AUXILIARY MODEL OF PARAMETRIC SENSITIVITY

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Abstract: The two-point boundary value problem of the differential equations of parametric sensitivity in addition to solving a two-point boundary value problem of the differential equations of state is solved. A joint monodromy matrix is defined by the product of matrix coefficients of the equations of state and the matrix of so-called auxiliary model of parametric sensitivity as the equations of first variation of the incomplete equations of state. The computational results are presented.

Keywords: parametric sensitivity, mathematical model, steady-state process.

1. Introduction

The auxiliary model of parametric sensitivity is the very key that made it possible to use the latest achievements of the theory of ordinary nonlinear differential equations to analyze three of the four main stages of the analysis of any physical system, namely: the calculation of steady-state processes, determination of their static stability and parametric sensitivity. The auxiliary model of parametric sensitivity summarizes the auxiliary model of sensitivity to initial conditions. For the first time, the idea of this model was proposed by us in [1]. It proved to be a very effective method for the analysis of the most difficult practical problems [2].

2. Mathematical model

We write the system of differential equations for a physical system in vector form

$$\frac{dx}{dt} = f_1(x,t); \quad 0 \le t \le \infty, \qquad (1)$$

with $f_1(x,t)$ being T– periodic, $x = (x_1, x_2, \dots, x_n)$.

We consider that the periodic solution to the equation (1) $x(t) = x(t+T)$ exsists. There are such initial conditions $x(0)$, which, when integrating (1) in the time interval [0, T], allow entering directly into the periodic solution, passing over the transient response. These initial conditions we consider as an argument for the following periodic equation

$$f(x(0)) = x(0) - x(x(0),T) = 0, \qquad (2)$$

The expressions (1), (2) are the T-periodic two-point boundary value problem for nonlinear differential equations of state.

The nonlinear transcendental equation (2) should be solved by Newton's iterative method

$$x(0)^{s+1} = x(0)^s - f'\left(x(0)^s\right)^{-1} f\left(x(0)^s\right), \qquad (3)$$

We will obtain Jacobi's matrix $f'(x(0))$ by differentiating it with respect to $x(0)$ of the target function (2)

$$f'(x(0)) = E - \Phi(T), \qquad (4)$$

where

$$\Phi(T) = \left.\frac{\partial x(x(0),t)}{\partial x(0)}\right|_{t=T}, \qquad (5)$$

$\Phi(T)$ is the monodromy matrix. It is obtained from the equation of first variation by differentiating (1) with respect to $x(0)$:

$$\frac{d\Phi}{dt} = \frac{\partial f(x,t)}{\partial x}\Phi. \qquad (6)$$

In the s-th iteration of Newton's formula (3), the linear variational equation (6) is subject to compatible integration with the nonlinear (1) in the time interval $[0,T]$. The process of iteration is over when you reach a given accuracy of the entry into the periodic solution

$$\left| f\left(x(0)^s\right) \right| \le \varepsilon, \qquad (7)$$

where ε is the vector of the given accuracy.

The monodromy matrix Φ (5) is essentially a matrix of sensitivity to initial conditions. Each of the lines can be considered as a gradient of a certain variable in the space of initial conditions, with each of its columns describing the sensitivity of the whole set of variables to the same initial conditions. Therefore, the differential equation (6) can be considered as a model of sensitivity to initial conditions.

Multipliers (eigen values) of the matrix $\Phi(T)$ determine the static stability of the process found. To do this, their modules must be less than one!

The easiest way to solve the problem concerning the parametric sensitivity calculation is using variational methods as a simple addition to the algorithm of accelerated search for periodic solutions to nonlinear differential equations based on Newton's iteration (3).

We denote the vector of constant parameters as

$$\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n). \qquad (8)$$

Then, the matrix of parametric sensitivities is determined as the partial derivative

$$S = \frac{\partial x}{\partial \lambda} . \qquad (9)$$

The element of matrix λ can be any constant parameter of the target system.

The argument x is found from the equation (1) which we write in more general form:

$$dx / dt = f_1(x, \lambda, t) . \qquad (10)$$

Differentiating (10) with respect to λ, we will obtain a linear parametric equation

$$\frac{dS}{dt} = \frac{\partial f_1(x, \lambda, t)}{\partial x} S + \frac{\partial f_1(x, \lambda, t)}{\partial \lambda} . \qquad (11)$$

In the steady state $x(0) = x(T)$, so the equation (11) also has the $S(t)$ periodic solution.

Obtaining partial derivatives with respect to x and λ in the right part of (6) and (11) is rather a difficult task, or even unsolvable. Therefore, we introduce *the matrix of auxiliary parametric sensitivities* χ in relation to another vector y:

$$\chi = \frac{dy}{d\lambda} . \qquad (12)$$

The equation of state of the **target** object in relation to the vector y we also write in general form:

$$dy / dt = f_2(x, \lambda, t) = f_3(y, \lambda, t) , \qquad (13)$$

f_2, f_3 are T– periodic with respect to t.

Differentiating (13) with respect to λ, and taking into account (5) and (6), we obtain

$$\frac{d\chi}{dt} = \frac{\partial f_2(x, \lambda, t)}{\partial x} \xi + \frac{\partial f_2(x, \lambda, t)}{\partial \lambda} . \qquad (14)$$

The equation (14) also has a periodic solution $\chi(t)$. The function $\chi(t)$, besides performing a supporting role, is often of an independent interest.

The replacement of x with y should be carried out in such a way to make the equation (13) simpler than the equation (10). Such a substitution is reasonable only if the connection between x and y is known. In general, it can be represented as:

$$y = G^{'} x + H , \qquad (15)$$

where $G^{'} = G^{'}(x)$ is the matrix of static parameters; $H = = H(t)$ is a vector.

For example, in differential equations of an electric circuit, inductance coil currents and capacitor voltages are considered as components of the vector x, and their linkages and charges, respectively, as components of the vector y. In differential equations of motion, coordinates and velocities are considered as components of the vector

x, and generalized impulses – as components of the vector y, etc. The connection between x and y must be known in any system under research as its internal setting.

Let us establish the connection between the functions $f_1(x, t)$ and $f_2(x, t)$. For this, we differentiate (15) with respect to time and substitute the derivatives of (10) and (13) into the result obtained:

$$f_2(x, t) = \left(G^{''} + \frac{\partial H}{\partial x} \right) f_1(x, t) + \frac{\partial H}{\partial t} , \qquad (16)$$

where $G^{''} = G^{''}(x)$ is the matrix of differential parameters. Simplifying the right-hand parts of the differential equation (13) in comparison with (10) is achieved just by these two operations – multiplication and addition.

Establish the connection between the matrices of the parametric sensitivities S and χ. For this purpose, we differentiate (15) with respect to λ, and substitute the derivatives of (9) and (12) into the result obtained:

$$\chi = \left(G^{''} + \frac{\partial H}{\partial x} \right) S + \frac{\partial G^{'}}{\partial \lambda} x + \frac{\partial H}{\partial \lambda} . \qquad (17)$$

Having solved (17) with respect to S, we finally obtain

$$S = A \left(\chi - \frac{\partial G^{'}}{\partial \lambda} x - \frac{\partial H}{\partial \lambda} \right); \ A = \left(G^{''} + \frac{\partial H}{\partial x} \right)^{-1} . \qquad (18)$$

The structure of the matrix $G^{'}$ is much simpler than that of $G^{''}$, so it is easy to obtain a derivative of $\partial G^{'} / \partial \lambda$. The matrix A is typically a matrix of coefficients of the equations of state written in Cauchy's normal form.

On substituting (18) into (14), we obtain the required heterogeneous linear differential equation of auxiliary parametric sensitivity:

$$\frac{d\chi}{dt} = \frac{\partial f_2(X, \lambda, t)}{\partial X} A \left(\chi - \frac{\partial G^{'}}{\partial \lambda} x - \frac{\partial H}{\partial \lambda} \right) + \frac{\partial f_2(X, \lambda, t)}{\partial \lambda} . \qquad (19)$$

If we assume that $\lambda = x(0)$, then (19) degenerates into a homogeneous equation

$$\frac{d\chi}{dt} = \frac{\partial f_2(\chi, \lambda, t)}{\partial X} A \chi , \qquad (20)$$

which describes the model of sensitivity to the initial conditions (6).

The periodic solution to the equation of parametric model of sensitivity (19) is also found on the basis of (3). As a result of numerical calculation of the expressions (1), (3), and (20) we find a periodic solution to the equations of state of the object under research, and, therefore, a matrix of monodromy $\Phi(T)$ (5).

The matrix of parametric sensitivity (12) is divided into columns and written as a vector

$$\chi = (\chi_1, \chi_2, \ldots, \chi_m) , \qquad (21)$$

where m is the number of elements of the vector of constant parameters $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$, $\lambda = const.$, with

$$\chi_i = dy / \partial\lambda_i, \quad i = 1, 2, \ldots, m, \tag{22}$$

being the vectors of parametric sensitivities of the y-vector elements to individual constant parameters.

A conditon of the periodic solution to the differential equation (19) we write similarly to (2)

$$F(\chi_i(0)) = \chi_i(0) - \chi_i(\chi_i(0), T) = 0, \quad i = 1, 2, \ldots, n. \tag{23}$$

The equation (23) is solved by using Newton's iterative method, but since (19) is a linear equation, then, one iteration is enough to solve the equation. Provided the approximation equals zero, the equation (3) is as follows

$$\xi_i(0) = F'(\xi_i(0))^{-1}\xi_i(T), \quad i = 1, \ldots, n. \tag{24}$$

Jacobi's matrix is expressed by a certain matrix $\Phi(T)$, derived from calculating the periodic solution $x(t) = x(t + T)$:

$$F'(\xi_i(0)) = E - \Phi(T). \tag{25}$$

Integrating (19) of the computed according to (24) initial conditions, after previously changing, according to (18), from $S_i(0)$ to $\chi_i(0)$, we obtain a periodic solution $\chi(t) + \chi(t+T) = 0$. The sensitivity S we find from the same algebraic expression (18).

Time discretization of the given differential equations and differential equations of their sensitivity (to the initial conditions and constant parameters) is performed by the explicit or implicit methods. Their joint solution is especially harmoniously combined in the case of implicit methods, since Jacobi's matrixes of the basic equation and parametric equation are matched.

Let us demonstrate the essence of the method, using the two easiest examples involved in the theory of electric circuits.

Example 1. We will completely analyze a circuit formed from a resistive-inductive element r-L powered by a sinusoidal voltage $u = U_m \sin \omega t$. The differential equation of condition (10) of such a circuit is clear ($x = i$)

$$di / dt = (U_m \sin \omega t - ri) / L(i); \quad 0 \leq t \leq \infty, \tag{26}$$

where i is the current; $L(i)$ is the differential inductance Direct integration of (26) defines the transient process in the circuit.

Steady-state currents and parametric sensitivities are determined in accordance with the suggested method. We will write the vector (8) in the following way: $\lambda = (r, U_m, \omega)$. The parametric sensitivities have the following form:

$$S_r = di / dr, \quad S_{U_m} = di / dU_m, \quad S_\omega = di / d\omega. \tag{27}$$

Case: $L(i) = const$. In this case the equation (26) has an analytical solution

$$i(t) = (i(0) + I_m \sin \varphi)e^{-\alpha t} + I_m \sin(\omega t - \varphi)$$

$$I_m = \frac{U_m}{\sqrt{(r^2 + \omega^2 L^2)}}; \quad \varphi = \text{arctg}\frac{\omega L}{r}; \quad \alpha = \frac{L}{r}. \tag{28}$$

We assign an initial zero approximation $i(0)^0$. Then, according to (2), and (28)

$$f(i(0)^0) = 0 - i(0, T) = I_m \sin \varphi (1 - e^{-\alpha T}). \tag{29}$$

The monodromy matrix is found, using (5) and (28)

$$\Phi(T)^0 = \left.\frac{\partial i(t)}{\partial i(0)}\right|_{t=T} = e^{-\alpha T}. \tag{30}$$

By substituting (29) and (30) into (3), provided (4), we obtain

$$x(0) = 0 - \frac{I_m \sin \varphi (1 - e^{-\alpha T})}{1 - e^{-\alpha T}} = -I_m \sin \varphi. \tag{31}$$

We see the results of one iteration as expected.

The obtained value of the initial condition levels down the constant of integration in (28), and we directly obtain a periodic solution. In the case of non-linear equations, the algorithm requires several iterations.

We obtain any of the parametric sensitivities (27), using direct differentiation with respect to λ

$$S_r = -I_m \left(\frac{\sin 2\varphi}{\sqrt{r^2 + \omega^2 L^2}} + \frac{t \sin \varphi}{L}\right)e^{-\alpha t} -$$
$$- \frac{I_m}{\sqrt{r^2 + \omega^2 L^2}}\sin(\omega t - 2\varphi). \tag{32}$$

Case: $L = var$. We write the additional equation (13) ($y = \psi$) in linkage form

$$d\psi / dt = U_m \sin \omega t - ri. \tag{33}$$

In this case the coupling equation (15) is clear: $\psi = L'i$, L' is the steady-state inductance. Provided it is differentiated with respect to λ, we will obtain

$$S = \chi / L. \tag{34}$$

We obtain the equation (19) as a result of the differentiation of (33), taking into account (34)

$$\frac{d\chi}{dt} = -\frac{r}{L}\chi + \frac{\partial(U_m \sin \omega t - ri)}{\partial\lambda}. \tag{35}$$

The homogeneous differential equation (35) corresponds to (20). Its joint implementation with (26) in the interval $[0, T]$ of the iteration formula (3) makes it possible to find initial conditions $i(0)$ of the integration into the steady process, as well as, according to (35), the monodromy matrix $\Phi(T)$, which will be used in (24) and (25) to find initial conditions of integration into the steady process in compliance with the equation (35).

The equation (26) having been differentiated with respect to λ, the equations of parametric sensitivities, from the perspective of (27), will have the following form

$$\frac{d\chi_r}{dt} = -\frac{r}{L}\chi - i; \quad \frac{d\chi_{U_m}}{dt} =$$

$$= -\frac{r}{L}\chi + \sin\omega t; \quad \frac{d\chi_\omega}{dt} = -\frac{r}{L}\chi + tU_m c\,os\,\omega t. \quad (36)$$

3. Simulation results.

The figures below show the results of computation if $U_m = 310,5$, $r = 1$,

$$L(i) = \begin{cases} 0,14, \ if \ i \le 4,5; \\ 0,14 - 0,039(i - 4,5) + 0,00318(i - 4,5)^2, \\ \quad if \ 4,5 < i < 10,6; \\ 0,0208, \ if \ i > 10,6. \end{cases} \quad (37)$$
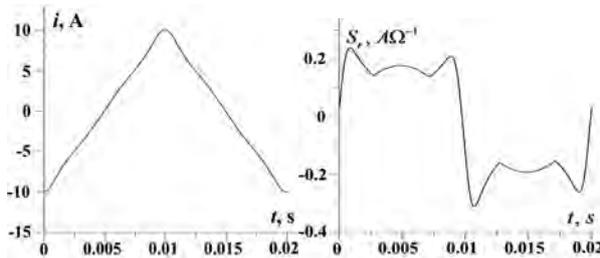
Fig. 1. Steady-state current
$i = i(t)$ in the interval $[0,T]$.
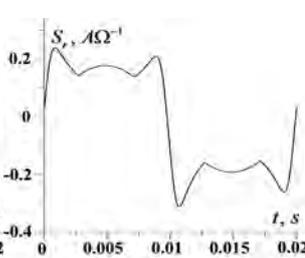
Fig. 2. Steady-state
parametric sensitivity
$S_r = S_r(t)$.
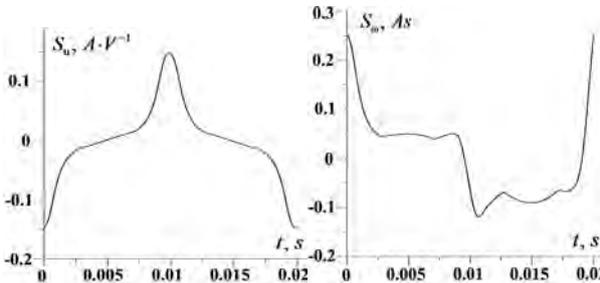
Fig. 3. Steady-state parametric
sensitivity $S_{U_m} = S_{U_m}(t)$.
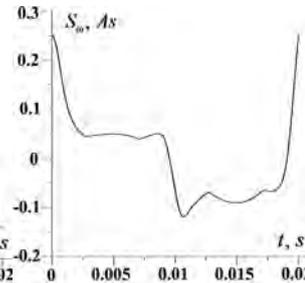
Fig. 4. Steady-state
parametric sensitivity
$S_\omega = S_\omega(t)$.

Example 2. Let us complicate the previous task by introducing a link r–L–C. In this case (26), (33), and (35) will become more complicated

$$\frac{di}{dt} = (U_m \sin\omega t - ri - u_c)/L(i); \quad \frac{du_c}{dt} = \frac{i}{C}, \quad (38)$$

Where u_c is the capacitor voltage; C is the capacity;

$$d\psi/dt = U_m \sin\omega t - ri - u_c. \quad (39)$$

$$\frac{d\chi}{dt} = -\frac{r}{L}\chi + \frac{\partial(U_m \sin\omega t - ri - u_c)}{\partial\lambda}, \quad (40)$$

with $\lambda = (r, U_m, \omega, u_c)$.

In the expanded form, (40) will be as follows:

$$\frac{d\chi^i}{dt} = -\frac{r}{L}\chi^i - S^{U_c} + \frac{\partial(U_m \sin\omega t - ri - u_c)}{\partial\lambda};$$

$$\frac{dS^{U_C}}{dt} = \frac{\chi^i}{LC} + \frac{\partial(i/C)}{\partial\lambda}. \quad (41)$$

The results of the joint use of (3)–(5), (38), and (41) are shown in Fig. 5–10.

Fig. 5–8 show the results of the calculations of ferro-resonant conditions of a circuit: two resistant and one non-resistant. Any of the conditions is reached by variation of a zero approximation in Newton's formula (3), specified in the captions. Input values: $U_m = 135$, $r = 0,1$, $C = 0,00001966$, as well as the curve (37). Curve 3 (Fig.7) is obtained from the transient process at the 1000th period. As we see, the process is still alive as the transient component can be seen
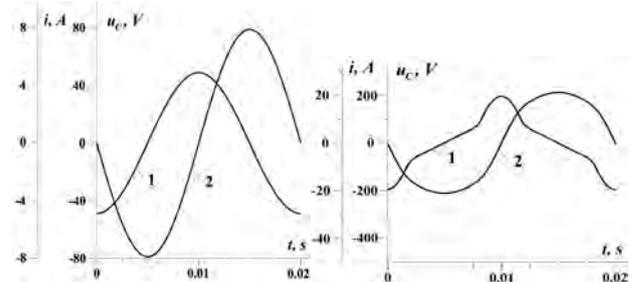

Fig. 5. Curves $i = i(t)$ (1),
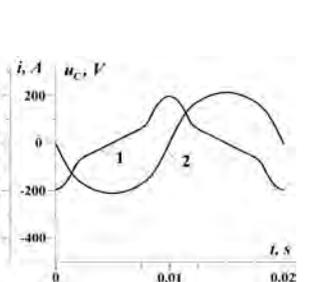*$u_C = u_C(t)$ (2) if $i(0)^0 = 0$.*

Fig. 6. Curves $i = i(t)$ (1),
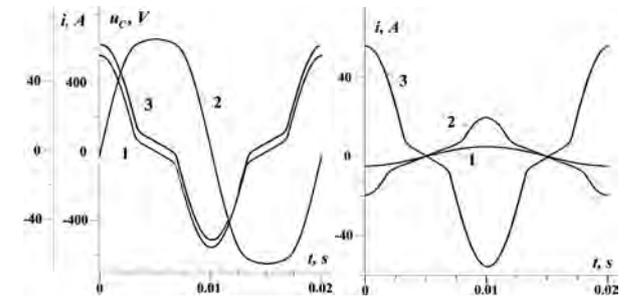*$u_C = u_C(t)$ (2) if $i(0)^0 = 0.08$*

Fig. 7. Steady-state curves
$i = i(t)$ (1), (3),
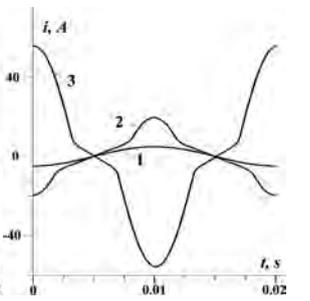*$u_C = u_C(t)$ (2) if $i(0)^0 = 100$.*

Fig. 8. Comparative curves
of constant currents Fig. 5
(1), Fig. 6 (2) and Fig. 7
(3).

Fig. 9–16 demonstrate the results of the parametric sensitivities computation. Input values: $U_m = 282$, $r = 1$, $C = 0,00055$, as well as the curve (37).
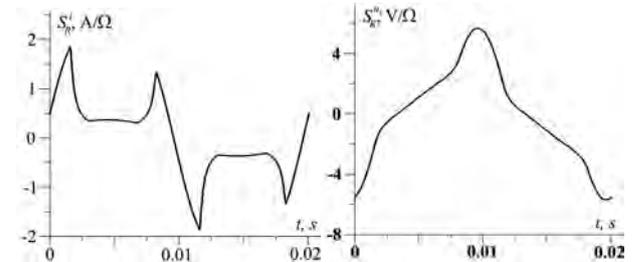

Fig. 9. Steady-state parametric
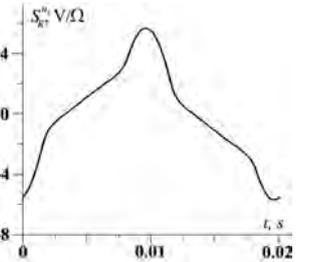*sensitivity $S_r^i = S_r^i(t)$.*

Fig. 10. Steady-state
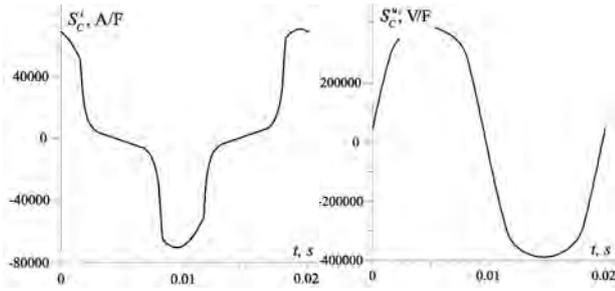*parametric sensitivity*
$S_r^u = S_r^u(t)$.

Fig. 11. Steady-state parametric
sensitivity $S_C^i = S_C^i(t)$.

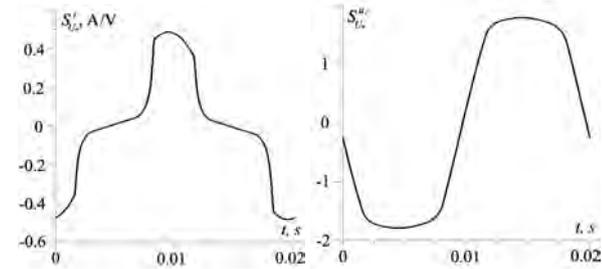Fig. 12. Steady-state
parametric sensitivity
$S_C^u = S_C^u(t)$.

Fig. 13. Steady-state parametric
sensitivity $S_{U_m}^i = S_{U_m}^i(t)$.

Fig.14. Steady-state
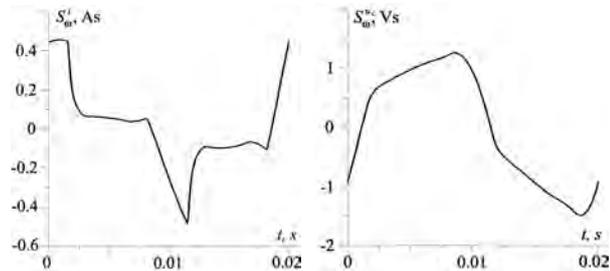parametric sensitivity
$S_{U_m}^u = S_{U_m}^u(t)$.

Fig. 15. Steady-state parametric
sensitivity $S_\omega^i = S_\omega^i(t)$.

Fig. 16. Steady-state
parametric sensitivity
$S_\omega^u = S_\omega^u(t)$.

The multipliers of the monodromy matrix Φ solve the problem of asymptotic stability of the steady state found [2]. Let us show the values of the matrixes and their multipliers for the existing ferroresonance conditions in Fig. 5–7 in succession.

$$\Phi(T)_5 = \begin{array}{|c|c|} \hline -0,7551 & -15,4034 \\ \hline 0,0271 & -0,7575 \\ \hline \end{array}; \quad \lambda_{12} = -0,7563 \pm j0,6463;$$

$$\Phi(T)_6 = \begin{array}{|c|c|} \hline 2,6974 & 32,4389 \\ \hline 0,2077 & 2,8566 \\ \hline \end{array}; \quad \lambda_1 = 0,1798; \ \lambda_2 = 5,3742;$$

$$\Phi(T)_7 = \begin{array}{|c|c|} \hline 01676 & 24,4782 \\ \hline -0,0389 & 0,0057 \\ \hline \end{array}; \quad \lambda_{12} = 0,0867 \pm j0,9724.$$

Taking the multiplier modules of all the three matrixes into consideration, we come to an clear conclusion that the existing ferroresonans conditions

presented in Fig. 5–7 are asymptotically consistent, and those in Fig. 6 are asymptotically inconsistent, that are well complied to physics of the prosess.

Generally, the existing parametric sensitivities are characterized by their root-mean-square values.

Taking into account a small value of the integration time step, it is expedient to substitute the integral for the formula of rectangles (1.46)

$$S = \sqrt{\frac{1}{n}\sum_{i=1}^{n} S(t_i)^2}, \qquad (41)$$

where n is the number of integration time steps in the period.

For example, we show the corresponding values for the parametric sensitivities in Fig. 9, 11, 13 and 15 respectively:

$$S = 0,7570; \ 40223,41; \ 0,2789; \ 0,2072.$$

It is interesting to know how a transitional parametric sensitivity behaves in time. For example, we show two of them, namely according to the formula (32) as a linear case,
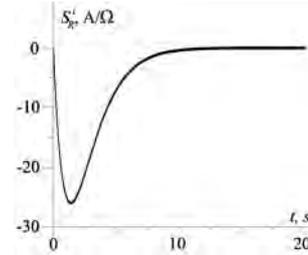

Fig. 17. Transitional sensitivity $S_r = S_r(t)$
that corresponds to Fig. 2 if $L = const.$
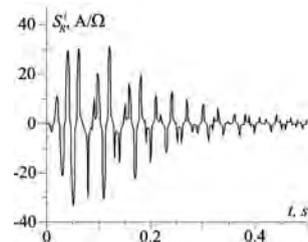
And according to Fig. 9, as a non-linear case

Fig. 18. Transitional sensitivity $S_r = S_r(t)$
that corresponds to Fig. 9.

The effectiveness of using the results of calculation of parametric sensitivity can practically be judged by the curves presented in Fig. 19. Curve 1 characterizes the state of the circle at the given input data for simulation. Curve 2 illustrates the extent to which maximum of current at 4-fold increase in the resistor's resistance gets reduced. Curve 3 illustrates the same, but at a similar increase in capacity of the capacitor.

According to the root-mean-square value of the corresponding parametric sensitivities of current S_r^i (0,7570) and

S_C^i (40223,41), we should be interested in how the capacitance must be increased to ensure maximum of current which is reached by 4-fold increase in the resistor's resistance. The answer is given by the curve in Fig. 4 – only by 7%.
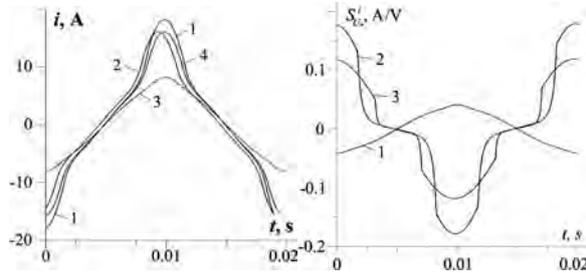

Fig. 19. Steady-state value of current in the state that corresponds to Fig. 9-16. 1 – at the given parameters of circle; 2 – at 4-fold increase in resistance of the resistor; 3 – at 4-fold increase in the capacitor capacitance ; 4 – at 1.07 times increase in the capacitor capacitance.

Fig. 20. Steady-state values of parametric sensitivity

$S_{U_m}^i = S_{U_m}^i(t)$ *, which correspond to ferroresonant states: 1 – Fig. 5, 2 – Fig. 6, 3 – Fig. 7.*

It is interesting to observe that, in ferroresonance states, not only currents and voltages depend on the initial conditions, but also certain parametric sensitivities.

Example 3. The previous example is complicated in the case of a nonlinear capacitor: matrices (9), (12) will be as follows

$$S = diag(\xi, \sigma); \quad \chi = diag(z, g), \qquad (42)$$

where

$$\xi = \frac{\partial i}{\partial \lambda}; \quad \sigma = \frac{\partial u_C}{\partial \lambda}; \quad z = \frac{\partial \psi}{\partial \lambda}; \quad g = \frac{\partial q}{\partial \lambda}, \qquad (43)$$

where q is the capacitor charge.

The differential equations, that correspond to the last two expressions (43), take a clear form

$$\frac{d\psi}{dt} = U_m \sin \omega t - ri - q/C; \quad \frac{dq}{dt} = i. \qquad (44)$$

The corresponding variational equations (44) will be as

$$\frac{dz}{dt} = -\frac{r}{L}z - \frac{g}{C}; \quad \frac{dg}{dt} = z. \qquad (45)$$

Communication of matrices (42) becomes apparent

$$S = A\chi; \quad A = diag(L^{-1}, C^{-1}). \qquad (46)$$

Since the formula (13) has two right parts, the function $f_2(x, \lambda, t)$ leads simpler to targets, but with the possible loss of some constant parameters in an explicit form in the equations of the auxiliary parametric sensitivity. In this case, we must use the function $f_3(y, \lambda, t)$. Then equations (44) will take the following form

$$\frac{d\psi}{dt} = U_m \sin \omega t - rL^{-1}\psi - q/C; \quad \frac{dq}{dt} = L^{-1}\psi. \quad (47)$$

4. Conclusions

In complex circuits which are formed of multipolar nonlinear elements, such as electromechanics, method of auxiliary parametric sensitivity (including sensitivity to initial conditions) is only one means of achieving the goal, i.e. developing common algorithms of analysis of transition processes, steady-state processes, identification of asymptotic stability of established processes and, finally, calculating parametric sensitivities.

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ДОПОМІЖНА МОДЕЛЬ ПАРАМЕТРИЧНОЇ ЧУТЛИВОСТИ

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Розв'язується доточкова крайова задача для диференціальних рівнянь параметричної чутливости як

додаток до розв'язання доточкової крайової задачі для диференціальних рівнянь стану. Матриця монохромії визначається добутком матриці коефіцієнтів диференціальних рівнянь стану і матриці так званої допоміжної моделі параметричних чутливостей як рівнянь першої варіації неповних рівнянь стану. Подаються результати комп'ютерної симуляції.

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