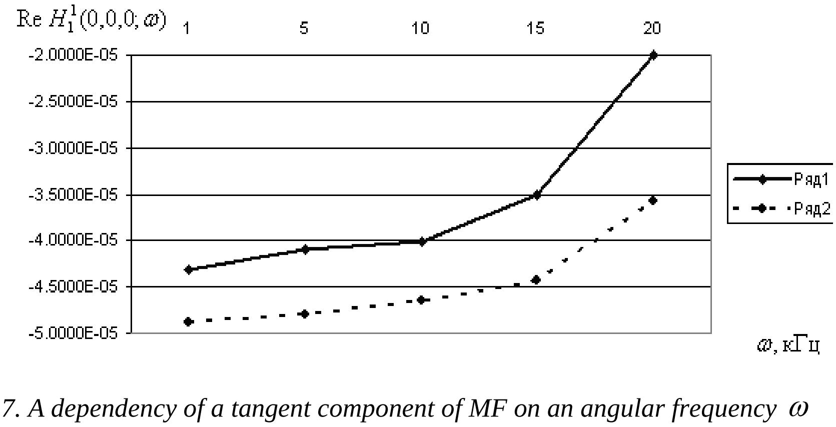

Fig 7. A dependency of a tangent component of MF on an angular frequency ω

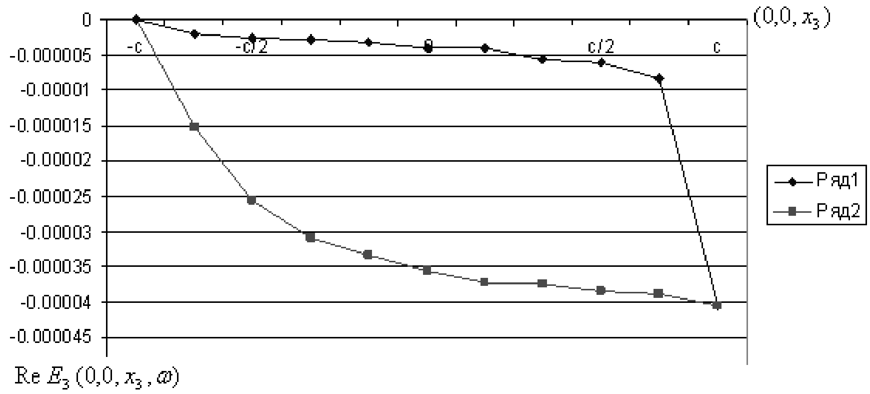

Рис. 8. A dependency of a normal component EF on coordinates.

Conclusions

From the standpoint of an iterative process, a direct problem of inductive sounding is very important and key and this problem should be optimized. It is important to note that an interpretation of data, concerning to horizontal-homogeneous environments, by means of apparent parameters one-dimensional models is settled yet. Nowadays it is sensible to use three-dimensional models. As it's known, an inverse problem is solved on the base of analysis of sensibility of a direct problem to changes of geoparameters. So algorithm of a direct problem in this case is very useful.

It's also reasonable to consider a stratified model with an inclusion of various forms because it's important for inductive impulsive electrical prospecting. But otherwise for processing bulky data at the process of solving an direct geoelectric problem, we need to find interpretating formulas on the base amplitude and phase curves of an apparent resistance.

The results of introduced mathematical modeling shows a reasonable of usage of inductive impulse electro prospecting methods of frequency sounding for detection of high-conductive and high-resistive inclusions.

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