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MATHEMATICAL AND SOFTWARE ASPECTS OF MODELLING AGE STRUCTURE OF DEVELOPMENT OF TWO TYPES OF FOREST

Simulation is carried out by numerical analysis of the dynamic system equations by the Runge-Kutta method. Consistently described the construction of a model that takes into account both interspecies competitionand ranks of other factors: light, water-logging, age structure, rainfall, external influences. The results of simulation obtained on the created software. The possibilities of using the created model local level to ensure the development of an information and decision support in forest management.

Keywords: forestry, forecasting, information systems, modeling.

Statement of the problem

Forest as an object of human activities, is also the subject of research, as forest plantations - a complex natural formation. Forest - part of the geographical landscape and consists of a set of different species of trees, shrubs, grasses, mosses, animals and micro-organisms that are interdependent and affect each other and the environment. However, the forest should be considered not only in space but also in time, given its own development. These dynamic processes are the result of complex relationships between the components of his body (the struggle for existence and natural selection, constant updating and development, changes in age structure, metabolism and energy). The study of forest ecosystems should be done together, in a variety of relationships between its parts and processes that take place within it. This makes it necessary to use a systematic approach, ie the principle of consistency in the knowledge of biological phenomena of nature, both in detail and as a whole. It should be noted that forest ecosystems are characterized by high variability in space because conditions microenvironment within them always slightly different from each other. Great variability and signs caused by humans, it is greatly complicates studies [9].

Simulation of forest ecological systems to predict the consequences of a scenario, by selecting less dangerous and more useful for the studied system. With models can quickly predict effects of actions directed for many years, even centuries. Moreover, the undisputed advantage of modeling and concomitant prediction is the lack of negative or even catastrophic consequences ekspery¬mentuvannya on the environment.

Age structure is one of the important indicators of forest fund in the evaluation of forest resources and their raw potential. It gives an idea of the forest area within age groups and the timber resources that can be used both in this and in the following decades.

Stability of forest to drought, waterlogging, shade and pollution as well as to a number of anthropogenic factors largely depends on the age of the plants. In addition to economic value, taking into account the age distribution of mathematical models, it is important for the environment and the area in which there is forest.

Analysis of recent publications and research

In the EU, research, and related software development on prediction of forest plantations, decision support implemented and used both at the regional and national levels. However, complex systems of forest management planning for the most various reasons a number of simplifications used in the calculations do not include age particular distribution (eg, counting change the diameter of the trunk, and on this basis using empirical relationships calculate other characteristics including age (Busing, 1991; Chave, 1999; Bartelink, 2000). Among the exceptions model SILVA2, where the focus is on growth height

(Pretzsch, 1992) and model Moravie MA et al. (1997), where the first calculated increase the radius of the crown, and then the diameter and height the characteristics of the crown [5].

HARVEST - decision support system (DSS) that was developed in the United States as a strategic research and planning tool, allows estimation of the spatial structure of long-term effects of timber management strategies. Model suitable for evaluation of alternative strategies, providing predictions about what alternative effects influence the age and spatial distribution of forest structure resulting forest landscape (Gustafson, Crow, 1999). However, the model simulates forest harvesting activities by providing arrays by a value of zero, assuming that the age structure of the forest does not change during recovery.

The issue of forest management performance with consideration of age structure was discussed in [7] estonian scientists K.Kiviste, A.Kiviste (2009). Using databases forestry enterprises Estonia was derived algebraic equation of the stem height, diameter volume depending on the type of forest area, the dominant species and their age. The model is included in the Estonian public information system on forest and several software products for forest inventory data.

The latest research in this topic should pay attention to the system PlanWise [8] (developers, scientists Swedish University of Agriculture Sciences and economic A.Korosuo, P. Wikström, 2011), where the approach of multi decision analysis (Multi-criteria decision analysis or (MCDA), this approach allows for the planning and modeling of multi-age forest take into account non-monetary values (include sustainable development).

The purpose of research

The purpose of the publication is to show the possibility of application of computer simulation models to predict the dynamics of forest ecosystems in different scenarios of human impact. Conduct improving mathematical model of two types of forest described and discussed in [4], taking into account the age structure of the forest and its present software implementation.

The main material

Age of plants and its structure are important attributes that reflect the dynamics of the forest. They are also largely characterize plant resistance to pollution, excessive moisture, drought, disease. Based on the considered in [4] system of equations (1), which examined the evolution of plants of the same age, were make changes to reflect the age distribution in the forest.

$$\begin{cases} \dot{P} = \sum_i (A_i - V_i)u_i + B(P^0 - P) - W, \\ \dot{u}_i = (k_i C_i V_i - D_i)u_i \end{cases} \quad \text{i} = \{\text{х}, \text{л}\}. \qquad (1)$$

де

$\dot{P}$ - generalized indicator of fertility - the resource density (kg/ м^2);

$u_{л}$ - biomass density hardwood (kg/ м^2);

$u_{х}$ - Biomass density softwood (kg/ м^2);

A_i - coefficient of restitution of the soil by rain i-th specie;

B - coefficient of self ground (1/ year);

P^0 - asymptotic value of fertility in the absence of forest (kg/ м^2);

V_i - the rate of consumption of resources (trophic function) (1/ year);

C_i - adjustment factor that describes the competition;

k_i - growth rate of the i-th specie;

D_i - rate of natural mortality of trees (1/ year);

W - the impact of external factors, often negative, so a negative sign, (kg/h · м^2);

t^0 - average time of maturation of forests (years).

Given that the interval age groups differ significantly from that adopted in the model [4] step (one) time, depending on the rate tree species to be added differentiation by age within the range of the age group.

Age structure of plants display the advantage taken in forest management division into age groups (young class 1, young 2nd class, middle-aged, prystyhayuchi, mature, overmature) we assume that two types structure formed beech forest (both deciduous species) white spruce (both coniferous) with an appropriate age class.

In constructing age models will lean on the model [3], which is the basis of the classical model of Voltaire, with boundary and initial conditions:

$$u(\mathrm{t},0) = \int_{\tau}^{T_m} \upsilon(u,T')u(t,T)dT \tag{2}$$

$$u(0,\mathrm{T}) = \psi(T) \tag{3}$$

If the age group includes years (depending on age class) and is considered mode in which the input group enters the same value of biomass u_0, at each year of age we will receive such biomass [3]:

1-th year - $u_0(1+C)$;

2-th year - $u_0(1+2C),...$;

p-th year - $u_0(1+pC)$;

C-coefficient that shows how increasing the initial density of biomass per year. Its value is usually 0.1 to 0.18 depends on the group, the density of biomass fertility. However, within the group, it changes little. If the group according to class age is 10 years, the law adopted linear increase in the group is justified.

Considering that full amount of biomass in the group is the sum of all amounts by year:

$$U[j] = \left[p + 0{,}5(1+p)pC[j]\right] \cdot u_0, \tag{4}$$

the share of biomass weight group for each year into another group can be defined by the ratio:

$$Q[j] = \frac{1 + pC[j]}{p + 0{,}5(1+p)pC[j]} \tag{4}$$

At the same time introduced in (4) function C does not match the function defined in [4], because it contains k, and trophic function, with explicitly indicated a dependence C and j.

In general, the function $u_i[j]$ can be described by equations similar to system (1), but instead of coefficients $C^0{}_i$ and $D^0{}_i$, input arrays $C_i{}^0[j]$ i $D^0{}_i[j]$, respectively.

When $p = 10$, we get:

$$Q_i[j] = 0{,}1 + \frac{4{,}5C_i^0[j]}{10 + 5{,}5C_i^0[j]} \tag{5}$$

When taking into account the age structure, can not fail to consider the factor that young and mature trees differently related to drought, waterlogging and other factors. Therefore modify the correction function (6), presented in [4] with depending on age j.

$$C_i[j] = C_i^0 \mathrm{Sun}_{\mathrm{i}}[j] Bog_i[j] \cdot Ext[j] - Rain_i[j] \text{ для } j \neq 0 \tag{6}$$

For $j = 0$, in the performance of one of two conditions: $N^{btot} > 0$, $N > \sum_{i,j} \prod u_i[j]$ - implemented the same expression, but with, $j = 0$ in the case, when both conditions are violated, then $C_i[0] = 0$.

Let : $\mathrm{N}^{\mathrm{btot}}$ - full "water layer", which was accumulated during the existence of forest , N^{cr} - critical "thickness" in which there is a sharp deterioration of the forest.

$$\mathrm{Bog_i}[\mathrm{j}] = \mathrm{Bog_i}(\mathrm{N_i^{cr}}[\mathrm{j}], \mathrm{N^{btot}}) \tag{7}$$

The rate of resource consumption we define as:

$$V_i[j] = V_i^p[j] \cdot p \tag{8}$$

The function $Rain_i(\vec{u}, N)$ takes into account the negative impact of drought in the annual growth of biomass. This function describes the total rainfall for the year $N(t)$ and takes into account the necessary expenses of the rocks on the internal evaporation and water consumption. It is believed that hardwoods for a normal life without growth need more water than pine [4]. Drought expose the equation:

$$Rain_i[j] = \begin{cases} Rain_i \cdot \dfrac{V_i[j]}{\sum\limits_{i'j'} \mathrm{V}_i[j'] \cdot u_i[j']}, \; j \neq 0, \\ 0, \; j = 0 \end{cases} \tag{9}$$

Consider that an increase in generalized indicator of fertility is due to the processing of annual rainfall (A_i^0)and dead biomass that is equal to the sum [4]:

$$A_i[j] = A_i^0 + A_i^1 D_i^0[j] \tag{10}$$

To record the system of equations we use the following abbreviations: product functions in the same structure will use indexes only the right factor, recording such designation $C_i[0] \cdot V_i[0] \cdot u_i[0]$ equivalent $C \cdot V \cdot u[0]$. In such notation we obtain the system of equations (11) [3]:

$$\begin{cases} \dot{u}_i[0] = \sum CV_i[0] \cdot k_i(u'', j) - Qu_i[0] + CkVu_i[0] - Du_i[0] - Ext_i[0], \\ \dot{u}_i[j] = Qu_i[j-1] - Qu_i[j] + CkVu_i[j] - Ext_i[j]), \\ \dot{p} = \sum\limits_{ij} (A - V) u_i[j] + \mathrm{B}(p^0 - p) - W \end{cases} \tag{11}$$

Simulation results

One of the main stages in the development of mathematical models of stands is the analysis, evaluation and simulation of dynamic processes that occur in them. Adequate reflection of these processes in the form of mathematical models in the future will help to develop more reliable models to predict their growth and development. Stands for the main types lisotvirnyh Carpathian region in articles accepted for basic features such as age structure and stand density biomass species.

According to the mathematical model described above was supplemented by the software described in [4]. To show the adequacy of the established model was performed numerical calculations, in possible conditions. The calculation parameters for the model: the number of age groups - 6 interval age group - 10 years, 1 year time step, the simulation -150 years, rainfall for the year to 800 mm.

On fig.1 depicted biomass u biomass density hardwood (beech) and softwood (spruce white) rocks that form of two types of forest structure according to age class. As shown on fig.1 resource density P (kg/м^2), growing rapidly to reaching growth species, followed by a slowdown in growth.

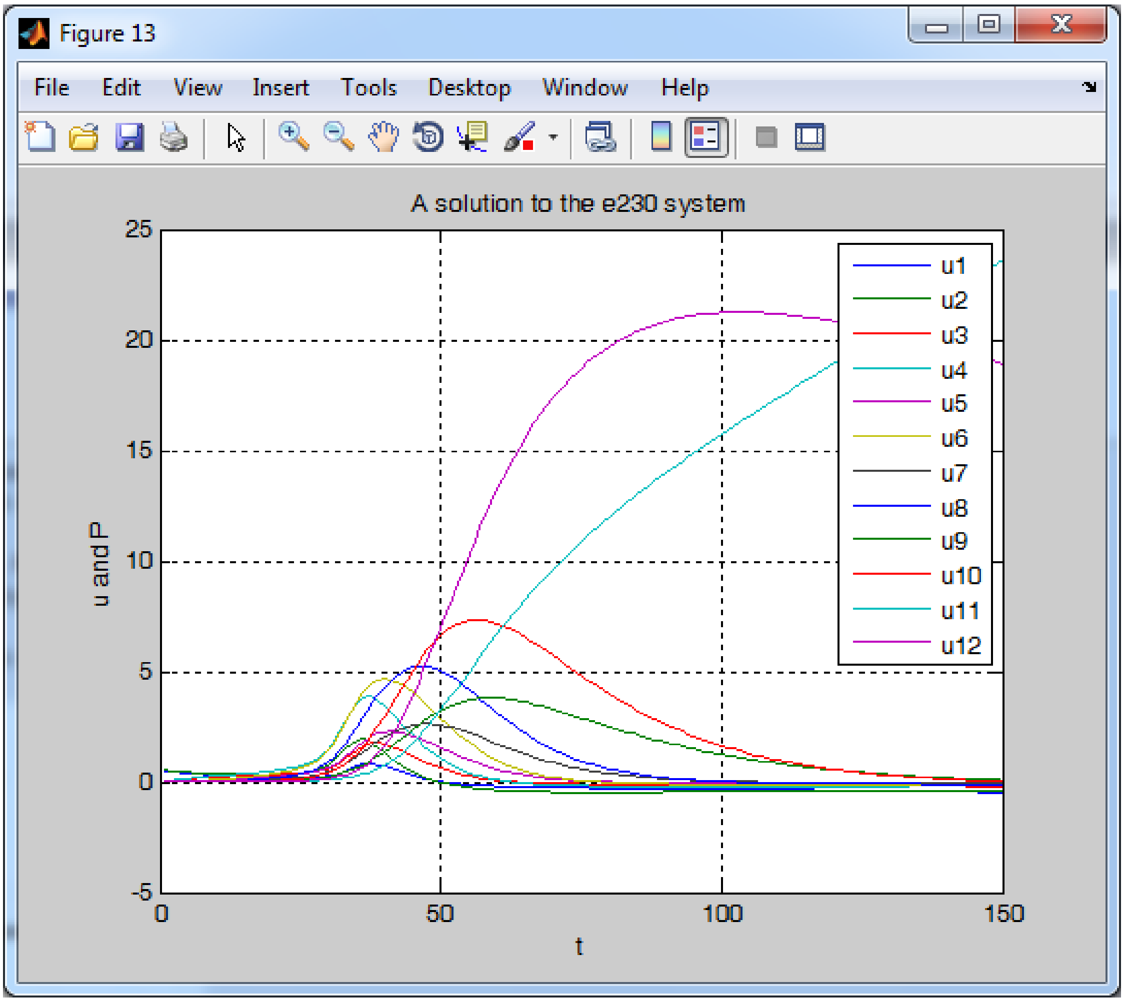

Figure 1. The development of conifers and deciduous according to age group.

Figure 2 shows the evolution of the forest in the area drop-off (in the center point of the array) depending on time. From the figure shows that the first on the landing is fast growth hardwood and distribution to free neighboring areas. We can assume that over time conifers in the middle finger actively replacing broad that leads to the formation of ring structures: in the middle of an old pine forest, and on the outer circle of light leaf area.

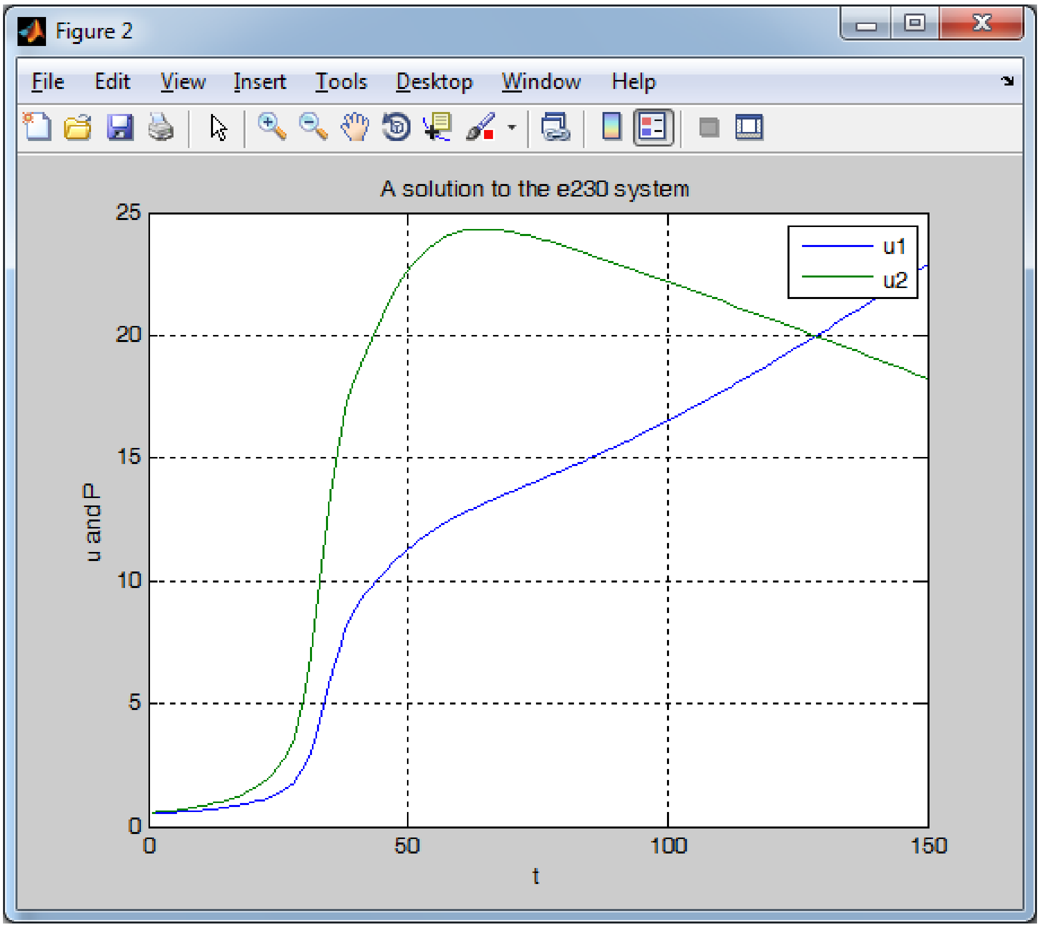

Fig.2. Dynamics of conifers and deciduous in the center point of the array.

The work of mathematical model and accompanying program complex was tested for a large number of inputs. It was modeled impact of insufficient rainfall, flooding and various anthropogenic factors. In comparing the results obtained by simulation of tables go growth [1], we can draw conclusions about the adequacy of the data.

The essence of the approach in the construction of mathematical model consists mainly of the following:

- modeling aimed at practical application (analysis and forecast);
- the possibility of using the model as a basis for complex spatial models, taking into account local ecological orientation;
- tasks pertaining to a particular territory, refers, in general, in the form of integro-differential equations with appropriate boundary conditions. This opportunity requires constant correction of initial conditions according to the actual changes that occur in the system;
- a special role for the construction of scenarios of the ecosystem at a given external influences (anthropogenic or natural);
- modeling aimed at short-term and medium-term forecast. It should be noted that the system with this set of parameters, there is a short period of time and the parameters that determine the behavior of the model is continuously changing.

Conclusions

As a result of this work was developed practical model of two types of forest from the appropriate software package. Mathematical model adequately takes into account both interspecific competition and other factors: light, water-logging, age structure, rainfall, external influences and can be used as a basis for building models of a higher level of detail and spatial coverage.

Today's team is working to create a custom program that takes into account the characteristics of the area and the initial conditions, which were obtained experimentally for a given area. Using available software package discussed in this article, you can spend testing the model on the events of the past years and get possible scenarios of forest management. One of the applications of the model may be its use during the reconstruction of the park.

Practical application program complex built on the implementation of this type of models in forestry Ukraine will contribute to sustainable development and adaptation mechanisms driving forest work and business processes to international standards of management.

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