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Petro Kravets
Lviv Polytechnic National University,
Department of information systems and networks

ADAPTIVE GAME METHOD OF SIGNALS SYNCHRONIZATION OF THE DISTRIBUTED SYSTEMS

In this paper the adaptive game method for signals synchronisation of the distributed systems is offered. The method is constructed on the basis of stochastic approximation of a complementary slackness condition which describes the game solution on Nash in the equilizing mixed strategies. The game algorithm is developed and computer modelling of stochastic game for alignment of phases of harmonious signals is executed. Influence of parameters of a game method on quality of signals synchronisation of the distributed system is defined.

Keywords – synchronisation of the signals, the distributed system, the adaptive game method, the complementary slackness condition.

Introduction

Synchronisation is a searching by objects of the distributed system of a uniform rhythm of work. Synchronisation is reached by the coordination of frequencies, phases or other characteristics of signals which are generated by co-operating oscillatory systems.

Frequency synchronisation consists that system of generators, each of which has own frequency $\{\omega_i \,|\, i=1..L\}$, in the course of work starts to fluctuate with uniform frequency for all elements $\omega \in [Inf\{\omega_i\}, Sup\{\omega_i\}]$.

Phase synchronisation consists in an establishment of stationary values of differences of phases between signals of generators: $\Delta\varphi = \varphi_i - \varphi_j = const$ where $i, j = 1..L$.

Synchronisation of frequency and phase of signals are dependent among themselves. So, in devices of automatic frequency control regulation of frequency of a controlled signal is carried out on a difference of phases of basic and reference signals.

Uncontrollable external factors or noise can influence processes of measurement of frequencies or phases of signals. Complication of methods of signals synchronisation is result of it.

The phenomena of synchronisation of systems often occur in the nature, the technician and a society [1–2]. So, in the electrical engineer and electronics synchronisation is used for stabilisation of frequency of electric, electromagnetic and quantum generators, synthesis of frequencies, demodulation of signals, control of the phased-array antennas, in Doppler systems, in exact times systems, at different ways of an information transfer, etc.

Synchronisation processes in systems of the different nature have much in common and can be studied with use of the general mathematical and computing tools. In a role of electrotechnical models of synchronisation of system of the distributed objects as a rule accept synchronisation of oscillator networks. For synchronisation studying use methods of the control theory, fluctuation theory, phase dynamics, mapping theory, nonlinear environments and networks, chaos theory, fractal theory, cellular automatic machines and others [3–5].

In the synchronisation theory two basic parts allocate:

- The classical theory of synchronisation which studies the phenomena in the connected periodic self-oscillatory systems;
- The theory of chaotic synchronisation which studies co-operative behaviour of chaotic systems.

Among systems of chaotic synchronisation allocate three main types:

- Full (or identical) synchronisation when states of the connected objects completely coincide;
- The generalised synchronisation when exits of objects are connected through some function;
- Phase synchronisation is an establishment of some parities between phases of co-operating objects, result of that is coincidence of their characteristic frequencies or characteristic time scales.

Distinguish such kinds of synchronisation of objects:

- Compulsory synchronisation of objects by means of an external source of signals, for example, the generator of clock frequency;
- The free spatially-distributed synchronisation of objects at the expense of their interaction among themselves.

In this work synchronisation of signals of objects of the distributed system is offered to be carried out on the basis of model of stochastic game of agents [6, 7]. The agent is an independent system of decision-making on the basis of artificial intellect methods. Set of agents which co-operate among themselves during the common problem solving, form the multiagent system.

Game synchronisation is the actual scientifically-practical problem not enough studied now. Unlike synchronisation oscillator networks which are described by systems of the differential equations, stochastic game synchronisation investigates difficult behaviour of networks of agents with various models of decision-making in the conditions of uncertainty on the basis of artificial intellect methods. During stochastic game there is a self-learning of agents to choose optimum on the average pure strategies (actions), reconstructing own vectors of the dynamic mixed strategies (conditional probabilities of variants of actions). Under certain conditions, which are defined by parametres of environment, in parametres of a game method and criteria of a choice of variants of decisions, self-learning of stochastic game provides synchronisation of strategies of agents.

Construction of a stochastic game method of free phase synchronisation of signals of the distributed system is the purpose of this article. For purpose achievement it is carried out the formulation of a stochastic game problem, the method is offered and the algorithm is developed for its solving, results of computer modelling of stochastic game are analysed.

Statement of a game problem of signals synchronisation

Let's consider set of objects D each of which $i \in D$ generates a harmonious signal $y_i(t) = A_i \sin(\omega t + \varphi_i)$ where t it is a continuous time, A_i it is an amplitude of a signal, ω it is a frequency of a signal, φ_i it is a controlled phase of a signal.

For object with number i are accessible to measurement of a phase of signals from a local subset of objects $D_i \neq \varnothing \quad \forall i \in D$. Set of subsets $\{D_i \mid \bigcup_{i \in D} D_i = D\}$ form a network of synchronisation of objects.

For each object of such network we will put in conformity of the game agent with the ordered set of pure strategies $\Phi^i = \{\varphi^i(1), \varphi^i(2), ..., \varphi^i(N)\}$ where $\varphi^i(j) = -\pi + j\dfrac{2\pi}{N}$ ($j = 1..N$) it is a discrete value of a phase of a signal, $N \geq 2$ it is a quantity of pure strategies.

Let the choice of pure strategies is carried out by agents during the discrete moments of time $n = 1, 2, \ldots$. After termination of a collective choice of strategies $\varphi_n^i \in \Phi^i \quad \forall i \in D$ agents calculate a deviation of current phases of signals:

$$\xi_n^i = |D_i|^{-1} \sum_{j \in D_i} |\varphi_n^i - \varphi_n^j| + \mu_n^i \quad \forall i \in D, \tag{1}$$

where μ_n^i it is a white noise which simulates an error of measurement of phases of signals.

Current losses of agents $\xi_n^i = \xi_n^i(\varphi_n^{D_i})$ are functions of their general strategies $\varphi^{D_i} \in \Phi^{D_i} = \times_{j \in D_i} \Phi^j$ from local subsets $D_i \subseteq D \quad \forall i \in D$. Stochastic characteristics of random losses are not known to agents a priori.

Efficiency of signals synchronisation is defined by functions of average losses:

$$\Xi_n^i = \frac{1}{n}\sum_{\tau=1}^{n}\xi_\tau^i \quad \forall i \in D, \tag{2}$$

where ξ_τ^i it is calculated according to (1).

The game purpose is minimisation of functions of average losses:

$$\overline{\lim_{n\to\infty}}\,\Xi_n^i \to \min \quad \forall i \in D. \tag{3}$$

So, on the basis of calculations of random current losses $\{\xi_n^i\}$ players should choose pure strategies $\{\varphi_n^i\}$ so that with time course $n = 1, 2, ...$ to provide performance of system of the purposes (3). Depending on a way of formation of sequences $\{\varphi_n^i \mid \forall i \in D,\ n = 1, 2, ...\}$ the multicriterion problem (3) has solutions which satisfy to one of conditions of a collective optimality: on Nash, Pareto, etc. [7–9].

For the game solving it is necessary to construct such method of generating of sequences pure strategies $\{\varphi_n^i\}\ \forall i \in D$ that synchronisation of signals within local subsets $D_i \in D \ \ \forall i \in D$ has led to global synchronisation of signals from set D.

Adaptive method of a game problem solving

For a game choice of pure strategies in the conditions of uncertainty Marcovian adaptive methods are applicable.

Formation of sequence of variants of decisions $\{\varphi_n^i\}$ we will execute on the basis of dynamic vectors of the mixed strategies $p_n^i = (p_n^i(1), p_n^i(2), ..., p_n^i(N))\ \ \forall i \in D$ which elements $p_n^i(j)$, $j = 1..N$ are probabilities of a choice of pure strategies under condition of realisation of prehistory of strategies $\{\varphi_t^i \mid t = 1, 2, ..., n-1\}$ and reception of corresponding losses $\{\xi_t^i \mid t = 1, 2, ..., n-1\}$. The mixed strategies accept values on N-dimensional unit simplex:

$$S^N = \left\{ p \,\middle|\, \sum_{j=1}^{N} p(j) = 1;\ p(j) \ge 0 \ \ (j = 1..N) \right\}.$$

Values of pure strategies are defined from a condition:

$$\varphi_n^i = \left\{ \varphi^i(l) \mid l = \arg\min_{l} \sum_{k=1}^{l} p_n^i[k] > \eta \ (k, l = 1..N) \right\}, \tag{4}$$

where $\eta \in [0,1]$ it is a random variable with uniform distribution.

It is necessary to define a method of change of vectors of the mixed strategies $p_n^i\ \forall i \in D$ which according to (4) will provide generating of random pure strategies $\{\varphi_n^i\}$ so that in asymptotics time to provide performance of system of criteria (3).

Construction of a method of the stochastic game solving we will satisfy on the basis of stochastic approximation of a complementary slackness condition of the determined game, which it's true for the mixed strategies in Nash equilibrium point [8].

For this purpose we will define polylinear function of average losses of the determined game:

$$V^i(p^{D_i}) = \sum_{u^{D_i} \in U^{D_i}} v^i(u^{D_i}) \prod_{j \in D_i;\, u^j \in u^{D_i}} p^j(u^j),$$

where $v(u^{D_i}) = M\{\xi_n^i(u^{D_i})\}$.

Then the vector of a complementary slackness condition (CS, Complementary Slackness) will look like:

$$\vec{CS} = \nabla_{p^i} V^i(p^{D_i}) - e^{N_i} V^i(p^{D_i}) = 0 \qquad \forall i \in D,$$

where $\nabla_{p^i} V^i(p^{D_i})$ it is a gradient of average losses function; $e^N = (1_j \mid j = 1..N)$ it is the vector, which all components are equal 1; $p^{D_i} \in S^{M_i}$ it is the combined mixed strategies of players from the local sets D_i witch possess the value on M_i-dimentional convex unit simplex S^{M_i}, where $M_i = \prod_{j \in D_i} |D_i|$, and $|D_i|$ it is a cardinal number of set D_i.

For inclusion of solutions in tops of an unit simplex we will satisfy weighing of a complementary slackness condition by elements of vectors of the mixed strategies:

$$\vec{CS} = \nabla_{p^i} V^i(p^{D_i}) - e^{N_i} V^i(p^{D_i}) = 0 \qquad \forall i \in D, \tag{5}$$

where $diag(p^i)$ it is the square diagonal matrix of an order N_i constructed of elements of a vector p^i.

Considering that

$$diag(p^i)[\nabla_{p^i} V^i - e^{N_i} V^i] = E\{\xi_n^i [e(\varphi_n^i) - p_n^i] \mid p_n^i = p^i\},$$

where $E\{\}$ it is a expectation function, with (5) on the basis of a method of stochastic approximation we will receive next recurrent method:

$$p_{n+1}^i = \pi_{\varepsilon_{n+1}}^N \left\{ p_n^i - \gamma_n \xi_n^i \left[e(\varphi_n^i) - p_n^i \right] \right\}, \tag{6}$$

where $\pi_{\varepsilon_{n+1}}^N$ it is a projector on unit ε-simplex $S_{\varepsilon_{n+1}}^N \subseteq S^N$ [9]; $p_n^i \in S_{\varepsilon_n}^N$ it is the mixed strategy of i-th agent; $\gamma_n > 0$ it is a monotonously descending sequence of positive values which regulates size of a step of a method; $\varepsilon_n > 0$ it is a monotonously descending sequence of positive values which regulates speed of ε-simplex expansion; $\xi_n^i \in R^1$ it is a current loss of the agent; $e(\varphi_n^i)$ it is an unit vector-indicator of a variant $\varphi_n^i \in \Phi^i$ choice.

Projecting on expanded ε_n-simplex $S_{\varepsilon_{n+1}}^N$ provides performance of a condition $p_n^i[j] \geq \varepsilon_n$, $j = 1..N$ which is necessary for completeness of the statistical information on the chosen pure strategies, and parameter $\varepsilon_n \to 0$, $n = 1, 2, \ldots$ is used as an additional element for convergence control of a recurrent method.

Stochastic game begins from not learned vectors of the mixed strategies with values of elements $p_0^i(j) = 1/N$ where $j = 1..N$. During following moments of time dynamics of vectors of the mixed strategies is defined by a Marcovian recurrent method (6).

At the moment of time n each player $i \in D$ on the basis of the mixed strategy p_n^i chooses pure strategy u_n^i and to moment of time $n+1$ receives current loss ξ_n^i then calculates the mixed strategy p_{n+1}^i according to (6).

Thanks to dynamic reorganisation of the mixed strategies on the basis of optimum processing of current losses, the method (6) provides an adaptive choice of pure strategies in time.

Parameters γ_n and ε_n define conditions of convergence of stochastic game and can be set so:

$$\gamma_n = \gamma n^{-\alpha}, \ \varepsilon_n = \varepsilon n^{-\beta}, \tag{7}$$

where $\gamma > 0$; $\alpha > 0$; $\varepsilon > 0$; $\beta > 0$.

Convergence of strategies (6) to optimum values with probability 1 and in mean-square is defined by parities of parameters γ_n and ε_n which should satisfy base conditions of stochastic approximation [10].

Efficiency of synchronisation of signals is estimated by functions of average losses Ξ_n^i (2) and synchronisation factor K_n is a fraction of agents with the synchronised signals:

$$\mathrm{K}_n = \frac{1}{n\,|D|}\sum_{\tau=1}^{n}\sum_{i\in D}\chi\left(\varphi_\tau^i = \bar{\varphi}_\tau\right), \tag{8}$$

where $\chi() \in \{0,1\}$ it is an indicator function of event, $\bar{\varphi}_\tau = |D|^{-1}\sum_{i\in D}\varphi_\tau^i$ it is a current average value of phases of signals.

Algorithm of the problem solving

1. To set initial values of parameters:
$n = 0$ it is the initial moment of time;
$L = |D|$ it is a quantity of players;
N it is a quantity of pure strategies of players;
$\Phi^i = \{\varphi^i(1), \varphi^i(2), ..., \varphi^i(N)\}$, $i = 1..L$ are vectors of pure strategies of players;
$p_0^i = (1/N, ..., 1/N)$, $i = 1..L$ are initial mixed strategies of players;
$\gamma > 0$ it is a parameter of a step of learning;
$\alpha \in (0,1]$ it is an order of a step of learning;
ε it is a parameter of ε-simplex;
$\beta > 0$ it is an order of rate of ε-simplex expansion;
$d > 0$ it is a dispersion of noises;
$n_{\max}$ it is a maximum quantity of steps of a method.
2. To choose value of phases of signals $\varphi_n^i \in \Phi^i$, $i = 1..L$ according to (4).
3. To receive value of current losses ξ_n^i, $i = 1..L$ according to (1). Current values of Gaussian white noise are calculated by the formula:

$$\mu_n = \sqrt{d}\left(\sum_{j=1}^{12}\eta_{j,n} - 6\right),$$

where $\eta \in [0,1]$ it is the valid random number with the uniform law of distribution.
4. To calculate values of parameters γ_n and ε_n according to (7).
5. To calculate elements of vectors of the mixed strategies p_n^i, $i = 1..L$ according to (6).
6. To calculate characteristics of quality of decision-making Ξ_n^i (2) and K_n (8).
7. To set the following moment of time $n := n + 1$.
8. If $n < n_{\max}$ then go to a step 2, else stop.

Results of computer modelling

Let's consider the distributed system with linear topology of placing L of generators of harmonious signals. The similar problem is known in cybernetic literature as a firing squad synchronization or a Myhill's problem [11]. Each agent $i \in D$ fixes phases φ_n^j $\forall j \in D_i$ of signals of the next agents and calculates their current deviation δ_n^i from a phase φ_n^i of own signal. For an example, we will choose a differential way of formation of a current deviation of phases of signals:

$$\delta_n^i = \begin{cases} |\varphi_n^{i-1} + \varphi_n^{i+1} - 2\varphi_n^i|, & if \ \ i > 1 \ \ and \ \ i < L; \\ |\varphi_n^{i+1} - \varphi_n^i|, & if \ \ i = 1; \\ |\varphi_n^{i-1} - \varphi_n^i|, & if \ \ i = L. \end{cases}$$

Current losses of players are calculated as a deviation of phases δ_n^i under the influence of white noise μ_n^i which models a measurement error:

$$\xi_n^i = \delta_n^i + \mu_n^i .$$

Game begins with not trained mixed strategies: $p_0^i[j] = 1/N$, $j = 1..N$, $i = 1..L$. Modelling of stochastic game is carried out throughout $n_{\max} = 10^5$ iterations.

On fig. 1 in logarithmic scale schedules of functions of average losses Ξ_n^i and fraction K_n of the synchronised signals are represented. Results are received for such parameters of a game method: $L = 4$, $N = 10$, $\gamma_0 = 1$, $\varepsilon_0 = 0.999/N$, $\alpha = 0.1$, $\beta = 1$, $d = 1$.

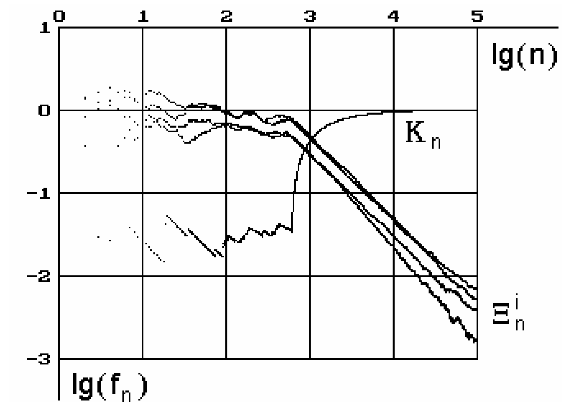

Fig. 1. Characteristics of synchronisation of stochastic game

Reduction of average losses Ξ_n^i, $i = 1..L$ and growth of synchronizing coefficient of agents K_n testify to convergence of a game method owing to performance of system of criteria (3). Growth of quantity of agents L and quantities of strategies N and a dispersion d of measurement of phases of signals lead to increase in time of stochastic game learning.

The experimental order of convergence rate is defined by a tangent of angle of linear approximation of functions of average losses Ξ_n^i, $i = 1..L$ on a concluding period of time of modelling. Apparently on fig. 1, for preset values of parameters the offered game method provides close to 1 order of speed of convergence.

On fig. 2 and 3 schedules of additive signals y_n of system of synchronisation accordingly for not learned and learned stochastic game are represented. Schedules are averaged by quantity of generators of harmonious signals:

$$y_n = \frac{1}{L}\sum_{i=1}^{L} y_n^i .$$

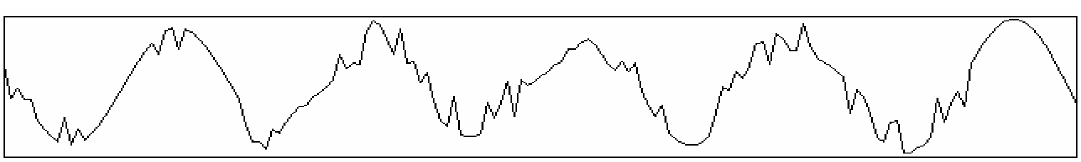

Fig. 2. An average signal of system for not learned stochastic game

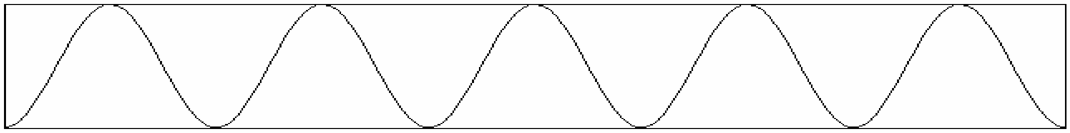

Fig. 3. An average signal of system for the learned stochastic game

The random nature of an additive signal on fig. 2 is caused by different phases of harmonious signals of a network of synchronisation for not learned game.

During learning alignment of phases of signals of objects of the distributed system is carried out. Result of learning is that the synchronisation system generates a harmonious additive signal as it is represented on fig. 3.

Conclusions

The adaptive game method offered in this article provides phase synchronisation of signals of the distributed system in the conditions of action of stochastic noises. The recurrent method is constructed on the basis of stochastic approximation of a complementary slackness condition which describes the Nash game solution in the mixed strategies.

On each step of repeating game players carry out an independent random choice of pure strategies – values of discrete phases of signals on the basis of the dynamic distribution constructed on own mixed strategies. After choice of pure strategies completion by all players each of them finds a difference of values of phases own and the next signals which uses as current loss. The current step of game comes to the end with recalculation of the mixed strategies of all players according to offered recurrent method. Such course of game in asymptotics time provides adaptive phase synchronisation of signals of the distributed system. For practical applications the moment of game finish is limited to achievement of the prescribed accuracy of the game solution.

Convergence of a game method is provided with base conditions of stochastic approximation. Optimum values of parameters of a game method are specified experimentally. Growth of quantity of players, quantities of pure strategies and a dispersion of losses leads to reduction of an order of convergence rate of stochastic game.

Intellectual possibilities of the developed game agents are based on self-learned stochastic automatic machines with variable structure. For their improvement it is possible to use other methods of self-learning, in particular what are based on results of the theory of an artificial intellect.

Results of work can be used for synchronisation of work of components of sensor networks, cybernetic automatic machines and multiagent systems, construction of reports of interaction between intellectual agents, maintenance of conditions of self-organising of the active distributed systems.

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