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DEVELOPMENT METHOD OF PROTECTING IMAGES WITH AMPLITUDE-MODULATED SCREENING USING ATEB-FUNCTIONS

Proposed method of protection can be used for protection of printed documents and it is not expensive. The theory of Ateb-functions, which is && of classic trigonometry, was used as a mathematical apparatus to develop method of rasterization. A rasterization with amplitude-modulated method for offset printing is developed.

Key words: rasterization, Atheb-functions, protection of printed documents, amplitude modulation.

1. Introduction

Today new systems for protection of information in electronic form is developed. However, the necessity to develop technologies for the protection of printed documents is no less relevant, since printing tehnohigies of reproduction are constantly being improved and the new ones are developed.

There are two methods of shades of color reproduction in printing: - changing the thickness of the paint layer, or changing areas of painted and unpainted areas on the print. The first method is used in deep print by changing the size of the printing elements. The most common today is the offset printing. With the implementation of this printing method ink thickness is always constant, since a change in the thickness of the paint is technologically impossible, and the printing is based on the properties of paint adhesion-cohesion ink to printing elements and whitespace. Therefore, in the offset printing method the amplitude modulated rasterization is used. Rasterization is a process of parting an image into massive of points.

2. Basics of rasterization of printed pictures

The order of points in the picture or raster elements is called raster.This type of rasterization is used to implement amplitude-modulated method [11]. For monochrome image raster elements can take two values - "0" or "1".

The fig.1 shows a rastrer cell, which element is a square. This type of raster is called recangular raster. The most common is a round raster fig. 1b.
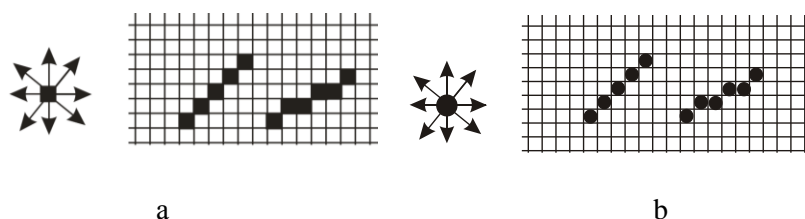

a   b
Fig.1. Rasterization with *a* – rectangular raster cell, *b* – round raster cell

All figures should be the same and they should fill all space without gaps during the process of rasterization.

As it is shown in Fig. 1 a round black raster point (completely filled) forms a gap, when an angle of 450 is printed, which at first glance seem subtle. When mode with 256 shades of gray is played, these distortions are more visible. Raster points destroys contours and fine details of the original gray-scale, reducing the quality of prints [5].

If raster point is of square form, it touches the neighbor four sides (see. Fig. 1), and this point creates a great unevenness in the image. Round point has the same drawback. Structure, which is based on superellipse is dependent on the direction of image details. As a result, you can achieve quality improvements.

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3. Modelling of area of the raster ellements

Modelling square raster cell with curve Lame

The equation of generalised superellipse is in [10]. Finding the value of variable y we can write

$$y = b \cdot \sqrt[q]{1 - \left(\frac{x}{a}\right)^p}.$$

If we take into account famous Newton-Leibniz formula [1], we will calculate area of generalized superellipse

$$s = \int_0^a b \cdot \sqrt[g]{1 - \left(\frac{x}{a}\right)^p}\, dx, \tag{1}$$

To simplify the expression, we assume $p = q = n$, $n > 0$, then

$$s = \int_0^a b \cdot \sqrt[n]{1 - \left(\frac{x}{a}\right)^n}\, dx. \tag{2}$$

If we take into account parametric equations it immediately follows that

$$\begin{cases} x = a\, \sin^{\frac{2}{n}}\varphi \\ y = b\, \cos^{\frac{2}{n}}\varphi \end{cases} \tag{3}$$

, where $0 < x < a$, a $0 < \varphi \le \frac{\Pi(n,n)}{2}$.

Differentiating first expression in (3), we obtain

$$dx = \frac{2a}{n}\, \sin^{\frac{2}{n}-1}\varphi \cdot \cos\varphi\, d\varphi \tag{4}$$

Substituting this into (3), (4) у (2), we get

$$s = \int_0^a y\, dx = \int_0^{\frac{\Pi}{2}} b\, \cos^{\frac{2}{n}}\varphi \cdot \frac{2a}{n}\, \sin^{\frac{2}{n}-1}\varphi\, \cos\varphi\, d\varphi =$$

$$= \frac{2ab}{n} \int_0^{\frac{\Pi}{2}} \sin^{\frac{2}{n}-1}\varphi \cdot \cos^{\frac{2}{n}+1}\varphi\, d\varphi$$

As we know from [3]

$$\int_0^{\frac{\Pi}{2}} \sin\varphi^{\alpha} \cdot \cos\varphi^{\beta}\, d\varphi = \frac{1}{2} B\left(\frac{\alpha+1}{2}, \frac{\beta+1}{2}\right), \text{ де } \alpha, \beta > -1.$$

Hence

$$\int_0^{\frac{\Pi}{2}} \sin^{\frac{2}{n}-1}\varphi \cdot \cos^{\frac{2}{n}+1}\varphi\, d\varphi = \frac{1}{2} B\left(\frac{\frac{2}{n}-1+1}{2}, \frac{\frac{2}{n}+1+1}{2}\right) =$$

$$= \frac{1}{2} B\left(\frac{1}{n}, \frac{1}{n}+1\right) = \frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{1}{n}+1\right)}{2\Gamma\left(\frac{1}{n}+\frac{1}{n}+1\right)} = \frac{n}{2} \frac{\left\{\Gamma\left(\frac{1}{n}+1\right)\right\}^2}{\Gamma\left(\frac{2}{n}+1\right)} \tag{5}$$

Thus, area of superellipse is equal to

$$s = \frac{ab}{n} B\left(\frac{1}{n}, \frac{1}{n}+1\right) = \frac{ab}{n} \frac{\left\{\Gamma\left(\frac{1}{n}+1\right)\right\}^2}{\Gamma\left(\frac{2}{n}+1\right)}. \tag{6}$$

Let's find the area of generalised superellipse. Analogically using parametric equation, we obtain

$$\begin{cases} x = a\, \sin^{\frac{2}{n}}\varphi \\ y = b\, \cos^{\frac{2}{m}}\varphi \end{cases}$$

where $0 < x < a$, a $0 < \varphi \le \frac{\Pi}{2}$.

It immediately follows

$$s = \int_0^a y\,dx = \int_0^{\frac{\Pi}{2}} b\,\cos^{\frac{2}{m}}\varphi \cdot \frac{2a}{n}\,\sin^{\frac{2}{n}-1}\varphi\,\cos\varphi\,d\varphi = \frac{ab}{n}B\left(\frac{1}{n},\frac{1}{m}+1\right) = \frac{ab}{n}\frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{1}{m}+1\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{m}+1\right)}$$

(7)

Finding the length of raster cells based on generalized superelipse and Lame curve

The length of curve $f(x)$ is calculated as

$$l = \int_0^{x_0}\sqrt{1+(f'(x))^2}\,dx,$$

(8)

where $f'(x)$ is a derivative of function $f(x)$.

Let's calculate the length of generalized superellipse

$$l = \int_0^a \sqrt{1+\left(\frac{bp}{xq}\right)^2\left(\frac{x}{a}\right)^{2p}\left\{1-\left(\frac{x}{a}\right)^p\right\}^{\frac{2}{q}-2}}\,dx$$

(9)

The length of Lame, when $p = q = n$, is equal to

$$l = \int_0^a \sqrt{1+\left(\frac{b}{x}\right)^2\left(\frac{x}{a}\right)^{2n}\left\{1-\left(\frac{x}{a}\right)^n\right\}^{\frac{2}{n}-2}}\,dx$$

(10)

Finding the area of superellipse based on Ateb-functions

Let's find the area of curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^m = 1$, using parametric equation with Ateb-functions

$$\begin{cases} x = a \cdot ca(m,n,\omega) \\ y = b \cdot sa(n,m,\omega) \end{cases}$$

For reasons of symmetry let's define an area of each quarter of superellipse

$$\frac{S}{4} = \int_0^{\frac{\Pi}{2}} y\,dx$$

Derivative of f ormula (9) is calculated as

$$dx = a \cdot \frac{2}{m+1}\,sa(n,m,\omega)^n\,d\omega$$

(11)

Hence

$$\frac{S}{4} = a \cdot b \cdot \frac{2}{m+1} \cdot \int_0^{\frac{\Pi}{2}} sa(n,m,\omega)^{n+1} \cdot d\omega$$

(12)

Applying the formula (9), proved in [4]

$$\int_0^{\frac{\Pi}{2}} sa(n,m,\omega)^s \cdot ca(m,n,\omega)^l\,d\omega = \frac{1}{2}B\left(\frac{s+1}{n+1},\frac{l+1}{m+1}\right)$$

where s, l – are values, which can be described as $\frac{p}{q}$ ($p \in Z, q = 2q_1 + 1, q_1 \in N$). If $l = 0$, and $s = n + 1$, then

$$\int_0^{\frac{\Pi}{2}} sa(n,m,\omega)^{n+1}\,d\omega = \frac{1}{2}B\left(\frac{n+2}{n+1},\frac{1}{m+1}\right) = \frac{1}{2}\frac{\Gamma\left(\frac{n+2}{n+1}\right)\Gamma\left(\frac{1}{m+1}\right)}{\Gamma\left(\frac{n+2}{n+1}+\frac{1}{m+1}\right)}$$

Considering the properties of Gamma - functions [3, 6] and formulas of period of ateb-functions(7) we get

$$\int_0^{\frac{\Pi}{2}} sa(n,m,\omega)^{n+1}\,d\omega = \frac{1}{2}\cdot\frac{m+1}{n+m+2}\cdot \Pi(m,n)$$

(13)

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Substituting in (3.13) formula (3.12) we obtain

$$S = 4 \cdot a \cdot b \cdot \frac{2}{m+1} \cdot \frac{1}{2} \cdot \varPi(m,n)\frac{m+1}{n+m+2}$$
$$= \frac{4 \cdot a \cdot b}{n+m+2} \cdot \varPi(m,n) \tag{14}$$

If $m = n$ we get formula (6).

Construction of raster elements

A new method of rasterization gives a chance to reproduce thin image elements more accurately which is important for protection. Forming a special structure of raster points better adapt to display halftones in the image.

Forming raster point $T(i,j)$ can be done with formula

$$T(i,j) = \left|\frac{ca(n,n,\omega)}{A(i,j)}\right|^{n+1} + \left|\frac{sa(n,n,\omega)}{A(i,j)}\right|^{n+1} \tag{15}$$

where i, j – current raster point coordinates, n – parameter of periodic $ca(n,n,\omega)$ та $sa(n,n,\omega)$ &&& по горизонталі та вертикалі. Area of raster elements is calculated with formulas (6), (7) та (14).

Thus, we get formulas to evaluate area of raster elements, , which allowed to simulate values of raster cells. New forms of raster points can be measured with variable devices, including microscope spectrophotometer. A new form of raster points required for visual and laboratory control while protecting images.

4. Modelling of rasterization process based on Ateb functions

It is suggested to rasterize with Ateb functions. In table 1 it is calculated percentage value of filling of raster cell. To change the area of raster elements we vary a, b and n. Since the rasterization is amplitude-modulated, raster cells area have equal areas, so let's apply $a = b = 1.$ and periodic Ateb functions, which have $m = n.$ We will calculate the length with formula (3.10), and area with formula (3.14).

Table 1

The calculation of Ateb-functions parameters for raster cell formation

Parametrs		Length	Area	Size
a, b - raster cell size	n	P, mm	S, mm^2	%
1	0.41	3,43	0,1	10
1	0.54	3.18	0,2	20
1	0.68	2.99	0,3	30
1	0.82	2.87	0,4	40
1	1.00	2.83	0.5	50
1	1.24	2.88	0.6	60
1	1.57	3.00	0.7	70
1	2.13	3.18	0.8	80
1	3.3	3.42	0.9	90
1	10	3.79	0,97	100

Let consider an item on the prints, which are raster elements. The best elements to display with are symmetrical, so let's apply periodic Ateb- functions. Block diagram of the implementation of the method is below. Digital image, which can be rasterized is loaded transformated in Grayscale mode. Then, the amplitude-modulated rasterization is happened with the developed method. Digital image is stored in PDF-format, which is implemented wth high printing quality.

If the form of raster point is changed we can achieve the effect that the gap between raster cells are smaller and thus raster distortion will be less noticeable.

Let's form headset of raster elements based on Ateb-functions. Headset is formed as a font in FontLab Studio, which is shown in Fig. 2. . One element of this headset is shown in Fig 3

It is generated appropriate forms of raster elements, that are presented in Table 2

Table 2

The size and shape of raster cells based on Ateb- functions depending on the size of raster points

	size and shape of raster cells rasterization is amplitude-modulated									
Size of raster cells	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Normal rasterization	.	●	●	●	●	●	●	●	●	●
Rasterization based on *Ateb*- functions	⋆	✦	✦	✦	◆	◆	●	●	●	■
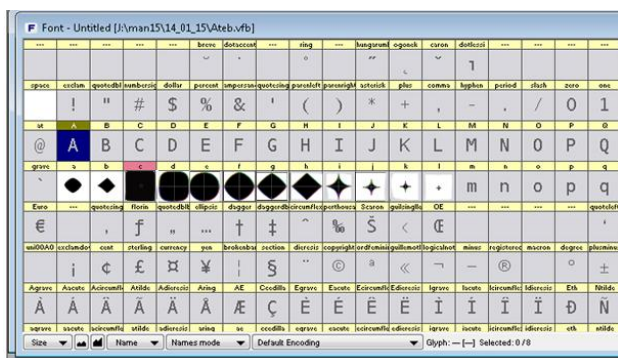
Рис. 2. Formed as a font Ateb.otf.
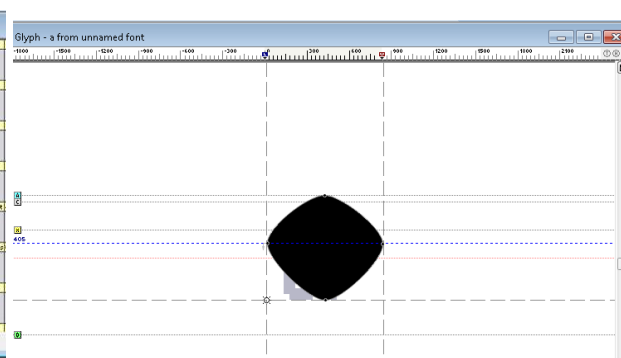
Рис. 3. Formation raster points size of raster dots 60%

Fig. 4 shows the algorithm of the method of rasterization. Let's load a bitmap, which can be rasterized, and determine the width L $i \in (1, \ldots, L-1)$ and height N, $j \in (1, \ldots, N-1)$. Transforming this image in raw format, we will avoid the loss of information. We will load headset, as a type with extension Ateb.otf. Let's develop function changing the baseline font that will be the basis for the derivation of raster cells. Then wi will evaluate the canvas size, which will display raster cells. Afterwards, the values of the new canvas are calculated, basing on read information about i-th row of pixels, and the values are converted into equivalent areas of Ateb- functions

Once the entire line is read and written, the transition to the adjacent column $j \in (1, \ldots, N-1)$ of picture is happened.

File with source code is written in the post-script format. It is converted to pdf-format with Acrobat distiller. It is known that pdf-format was developed for printing.

Figure 5 shows the original file with emblem of the city. Fig. 6 shows rasterized with developed method emblem of the city, and Fig. 7 shows enlarged rasterized image. Constructing the rasterized elements in more gradations we can get clearer and better image.
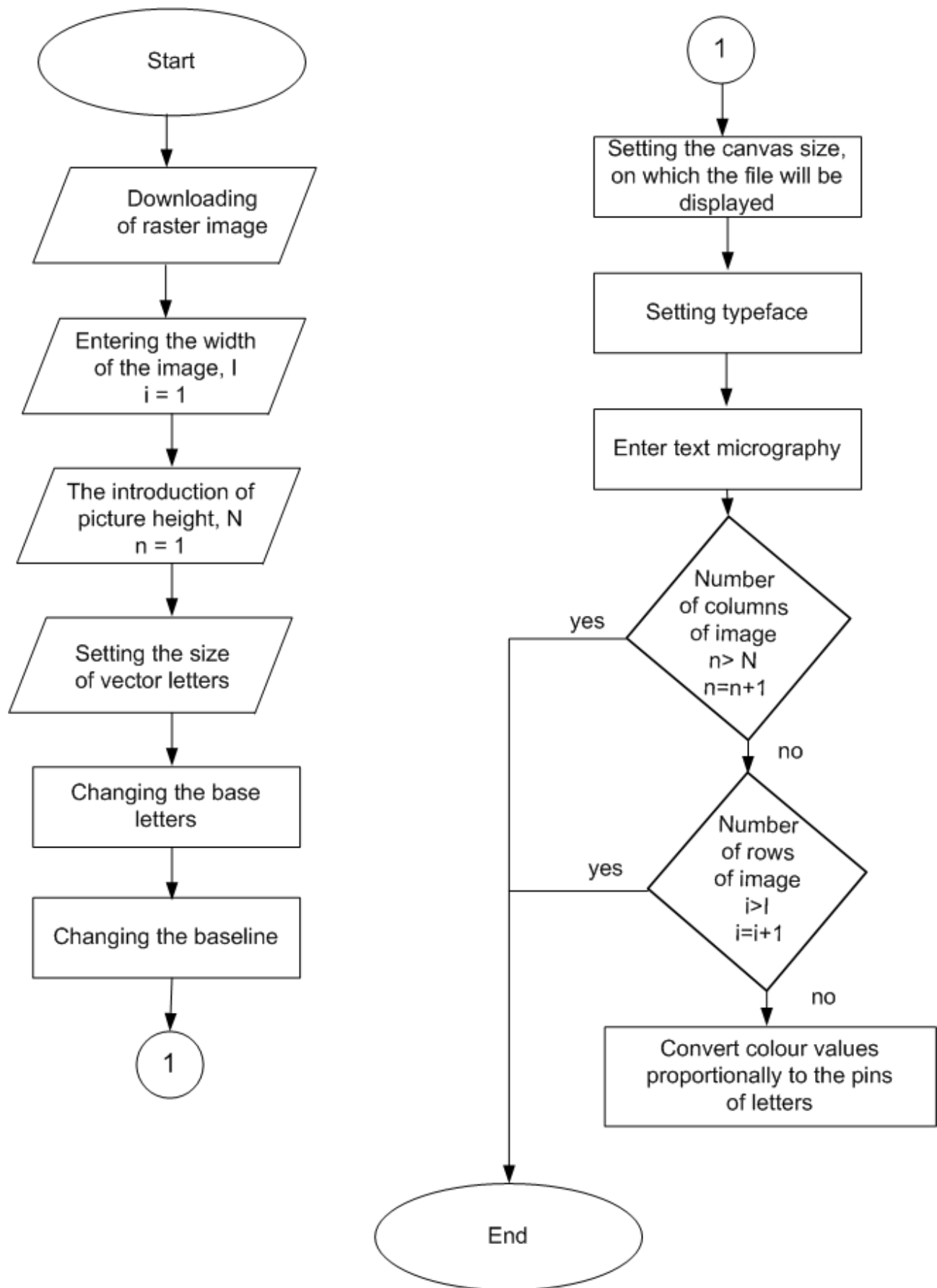.

Fig.4 Algorithm of rasterization based on Ateb-functions
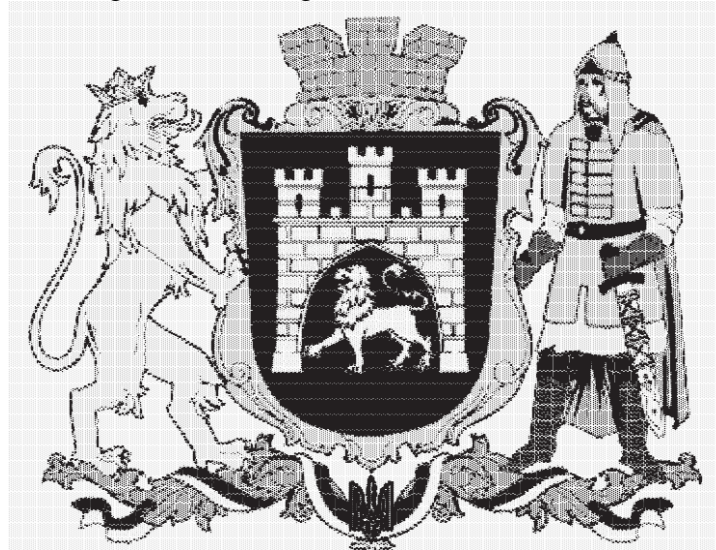
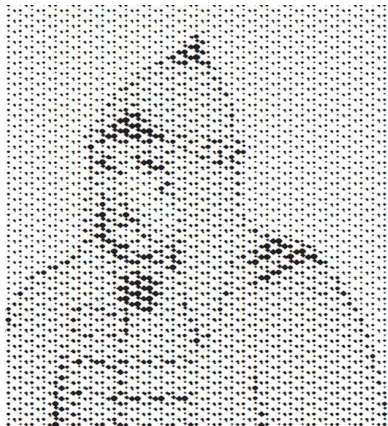Fig. 5 Entire image, which will be rasterized

Fig. 6 Rasterized image

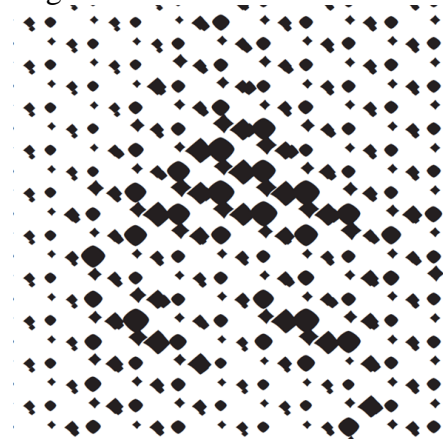Fig. 7. Enlarged fragment of rasterized image
in scale 200%

Fig. 7. Enlarged fragment of rasterized image
in scale 1000%

5. The method of protection - an attempt of falsification rasterized image

The goal of this protection is eo make copying of printed picture more expensive. Fig. 9 shows photographed picture, which was protected with the method of amplitude modulation, based on Ateb-functions. As we see from fig. 9, there aren't any rasterized points. Moreover, we see the massive of points, which are merged and make background picture. This happens, because rasterization is made with methods of vector graphic, this makes picture better.

Rasterized with this method picture can not be copied, because has other form of raster point. This method of rasterization makes picture of higher quality, because new form of raster element gives the

chance to avoid loss of clearness on the edge of raster cell. The advantage of this method is clearness of every raster cell, because rasterization is in vector format and is stored in pdf file, that is adapted for printing and multiplying. In future we plan to make rasterization of colourful picture.

Modelling forms of raster points, we can expand the number of gradations and therefore we get better picture quality.

Simulating forms of halftone dots, expanding the number of gradations and therefore get better picture quality. This method realizes ten forms of raster points. Increasing the number of raster points, we get greater amount of gradations, so we improve quality.
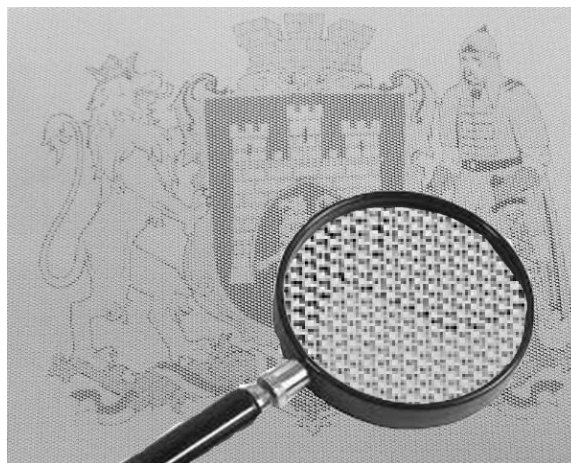

Fig. 9. Scanned image and its enlarged fragment

Summary

To protect printed images we need to develop new methods of rasterization. It was suggested to implement graphic construction of Ateb-function for rasterization problem. It was searched properties of figures, based on Ateb-functions. It was searched properties of cylinder, whose cross section is superellipse. It was calculated an area of superellipse, based on Ateb-functions. It was constructed a raster element. The picture was rasterized, using properties of Ateb-functions. Picture, which is rasterized with this method, is protected from copying. This method of rasterization makes picture of higher quality, because new form of raster element gives the chance to avoid loss of clearness on the edge of raster cell. Rasterization is in vector format. The file is stored in pdf file, that is adapted for printing the image.

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