UDC 004.94;674.047
Boretska I., Sokolovskyy Ya.
National forestry university of Ukrainian, Lviv, Chuprynkastreet, 103

THE MATHEMATICAL MODELING OF CONVECTIVE PROCESS OF WOOD DRYING WITH TAKING INTO ACCOUNT PHASE TRANSITIONS BOUNDARIES

The mathematical model of heat and mass transfer in capillary-porous bodies for drying with taking into account the zone of evaporation movement is given in the work. The analytical solutions of nonlinear problems of heat and mass transfer for unsteady regimes of drying are received. The effect of duration of drying regimes on the temperature of phase transition in wood is investigated.

Keywords: mathematical model, heat and mass transfer, phase transitions, process of drying.

Relevance of research

The choice of rational technologies of convective drying of materials with ensuring the maximum intensity of processes and minimum energy resources is an important task. The difficulty of drying process of capillary-porous materials, including wood, is conditioned by the occurrence of interrelated physical phenomena, heat and mass transfer and deformation in the condition of high variability of structural and physical properties of hygroscopic bodies.

The modern approach to the mathematical modeling of heat and mass transfer in the process of drying materials is based on using the theory of heat conduction. It allows to use the classical methods of mathematical physics for solving certain boundary problems. However, for considering the phase transitions, especially including the moving evaporation zone inside the material is a necessary to use investigation methods of non classical heat conduction. Therefore, the development of mathematical models of heat and mass transfer processes during the drying of capillary-porous materials, including wood, with considering the movement of phase transitions boundary, investigation of patterns of their change allow improving of existing one and developing new technologies of convective drying.

Analysis of research

The investigation of the mechanism of deepening the evaporation area of drying material was first investigated by T. Sherwood [1, 2]. Further theoretical and experimental confirmation of the deepening the evaporation surface are shown in the following works [1, 3-5]. There are several approaches to the modeling of heat mass transfer processes in materials during drying with taking into account the movement of deepening evaporation zone. The criterion of phase transition that is changing according to the coordinates of the body, is taken into account in the boundary conditions for some materials.

With using a different approach, the modeling of the moisture removal process is considered within the Stefan problem [6, 7]. From a mathematical point of view, the boundary problems of heat and mass transfer or generalized boundary problems are fundamentally different from the classical problems. The dependence of the characteristic size of the evaporation zone on the time, complicates the using of classical methods of variables separation or integral transformation. The analytical studies were conducted for limited cases of the pre-known law of the boundaries motion, such as linear or parabolic. For this purpose the methods of thermal potentials [8, 9], contour integration [10] power series [8, 11], "instant" eigenfunctions Greenberg [12] were used. The receiving of analytical solutions of boundary problem of the generalized type in the moving boundary are of the phase transition by a random law was reduced to integral-differential equation, including Voltaire integral equations of the 2nd kind with complex nuclei. Therefore, only qualitative results of the behavior of such systems were set.

In this aspect, the following works are important [6, 13]. The received functional transformation allowed to apply classical methods of separation of variables taking into account the various changes of boundaries motion for appropriate boundary conditions in the classical coordinate systems. The quite effective method of solving problems of heat and kinetics of drying wet materials is a method of differential series [8]. It allows to receive a numerical and analytical solutions of boundary problem of heat and mass transfer for boundary conditions of the third kind.

The using of numerical methods with a clear allocation of boundary phase transitions for multidimensional problems heat and mass transfer with phase transition in many cases is related to the algorithmic difficulty and considerable computational cost. In particular, to find an approximate solution, the method of "pass-through" calculation [7] with the use of generalized formation of classical Stefan problem, in which the unknown appears not temperature but enthalpy are widespread used. Difference schemes [14, 15] and its various modifications are used for the numerical realization of mathematical models.

The purpose of this work is to construct a mathematical model and investigate processes heat and mass transfer during the drying of capillary-porous materials with taking into account the movement of the evaporation zone for unsteady drying regime, given as a polynomial of exponential functions. The boundary problem of heat conduction in the case of three step linear change of drying agent was investigated in the works [16, 17].

The mathematical model of heat and mass transfer with considering boundary of phase transition

Let us consider the problem of heat and mass transfer capillary-porous plate which thickness is $2L$ ($-L \le z \le L$, z – is a coordinate) under the influence of its border of the unsteady convective heat flux. The plate is referred to Cartesian coordinate system. In the case of the dispersion of pore sizes, we should take into account the presence of dried, two-phase and liquid zone. The pores are opened and saturated with liquid in the initial time. During the drying process, the plate is in contact with the gaseous environment, which is a mixture of dry air and steam. During the convective drying, the heat that is given by drying agent is spent on evaporation of the liquid, heating the material and overcome the binding energy of the material. The heat partially returns to the environment with the evaporation damp. The velocity of heat and mass exchange, when internal diffusion resistance is small, depends only on the resistance of the boundary layer of drying agent. The thickness and hydrodynamic condition of the plate depend on the relative velocity of heated air. The uniformity and symmetry of two-way process of drying material is being created by the circulation of heated air.

The mathematical model of heat and mass transfer in the plate during the drying, taking into account changes over time of the boundary of phase transition L_m is constructed as follows.

The energy equation is:

$$\left[\Pi(c_v\rho_v + c_a\rho_a) + (1-\Pi)c_s\rho_s\right]\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_c \frac{\partial T}{\partial z}\right) + F\,, \quad L_m \le z \le L\,, \quad 0 \le L_m \le L\,, \tag{1}$$

where c_v, c_a, c_s – are specific heats of the steam, air, wood skeleton, ρ_v, ρ_a, ρ_s - are densities of the same components of the wet plate, F – is an internal source; Π – is a porosity material.

If the water vapor in the environment is not saturated, then the process of evaporation from the vapor space of the plate from the surface into the depths of the body. We consider that the pores are the same size and neglect the tape mass transfer mechanism. We consider that thermal conditions on the surfaces of the plate are identical and the drying process is symmetrical to the median surface. The drying process is the same throughout the cross-section of the body because of the absence of dispersion of pore size by radii. During the drying process, the area of drained pores and pore saturated with liquid occurs in the plate. The limit of contact L_m of these zones extends to the deep of plate's porous.

Note that drying can not be complete. The moisture stays in the form of a wall surface layer (bound moisture) in wet hydrophilic pores. We do not consider the mechanism of tape transfer fluid caused by the pressure gradient and the phenomenon of thermoosmosis. We consider that these processes have far less impact than phase transition processes. In the drying area, the air and water vapor are present in the plate's pores. Further, we assume that cross pore size significantly greater than the mean free path of existing molecules. It allows you to add expressions for the flow of air $\vec{j}_a$ and vapor $\vec{j}_v$ two-component mixture in the drying zone with expressions [18]: $\vec{j}_k = \rho_k\vec{\upsilon} - D'\vec{\nabla}\rho_k$, $k = a; v$, where D' is an effective diffusion coefficient in binary pores.

The flow of air inside the plate is quantitatively much smaller than the stream of vapor outside, caused by the phase transition "water-vapor." In this regard, we will continue to ignore the stream of air $\vec{j}_a$, considering that $\vec{j}_a = 0$. In the following, we assume a local change of the density of steam ρ_v as a constant, ie put $\partial\rho_v / \partial t = 0$. According to [18], we obtain the mass balance equation vapor $\vec{\nabla}\cdot\vec{j}_v = 0$.

The influence of porous structure of wooden plate is being counted by involving to the Stefan-Maxwell equations of efficient binary coefficients of interaction. The Stefan-Maxwell system of equations is complemented with Darcy filtration equations with effective viscosity μ_g and permeability K_g characteristics

$$\upsilon = -\frac{K_g}{\mu_g}\frac{\partial P_g}{\partial z}, \tag{2}$$

where P_g – is a gas pressure in the pores.

We assume that for the gas mixture, the ideal gas law is performed

$$P_g = \left(\frac{\rho_a}{M_a} + \frac{\rho_v}{M_v}\right)RT, \tag{3}$$

where M_a, M_v – are the molar mass of air and vapor; R – is a gas constant.

The equation of convective mass transfer in porous plate in the area $L_m < z < L$ will be the following:

$$\rho_a \frac{K_g}{\mu_g}\frac{\partial P}{\partial z} + D'\frac{\partial \rho_a}{\partial z} = 0,\ \frac{\partial}{\partial y}\left(\rho_v \frac{K_g}{\mu_g}\frac{\partial P}{\partial z} + D'\frac{\partial \rho_v}{\partial z}\right) = 0 \tag{4}$$

Using the equation of state for the gas mixture and Darcy law (2), we write the Stefan-Maxwell equation relatively to functions ρ_a, ρ_v - air and vapor densities

$$\rho_a \frac{K_g}{\mu_g}\vec{\nabla}\left(\frac{\rho_a}{M_a} + \frac{\rho_v}{M_v}\right)RT + D'\vec{\nabla}\rho_a = 0,$$

$$\vec{\nabla}\cdot\left[\rho_v \frac{K_g}{\mu_g}\vec{\nabla}\left(\frac{\rho_a}{M_a} + \frac{\rho_v}{M_v}\right)RT + D'\vec{\nabla}\rho_v\right] = 0, \tag{5}$$

where K_g – permeability coefficient that is depended on the radius and shape of pores, μ_g - the coefficient of dynamic viscosity of the gas.

The noted nonlinear system of differential equations (5) is valid in drying pores area, which is limited by limiting surface (S) and the surface (S*), that is determined by coordinate L_m.

For the further research we consider [2] that vapor density can be equal to the vapor density on the moving surface (S*), ie

$$\rho_v = \rho_{vn} \text{ on the surface } z = L_m, \tag{6}$$

Conditions on the surfaces $z = L$ of the gas zone is written as follows:

$$\rho_v \frac{K}{\mu_g}\frac{\partial P}{\partial z} + D'\frac{\partial \rho_v}{\partial z} = -j,\ \rho_a = \rho_{a0} \text{ on the surface } z = L, \tag{7}$$

where $j = \tilde{\beta}(\rho_v - \rho_{v0})$; $\tilde{\beta}$ is a mass transfer coefficient. Since the mass transfer problem is further solved in quasi setting (for medium drying atmospheric pressure), and T_m is the phase transition temperature that dependents on the saturation pressure, we assume $T_m = f(P_n)$, where P_n is the vapor pressure.

The equation of energy balance on the moving boundaries of phase transitions $z = L_m$ is written as:

$$-\lambda_c \frac{\partial T}{\partial z}\Big|_{z=L_m+0} = r_k \frac{K_g}{\mu_g}\frac{\partial P_g}{\partial z}\Big|_{z=L_m+0}, \tag{8}$$

$$T = T_m ,\tag{9}$$

where λ_c – the conductivity of dried zone, T_m – is yet unknown phase transition temperature, which depends on saturated vapor pressure P_n.

The linearized equation of state on moving boundary $z = L_m$ of phase transitions: $T_m = T_{mk} + \alpha_{mk} P_n$, where $T_{mk} = \frac{9T_k V_k}{8V}, \alpha_{mk} = \frac{3T_k V}{8V_k}$, T_k, V_k are critical temperature and pressure. In particular, in the work [18] $T_m = 83 + 16 \cdot 10^{-5} P_n$. In other cases, the coefficients T_{mk}, α_{mk} are determined by temperature conditions of drying the concrete material. Boundary conditions on the border $z = L$ express the heat exchange between the plate's surfaces and the drying agent by Newton's law

$$\lambda_c \frac{\partial T}{\partial z} + \tilde{\alpha}\left[T - T_a(t)\right] = 0 ,\tag{10}$$

where $\tilde{\alpha}$ – is a coefficient of heat transfer; $T_a(t)$ is a variable in the time temperature of drying agent. The function $T_a(t)$ can be represented as a polynomial of exponential functions

$$T_a(\tau) = T_1 + (T_2 - T_1)\sum_{i=1}^{N}\left(a_i e^{-b_i \tau}\right) = \alpha_0 + \sum_{i=1}^{N}\left(\alpha_i e^{-b_i \tau}\right).\tag{11}$$

We re-marked for convenience the following: $\alpha_0 = T_1$, $\alpha_i = (T_2 - T_1)a_i$; $D' = D^1{}_{ij} = \left(1/D_i^{\infty} + (1 - \alpha_{ij} z_i)/D_{ij}\right)^{-1}$; $D^1{}_{va} = D^1{}_{av} = D'$ – an effective diffusion coefficient; $D_{va} = D_{av} = D_{ij}$ – an effective binary diffusion coefficient in the macropores the second term in the expression $D_v{}^{\infty} = D^{\infty}$ $D_v{}^{\infty} = D^{\infty}$ takes into account the Knudsen flow effect of vapor in the micropores. Parameters α_i, b_i $(i = \overline{1, N})$ are determined by approximation a specific temperature regime of drying agent.

At the initial time the temperature of the plate satisfies the following

$$T(t = 0) = f(z),\tag{12}$$

where $f(z)$ – the function of temperature for the periods of constant and falling speeds of drying.

We define the relative humidity W_m of the porous plate as the ratio of mass m_L of the liquid in it at a time *t* to mass m_{Ln} of the liquid at a time $t = 0$ (moisture saturation)

$$W_m = \frac{m_L}{m_{Ln}} = \frac{\Pi L_m S \rho_L}{\Pi L S \rho_L} = \frac{L_m}{L} = \bar{z}_m ,\tag{13}$$

where S – is a square of pores; ρ_L – the density of water. The used during the drying mass Δm of liquid is defined as

$$\Delta m = m_{Ln} - m_L = \Pi S \rho_L L(1 - \bar{z}_m).\tag{14}$$

Thus, the relative saturation of pores by liquid in capillary-porous material under condition of acceptance the cylindrical capillaries model without dispersion of pores size matches the dimensionless coordinate of the interface "liquid-gas".

The speed of change of the fluid mass in the plate is determined by vapor flow j in it, ie

$$d\Delta m / dt = j\,S .\tag{15}$$

Then the equation of motion of boundary's phase transition is:

$$\frac{d\bar{z}_m}{d\tau} = -\frac{j(\bar{z}_m)}{\Pi \rho_L L}\tag{16}$$

for the initial condition

$$\bar{z}_m = 1 .\tag{17}$$

Equations (1) - (17) create constitute the mathematical model that describes the convective process of drying of capillary-porous body (plate) with consideration of moving boundaries of phase transitions. Note the nonlinearity of equations in formulated mathematical model.

The investigation of heat and mass transfer for unsteady regime of the drying agent

In the heat equation (1) of the mathematical model we pass to dimensionless coordinate system. We use the following replacement $\tau = a_T t / L^2, \bar{z} = z / L^1$, $\bar{z}_m = L_m / L$, where $\bar{z}_m$ – is a dimensionless coordinate of boundaries phase transition. The heat equation takes the following form:

$$\frac{\partial T}{\partial \tau}(\bar{z}, \tau) = \frac{\partial^2 T(\bar{z}, \tau)}{\partial \bar{z}^2} + F, \tag{18}$$

where $F = \dfrac{L^2 W_1}{a_T C\rho}$ – is an internal source; $a_T = \lambda_c / (\Pi[(1-\bar{z}_m)(C_v\rho_v + C_a\rho_a)] + (1-\Pi)C_s\rho_s)$

The boundary conditions on the border $\bar{z} = 1$ will be the following:

$$\partial T / \partial \bar{z} + H_T [T - T_a(\tau)] = 0, \; H_T = \tilde{\alpha} L / \lambda, \tag{19}$$

$\tilde{\alpha}$ – is the heat transfer coefficient, and on the verge of phase transition $\bar{z} = \bar{z}_m$ we consider that

$$T = T_m. \tag{20}$$

We construct the solution (18) - (20) for the initial condition: $T(\bar{z},0) = f(\bar{z})$.

We are looking for the solution of the heat transfer equation in the following form $T = T_1 + T^*$, where $T_1 = \chi_0 + \sum_{i=1}^{N} \chi_i e^{-b_i \tau}$ represents the solution of the heat equation that satisfies the boundary conditions of the problem without initial conditions, and T^* is the solution of the heat equation that satisfies the initial condition and uniform boundary conditions. We substitute this expression in equation (18) and compare similar terms in the equation and boundary conditions (19), (20) so that the values χ_0, χ_i satisfy equation and boundary conditions but function T^* – satisfies zero boundary conditions and touch up original condition. Then, to find χ_0 i χ_1 $(i = \overline{1,n})$, we obtain the differential equations:

$$d^2\chi_0 / d\bar{z}^2 = 0, \; d^2\chi_i / d\bar{z}^2 + b_i \chi_i = 0, \tag{21}$$

which satisfy the following conditions:

$$\partial\chi_0(1)/\partial\bar{z} + H_T[\chi_0(1) - \alpha_0] = 0, \; \chi_0(\bar{z}_m) = T_m;$$
$$\partial\chi_i(1)/\partial\bar{z} + H_T[\chi_i(1) - \alpha_i] = 0, \quad \chi_i(\bar{z}_m) = 0. \tag{22}$$

To obtain T^* we receive the boundary problem:

$$\partial^2 T^*(\bar{z}, \tau)/\partial\bar{z}^2 = \partial T^*(\bar{z}, \tau)/\partial\tau; \tag{23}$$
$$\partial T^*(1, \tau)/\partial\bar{z} + H_T T^*(1, \tau) = 0; \; T^*(\bar{z}_m, \tau) = 0;$$
$$T^*(\bar{z}, 0) = f(\bar{z}) - \chi_0(\bar{z}, 0) - \sum_{i=1}^{n} \chi_i(\bar{z}, 0). \tag{24}$$

To construct the solution of the boundary problem (23) - (24) we uses the influence function of the problem (18) - (20) with homogeneous boundary conditions in the absence of the function of internal sources. The influence function has the form

$$G(\bar{z}, \bar{z}_m, \xi, \tau) = \sum_{n=1}^{\infty} 2H_T \frac{\left[\dfrac{\sin\mu_n(\bar{z} - \bar{z}_m)}{\mu_n}\right]\left[\cos\mu_n(1-\xi) + H_T \dfrac{\sin\mu_n(1-\xi)}{\mu_n}\right] e^{-\mu_n^2 \tau}}{\cos\mu_n(1-\bar{z}_m)\left[(\mu_n^2 + H_T{}^2)(1-\bar{z}_m) + H_T\right]}, \tag{25}$$

where μ_n – are the roots of the transcendent equation $tg\mu_n(1-\bar{z}_m) = -\mu_n / H$.

Expressions for χ_0, χ_i we calculate as $\chi_0 = c_0 + d_0\bar{z}$; $\chi_i = c_i \cos\sqrt{b_i}\,\bar{z} + d_i \sin\sqrt{b_i}\,\bar{z}$.

Having found the expressions χ_i for the boundary conditions (22), we receive dependency for finding unknown quantities c_0, d_0, c_i, d_i $(i=\overline{1,n})$:

$$c_0 = T_m - H_T \frac{\alpha_0 - T_m}{\left[H_T(1-\bar{z}_m)+1\right]}\bar{z}_m,\ d_0 = H_T \frac{\alpha_0 - T_m}{\left[H_T(1-\bar{z}_m)+1\right]};$$

$$c_i = \frac{H_T\left[\sin\sqrt{b_i}(1-\bar{z}_m) - \alpha_i \cos\sqrt{b_i}\bar{z}_m\right]}{\sqrt{b_i}\cos\sqrt{b_i}(1-\bar{z}_m)}\frac{\sin\sqrt{b_i}(1-\bar{z}_m)}{\cos\sqrt{b_i}\bar{z}_m},$$

$$d_i = -\frac{H_T\left[\sin\sqrt{b_i}(1-\bar{z}_m) - \alpha_i \cos\sqrt{b_i}\bar{z}_m\right]}{\sqrt{b_i}\cos\sqrt{b_i}(1-\bar{z}_m)}. \qquad (26)$$

Note that c_i, d_i are functions of moving coordinate $\bar{z}_m$ of the phase transition.
Then

$$\chi_0 = T_m + H_T \frac{\alpha_0 - T_m}{\left[H_T(1-\bar{z}_m)+1\right]}(\bar{z}-\bar{z}_m);\ \chi_i = C_i \frac{\sin\sqrt{b_i}(\bar{z}-\bar{z}_m)}{\sqrt{b_i}}, \text{ where } (i=\overline{1,n}). \qquad (27)$$

From the (27) we can see that $\chi_i(\bar{z}_m) = 0$ for $i = 1,2\ldots N$.

The function $\sum_{i=1}^{N} \chi_i(\xi)$ we use note in the form $\sum_{i=1}^{N} \chi_i(\xi) = \sum_{i=1}^{N} C_i \frac{\sin\sqrt{b_i}(\xi-\bar{z}_m)}{\sqrt{b_i}}$, where

$$C_i\left(\sqrt{b_i}, \bar{z}_m\right) = \frac{H_T\left[\sin\sqrt{b_i}(1-\bar{z}_m) - \alpha_i \cos\sqrt{b_i}\bar{z}_m\right]}{\cos\sqrt{b_i}(1-\bar{z}_m)\cos\sqrt{b_i}\bar{z}_m}. \qquad (28)$$

To find $T^*(\bar{z}, \bar{z}_m, \tau)$ we use function (25), that satisfies the conditions $T_m(\bar{z}_m, \tau) = 0$, $T_a(1, \tau) = 0$. The solution of boundaries problem (23), (24) can be received with the use of impact function (25), if we use the following expression instead of $f(\bar{z})$

$$f_1(\bar{z}) = f(\bar{z}) - \overline{T}_0, \text{ where } \overline{T}_0 = \sum_{i=0}^{N} \chi_i(\bar{z}). \qquad (29)$$

Then, according to (21)-(22) we receive

$$T^*(\bar{z}, \tau) = \sum_{n=1}^{\infty} A_n \frac{\sin\mu_n(\bar{z}-\bar{z}_m)}{\mu_n}\exp\left(-\mu_n^2\tau\right), \qquad (30)$$

where

$$A_n = \frac{1}{\Delta_n(\mu_n, \bar{z}_m)}\int_{\bar{z}_m}^{1} f_1(\bar{z})\left[\cos\mu_n(1-\bar{z}) + \frac{H}{\mu_n}\sin\mu_n(1-\bar{z})\right]d\bar{z}. \qquad (31)$$

$$\Delta_n(\mu_n, \bar{z}_m) = \cos\mu_n(1-\bar{z}_m)\left[\left(\mu_n^2 + H^2\right)(1-\bar{z}_m) + H\right]$$

To find $T^*(\bar{z}, \bar{z}_m, \tau)$ we use function (21), that satisfies the conditions $T_m(\bar{z}_m, \tau) = 0$,$T_a(1, \tau) = 0$.

If the initial temperature distribution is even, which is $T(\bar{z}, 0) = T_0$, then with taking into account (21), the contribution to the initial temperature $T^*(\bar{z}, \bar{z}_m, \tau)$ will be as follows:

$$T_0^*(\bar{z}, \bar{z}_m) = \sum_{n=1}^{\infty} \frac{\sin\mu_n(\bar{z}-\bar{z}_m)}{\mu_n} A_n^*(\bar{z}_m) e^{-\mu_n^2\tau}, \qquad (32)$$

$$A_n^* = \frac{2T_0 H_T}{\Delta_n}\left\{\frac{\sin\mu_n(1-\bar{z}_m)}{\mu_n} - \frac{H_T}{\mu_n^2}\left[\cos\mu_n(1-\bar{z}_m) - 1\right]\right\} \qquad (33)$$

The contribution to $T^*(\bar{z}, \bar{z}_m)$ from the sum $\sum_{i=1}^{N} \chi_i(\bar{z})$ we can note in the form

$$T_\varphi^{\ *} = -\int_{\bar{z}_m}^{1}\sum_{i=1}^{N}\chi_i(\xi)\times G(\bar{z},\bar{z}_m,\xi,\tau)d\xi =$$

$$= -\sum_{n=1}^{\infty} B_n e^{-\mu_n^2\tau}\left[\frac{\sin\mu_n(\bar{z}-\bar{z}_m)}{\mu_n}\right]\int_{\bar{z}_m}^{1}\left[\cos\mu_n(1-\xi)+H_T\,\frac{\sin\mu_n(1-\xi)}{\mu_n}\right]\sum_{i=1}^{N} C_i\sin\sqrt{b_i}(\xi-\bar{z}_m)d\xi\,,$$

where

$$B_n = \frac{2H_T}{\cos\mu_n(1-\bar{z}_m)\left[\left(\mu_n^2+H_T^{\ 2}\right)(1-\bar{z}_m)+H_T\right]}; \tag{34}$$

$$\int_{\bar{z}_m}^{1}\left[\cos\mu_n(1-\xi)+H_T\,\frac{\sin\mu_n(1-\xi)}{\mu_n}\right]\sum_{i=1}^{N} C_i\sin\sqrt{b_i}(\xi-\bar{z}_m)d\xi = Z_{n1}-Z_{n2}\,,$$

where $Z_{n1} = \sum_{i=1}^{N} C_i\left\{-\frac{\cos\left[\sqrt{b_i}(1-\bar{z}_m)\right]}{2\left(\sqrt{b_i}-\mu_n\right)}-\frac{\cos\left[\sqrt{b_i}(1-\bar{z}_m)\right]}{2\left(\sqrt{b_i}+\mu_n\right)}+\frac{\cos\left[\mu_n(1-\bar{z}_m)\right]}{2\left(\sqrt{b_i}-\mu_n\right)}+\frac{\cos\left[\mu_n(1-\bar{z}_m)\right]}{2\left(\sqrt{b_i}+\mu_n\right)}\right\}$,

$$Z_{n2} = \sum_{i=1}^{N}\frac{C_iH_T}{\mu_n}\left\{\frac{\sin\left[\left(\sqrt{b_i}(1-\bar{z}_m)\right)\right]}{2\left(\sqrt{b_i}-\mu_n\right)-}-\sin\frac{\left(\sqrt{b_i}(1-\bar{z}_m)\right)}{2\left(\sqrt{b_i}+\mu_n\right)}-\frac{\sin\left[\left(\mu_n(1-\bar{z}_m)\right)\right]}{2\left(\sqrt{b_i}-\mu_n\right)}+\frac{\sin\left[\left(\mu_n(1-\bar{z}_m)\right)\right]}{2\left(\sqrt{b_i}+\mu_n\right)}\right\}$$

To finish the drying process $\bar{z}_m = 0$ we will receive $Z_n(0) = Z_{n1}(0) - Z_{n2}(0)$, where

$$Z_{n1}(0) = \sum_{i=1}^{N} C_i\left\{-\frac{\cos\left[\sqrt{b_i}\right]}{2\left(\sqrt{b_i}-\mu_n\right)}-\frac{\cos\left[\sqrt{b_i}\right]}{2\left(\sqrt{b_i}+\mu_n\right)}+\frac{\cos\left[\mu_n\right]}{2\left(\sqrt{b_i}-\mu_n\right)}+\frac{\cos\left[\mu_n\right]}{2\left(\sqrt{b_i}+\mu_n\right)}\right\},$$

$$Z_{n2}(0) = \sum_{i=1}^{N}\frac{C_iH_T}{\mu_n}\left\{\frac{\sin\left[\left(\sqrt{b_i}\right)\right]}{2\left(\sqrt{b_i}-\mu_n\right)}-\sin\frac{\left(\sqrt{b_i}\right)}{2\left(\sqrt{b_i}+\mu_n\right)}-\frac{\sin\left[\left(\mu_n\right)\right]}{2\left(\sqrt{b_i}-\mu_n\right)}+\frac{\sin\left[\left(\mu_n\right)\right]}{2\left(\sqrt{b_i}+\mu_n\right)}\right\}.$$

$$Z_n(1) = (Z_{n1}(1) - Z_{n2}(1) = 0$$

$$Z_{n1}(1) = \sum_{i=1}^{N} C_i\left\{-\frac{1}{2\left(\sqrt{b_i}-\mu_n\right)}-\frac{1}{2\left(\sqrt{b_i}+\mu_n\right)}+\frac{1}{2\left(\sqrt{b_i}-\mu_n\right)}+\frac{1}{2\left(\sqrt{b_i}+\mu_n\right)}\right\} = 0\,,$$

$$Z_{n2}(1) = \sum_{i=1}^{N}\frac{C_iH_T}{\mu_n}\left\{\frac{\sin 0}{2\left(\sqrt{b_i}-\mu_n\right)-}-\frac{\sin 0}{2\left(\sqrt{b_i}+\mu_n\right)}-\frac{\sin 0}{2\left(\sqrt{b_i}-\mu_n\right)}+\frac{\sin 0}{2\left(\sqrt{b_i}+\mu_n\right)}\right\} = 0$$

$$T_\chi^{\ *} = \sum_{n=1}^{\infty}\frac{\sin\mu_n(\bar{z}-\bar{z}_m)}{\mu_n}Z_n(\bar{z}_m)e^{-\mu_n\tau}\,, \tag{35}$$

$$Z_n(\bar{z}_m) = Z_{n1} - Z_{n2}\,,$$

The temperature in the unsteady convective heating (11) can be noted as

$$T(\bar{z},\tau) = \chi_0(\bar{z}) + \sum_{i=1}^{N}\left[\chi_i(\bar{z})e^{-b_i\tau}\right] + \sum_{n=1}^{\infty}\frac{\sin\mu_n(\bar{z}-\bar{z}_m)}{\mu_n}Z_n(\bar{z}_m)e^{-\mu^2{}_n\tau}\,, \tag{36}$$

where

$$Z_n(\bar{z}_m) = \frac{2T_0H_T}{\Delta_n}\left\{\frac{\sin\mu_n(1-\bar{z}_m)}{\mu_n}-\frac{H_T}{\mu_n^2}\left[\cos\mu_n(1-\bar{z}_m)-1\right]\right\} + (Z_{n1}-Z_{n2})\,. \tag{37}$$

With $T^* = T_0^* + T_\chi^*$ that satisfies the conditions $T^*(\bar{z}_m,\bar{z}_m) = T^*(\bar{z}_m,1) = 0$.

The equation (36) allows us to calculate the temperature at any point along the thickness of the plate and the arbitrary time depending on the coordinates of the phase transition.

The investigation of the mass transfer process for the symmetric convective drying

It is considered that the thermal conditions on the surface of the plate are the same and the drying process is symmetrical to the median surface. In connection with the symmetry of the problem, we will conduct a research for one of the halves of the plate. The mathematical model of mass transfer will be investigated (5-7). Introduce the following dimensionless variables $\rho_a = \rho_{a0}\bar{\rho}_a$, $\rho_v = \rho_n \bar{\rho}_v$, $z = L\bar{z}$ for a given temperature of the drying area $T_c = \frac{1}{\tau_\Pi}\int_0^{\tau_\Pi}\int_0^1 T d\bar{z} d\tau$, where ρ_{ao} – is an air density on the outer walls of the plate. Having integrated the second equation of the system (5), we receive:

$$\frac{d\bar{\rho}_a}{d\bar{z}} + b\frac{d\bar{\rho}_v}{d\bar{z}} + a\frac{1}{\bar{\rho}_a}\frac{d\bar{\rho}_a}{d\bar{z}} = 0, \tag{38}$$

$$\bar{\rho}_v\left(\frac{d\bar{\rho}_a}{d\bar{z}} + b\frac{d\bar{\rho}_v}{d\bar{z}}\right) + a\frac{d\bar{\rho}_v}{d\bar{z}} + \beta^* a(\bar{\rho}_{v1} - \bar{\rho}_{v0}) = 0, \tag{39}$$

The following boundary conditions (6), (7), (36), (37) take the following form

$$j = \beta'\rho_n(\bar{\rho}_{v1} - \bar{\rho}_{v0}),\ \bar{\rho}_a = 1 \text{ on the surface } \bar{z} = 1, \tag{40}$$

$$\bar{\rho}_v = 1 \text{ on the surface } \bar{z} = \bar{z}_m \tag{41}$$

Here $a = \frac{D' M_a \mu_g}{K_g \rho_{a0} R T_c}$, $b = \frac{\rho_n}{\rho_{a0}}\frac{M_a}{M_v}$, $\beta^* = \frac{L\beta'}{D'}$.

Integrating the second equation (38) by $\bar{z}$, we receive $\bar{\rho}_a + b\bar{\rho}_v + a\ln\bar{\rho}_a = C_1$.

The constant of integration $C_1 = 1 + b\bar{\rho}_{v1}$ was found from the second condition (40), where $\bar{\rho}_{v1}$ is an unknown dimensionless value of the vapor density on the surface $\bar{z} = 1$. After linearization with considering the fact that air flow in the pores is neglected, (then dimensionless density of the air in the pores differs little from unity), we get: $\frac{d\bar{\rho}_a}{d\bar{z}} = -\frac{b}{1+a}\frac{d\bar{\rho}_v}{d\bar{z}}$. Then equation (40) with considering (41) is written as:

$$\frac{b}{(1+a)}\bar{\rho}_v\frac{d\bar{\rho}_v}{d\bar{z}} + \frac{d\bar{\rho}_v}{d\bar{z}} + \beta^*(\bar{\rho}_{v1} - \bar{\rho}_{v0}) = 0.$$

The general integral of this equation is:

$$\frac{b}{(1+a)}\frac{\bar{\rho}_v^2}{2} + \bar{\rho}_v + \beta^*(\bar{\rho}_{v1} - \bar{\rho}_{v0})\bar{z} = C_2, \tag{42}$$

where $C_2 = \frac{b}{(1+a)}\frac{1}{2} + 1 + \beta^*(\bar{\rho}_{v1} - \bar{\rho}_{v0})\bar{z}_m$ is a constant of integration, which is defined by the condition (41).

Dimensionless density of the vapor for the thickness of layer is determined by the following formula

$$\bar{\rho}_v(\bar{z}, \bar{z}_m) = -A + \sqrt{A_2 - 2A_1(\bar{\rho}_{v1} - \bar{\rho}_{v0})(\bar{z} - \bar{z}_m)}, \tag{43}$$

where $A = (1+a)/b$; $A_1 = -\beta^* A$; $A_2 = (A+1)^2$.

The vapor pressure at any point $\bar{z}$ along the thickness of the plate is determined by the following formula:

$$P(\bar{z}, \bar{z}_m) = \left[-A + \sqrt{A_2 - 2A_1(\bar{\rho}_{v1} - \bar{\rho}_{v0})(\bar{z} - \bar{z}_m)}\right]\rho_n\frac{RT}{M_v}.$$

Then on the plate surface ($\bar{z} = 1$) occurs the following equation

$$\frac{b}{(1+a)}\frac{(\bar{\rho}_{v1}^2 - 1)}{2} + \bar{\rho}_{v1} - 1 + \beta^*(\bar{\rho}_{v1} - \bar{\rho}_{v0})(1 - \bar{z}_m) = 0.$$

Its solution determines the value of the dimensionless density vapor. Define $\bar{z}^*_m = 1 - \bar{z}_m$. $\bar{z}^*_m = W^*_m$ the change of the relative humidity of the drying process. The physically acceptable branch of solution is the following:

$$\bar{\rho}_{v1} = -(A + A_1\bar{z}_m^*) + \sqrt{A_2 + A_3\bar{z}_m^* + A_1^2\bar{z}_m^{*2}}, \tag{44}$$

where

$$A_3 = 2A_1\left(A + \bar{\rho}_{v0}\right). \tag{45}$$

With the known density of steam on the wall, under the conditions (39), the value of stream j will be presented in the following way:

$$j = H_m\left[-\left(a_1 + A_1\bar{z}_m^*\right) + \sqrt{A_2 + A_3\bar{z}_m^* + A_1^2\bar{z}_m^{*2}}\right], \tag{46}$$

where $a_1 = A + \eta_0$; $H_m = \beta'\rho_n$.

Define the relative humidity W_m porous plates according to formula (13), (14), where

$$\Delta m_L = \Pi S\rho_L L\left(1 - \bar{z}_m\right) = \Pi S\rho_L L\bar{z}_m^* \tag{47}$$

is the weight of the liquid that was spent during the drying.

From (46) we obtain the equation for determining the relative humidity changes over the time and equations of motion of the boundaries of separation of phases liquid-gas

$$\frac{d\bar{z}*_m}{dt} = \frac{j(\bar{z}*_m)}{\Pi\rho_L L} = \frac{\beta\rho_n}{\Pi\rho_L L}\left[-\left(a_1 + A_1\bar{z}_m^*\right) + \sqrt{A_2 + A_3\bar{z}_m^* + A_1^2\bar{z}_m^{*2}}\right] \tag{48}$$

for the initial conditions

$$\bar{z}_m^* = 0 \text{ to } \tau = 0. \tag{49}$$

For the receiving the solution of the Cauchy (48), (49) we use the substitution of variables

$$\sqrt{A_2 + A_3\bar{z}_m^* + A_1^2\bar{z}_m^{*2}} = \phi + \bar{z}_m^* A_1. \tag{50}$$

Taking into account $\bar{z}_m^* = \dfrac{\phi^2 - U}{A_3 - 2\phi A_1}$ and $\dfrac{d\bar{z}_m^*}{d\phi} = 2\dfrac{\left(-A_1\phi^2 + A_3\phi - A_1A_2\right)}{\left(A_3 - 2A_1\phi\right)^2}$, will result equation (48) to the form:

$$2\left(\frac{-A_1\phi^2 + A_3\phi - A_1A_2}{\left(A_3 - 2A_1\phi\right)^2}\right)\frac{d\phi}{d\tau} = H\left(\phi - a_1\right). \tag{51}$$

Taking into account $\phi - a_1 = \dfrac{2A_1\phi - A_3}{A_1}$, the last equation will be the following:

$$4A_1\left(\frac{A_1\phi^2 - A_3\phi + A_1A_2}{\left(2A_1\phi - A_3\right)^3}\right)d\phi = Hd\tau. \tag{52}$$

The general integral of the equation (52) has the form:

$$\frac{1}{2A_1}\ln\left|-A_3 + 2A_1\phi\right| + \frac{A_3^2 - 4A_1^2A_2}{4A_1}\frac{1}{\left(-A_3 + 2A_1\phi\right)^2} = H_m\tau + A_5, \tag{53}$$

where the sustainable integration A_5 will be received from the condition (52):

$$A_5 = \frac{1}{2A_1}\ln\left|-A_3 + 2A_1\sqrt{A_2}\right| + \frac{A_3 + 2A_1\sqrt{A_2}}{4A_1\left(A_3 - 2A_1\sqrt{A_2}\right)}. \tag{54}$$

From the obtained relationships can be determined the time for which the relative saturation reaches the value $\bar{z}_m^*$. It can be determined by:

$$\tau = \frac{\left\{\ln\left|\dfrac{2A_1\phi - A_3}{2A_1\sqrt{A_2} - A_3}\right| + \dfrac{1}{2}\left[A_3^2 - 4A_1^2A_2\left[\dfrac{1}{\left(2A_1\phi - A_3\right)^2} - \dfrac{1}{\left(2A_1\sqrt{\phi}A_2 - A_3\right)^2}\right]\right]\right\}}{2H_mA_1}. \tag{55}$$

The complete time of the drying process is given by:

$$\tau_{\Pi} = \frac{1}{2H_m A_1^2}\left\{\ln\left|\frac{\phi_0 - \left(A + \bar{\rho}_{v0}\right)}{\sqrt{A_2} - \left(A + \bar{\rho}_{v0}\right)}\right| + \right.$$

$$\left. + \frac{1}{2}\left[\left(A + \bar{\rho}_{v0}\right)^2 - \left(A+1\right)^2\right]\left[\frac{1}{\left[\phi_0 - \left(A + \bar{\rho}_{v0}\right)\right]^2} - \frac{1}{\left[\sqrt{A_2} - \left(A + \bar{\rho}_{v0}\right)\right]^2}\right]\right\}, \qquad (56)$$

where $\phi_0 = -A_1 + \sqrt{A_1^2 + A_2 + A_3} = -A_1 + \sqrt{A_1^2 + \left(A+1\right)^2 + 2A_1\left(A + \bar{\rho}_{v0}\right)}$.

This expression shows that drying time (55), and complete drying time (56) depend on the relative saturation vapor of the environment $\bar{\rho}_{v0}$ and received coefficient of mass transfer A_1. Minimizing the full time of drying with these parameters and knowing the dependence of mass transfer coefficients on Reynolds criterion, you can determine the blower velocity and the saturation moisture drying agent that minimize the time of drying porous body.

Conclusions

The one-dimensional nonstationary mathematical model of heat and mass transfer process for the drying of capillary-porous materials taking into account the borders of deepening evaporation area was presented. Analytical expressions for the study of temperature and moisture fields in drying plate at an arbitrary time, depending on the coordinates of the phase transition and changing the parameters of drying agent were received.

The method of research of established patterns of drying process of wet plate can be used to develop the computer-aided modeling systems and analysis of these processes, and for choosing rational technologies of drying.

List of references

1. Лыков, А. В. Теория сушки / А. В. Лыков. //− М.: Энергия, 1968 . − 472 с. 2. Шервуд Т.К. Сушка твердых тел, Гослесиздат., 1936 г. 3. Шубин Т.С. Сушка и тепловая обработка древесины // М.: Лесная промышленность, 1990.- 336 с. 4. Гайвась, Богдана. Математичне моделювання конвективного сушіння матеріалів з урахуванням механотермодифузійних процесів / Б. Гайвась // Фіз.-мат. моделювання та інформ. технології : наук. зб. – Львів : ЦММ ІППММ ім. Я. С. Підстригача НАН України. – 2010. – Вип. 12. – С. 9–37. 5. Гринчик Н. Н. Інтенсивність фазових переходів // Н.Н. Гринчик, В.А. Цурко // Инженерно-физический журнал. – 2002. – Т. 75, № 3. – С. 135–141. 6.Тихонов А.Н., Самарский А.А. Уравнение математической физики: учеб. Пособие. – М.:Изд-во МГУ, 1999, 799с. 7. Данилюк И.И. О задаче Стефана // Успехи математических наук, 1985, т.40, вып.5 (245), с.133 -185. 8. Любов Б.Я. Метод решения краевых задач диффузии для областей с границей, движущейся по произвольному закону / Б.Я. Любов, Э.М. Карташов // Изд. вузов. Серия Физика. – 1970, №12. – С.97-101. 9. Карташов Э.М. Метод решения обобщенных краевых задач уравнения теплопроводности в области с границей, движущейся по произвольному закону / Э.М. Карташов, Г.М. Бартенев, Б.Я. Любов // В кН. Тепло и массоперенос. – Минск. – 1972, т.8 – С. 274-285. 10. Квальвассер В.И. Методы нахождения функции Грина краевых задач уравнения теплопроводности для отрезка прямой с равномерно движущимися границами / В.И. Квальвассер, Я.Ф. Рутнер // Докл. АН СССР. – 1964, т. 165, №6. – С. 1273-1276. 11. Петухов Б.С. Теплообмен в ядерных энергетических установках / Б.С. Петухов, Л.Г. Генин, С.А. Ковалеви др. – М.: Издательство МЭИ, 2003. – 548 с. 12. Гринберг Г.А. Об одном возможном методе подхода к рассмотрению задач теории теплопроводности, диффузии, волновых и им подобных при наличии движущихся границ и о некоторых иных его приложениях / Г.А. Гринберг // Прикладная математика и механика. – 1967, т.31, №2. – С. 393-403. 13. Дульнев Г.Н., Новиков В.В. Процессы переноса в неоднородных средах. – Л.: Энергоатомиздат, 1991, 248 с. 14. Галанин М.П., Савенков Е.Б. Методы вычислений. Уравнения в частных производных. Интегральные уравнения: Учеб. пособие. – М.: Изд-во ИПМ им. М.В. Келдыша РАН, 2006, 72 с. 15. Патанкар С.В. Численное решение задач теплопроводности и конвективного теплообмена при течении в каналах. – М.: Изд-во МЭИ, 2003, 312 с. 16. Гайвась, Б.І. Осушення пористих тіл в сушильних установках при м'яких режимах / Б.І. Гайвась, І.Б Борецька // Науковий вісник НЛТУ України: Зб. наук-техн.праць. – Львів: НЛТУ України. -2011. Вип.21.9 – С.317-324. 17. Гайвась, Б. Вплив режиму сушильного агента на осушення пористих тіл. / Б. Гайвась, І. Борецька // Комп'ютерні технології друкарства – 2011.-

№26.Зб. наук праць – Львів.- С.231-240. 18. Дерягин Б.В. Поверхностные силы / Б.В. Дерягин, Чураев Н.В., Муллер В.М. // - М: Наука., 1985. -398с.