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CONDITIONS FOR APPLICATION OF ASYMPTOTIC METHOD TO ELECTROMAGNETIC FIELD ANALYSIS IN THE SYSTEM OF“A CURRENT LOOP–AN ELECTROCONDUCTING BODY”

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Abstract: A study of the conditions of application of an approximate method for analysis of a three-dimensional pulse electromagnetic field with an arbitrary current loop, which is located near the flat surface of a conductive body, has been performed. It has been shown that each term of the asymtotic series is calculated with some accuracy, and therefore it has its own minimum cutoff frequency. An assessment of cutoff frequencies has been done and appropriate intervals from the pulse start have been obtained depending on the number of the series' term and its chosen tolerable error.

Key words: three-dimensional pulse electromagnetic field, induced current, arbitrary current loop, asymptotic method.

1. Introduction

The task of electromagnetic field analysis, distribution of its Joule's losses and electromagnetic forces in the system of “a current loop of arbitrary spatial configuration – a conductive body” was fairly well investigated [1, 2], but despite this, the practical needs of developing new devices with specific conditions of field distribution are still of interest to researchers of this area of expertise. The discussed devices comprise electro-hydraulic pulse installations [3], appliances for magnetic pulse processing of metals [4], equipment for induction heat processing of metals [5, 6] and others. A variety of geometrical shapes of electrical devices with three-dimensional structure of their electromagnetic field raises the need to develop special methods of calculation for the each case based on a number of simplifying assumptions that take into account the most important features of electromagnetic process.

In these examples, a source of external alternating field comprises currents flowing through conductors that, in general case, form loops of arbitrary spatial configurations. In case of fast pulsing or high-frequency processes a quite strong skin effect arises in conducting bodies, whose field penetration depth is considerably less than the size of a typical electromagnetic system (size of circuits, distance from them to the surface or points where the field is computed, curvature of interfaces between media, etc.). Moreover, if the electromagnetic wave length is much greater than the characteristic size of a system, then, for engineering calculations, the electromagnetic process can be regarded as a quasi-stationary process for which the condition of continuity of current is fulfilled, including the closure of the current circuit.

The purpose of this paper is presenting a linear solution for, in general case, a three-dimensional problem of quasi-stationary electromagnetic field analysis, induced by pulse current flowing in a loop of arbitrary configuration located over the conductive half-space characterized by conductivity γ_i and relative permeability μ_i. It is stated that with the loop with the current is placed in a non-magnetic medium.

2. Mathematical model

In general, the problem is described by electromagnetic field equations and boundary conditions at the border surfaces between different media for vectors of magnetic field $\boldsymbol{B}$ and electric field $\boldsymbol{E}$, when current density $\boldsymbol{j}_e$ of external sources in loop elements is given and current density $\gamma\boldsymbol{E}$ of induced currents in the conductive body is taken into account [2]:

$$\mathrm{rot}\,\boldsymbol{B}=\mu\mu_0\boldsymbol{j}_e+\mu\mu_0\gamma\boldsymbol{E};\ \mathrm{rot}\,\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t};\ \mathrm{div}\,\boldsymbol{B}=0;\ \mathrm{div}\,\boldsymbol{E}=0. \quad (1)$$

In the case of pulse current $i(t)$ acting in linear media it is convenient to use the Duhamel integral to determine the time dependence of vector potential, magnetic field or functions that define them [7]:

$$v(t)=\int_0^t \left.\frac{dv_1(\varsigma)}{d\varsigma}\right|_{\varsigma=t-\tau} i(\tau)\,d\tau\,, \quad (2)$$

where $v_1(t)$ is a system response (time dependence of an appropriate function) to unit pulse current:

$$u(t)=\begin{cases}0, & t<0\\ 1, & t\geq 0.\end{cases}$$

To determine $v_1(t)$, the known solution for electromagnetic field induced by a sinusoidal current flowing in an arbitrary loop located over conductive half-space has been used [2, 8]. In the area where the sinusoidal current with the complex amplitude $\dot{I}$ flows the complex amplitudes of vector potential $\dot{\boldsymbol{A}}$ and magnetic field $\dot{\boldsymbol{B}}$ are determined by using appropriate expressions of integration along the loop l:

$$\dot{\boldsymbol{A}} = \frac{\mu_0 \mu_e \dot{I}}{4\pi} \int_l \left(\frac{\boldsymbol{t}}{r} - \frac{\boldsymbol{t}_1}{r_1} - \boldsymbol{\lambda} \times \operatorname{grad} \dot{G} \right) dl\ , \qquad (3)$$

$$\dot{\boldsymbol{B}} = -\frac{\mu_0 \mu_e \dot{I}}{4\pi} \int_l \left(\frac{\boldsymbol{t} \times \boldsymbol{r}}{r^3} - \frac{\boldsymbol{t}_1 \times \boldsymbol{r}_1}{r_1^3} - (\boldsymbol{\lambda} \cdot \nabla) \operatorname{grad} \dot{G} \right) dl\ . \quad (4)$$

where
$$\dot{G} = 2\int_0^\infty \frac{e^{-(z+h)\vartheta} J_0(\vartheta\rho)}{w} d\vartheta\ . \qquad (5)$$

To explain the corresponding geometry, a local element $\boldsymbol{dl}$ of the loop carrying the current $\dot{I}$ and its mirror image $\boldsymbol{dl}_1$ (mirrored with respect to the flat surface of the conducting body) are shown in Fig. 1. The expressions (3) - (5) include the following denotations: $\boldsymbol{r}$, $\boldsymbol{r}_1$ are radius-vectors joining the observation point Q and the current element and its image, correspondingly; $\boldsymbol{t}$, $\boldsymbol{t}_1$ are tangential orts with respect to the loop and its mirrored image; $\boldsymbol{\lambda} = \boldsymbol{e}_z \times \boldsymbol{t}$; $w = \vartheta + \frac{1}{\mu_i}\sqrt{\vartheta^2 + i\omega\mu_0\mu_i\gamma_i}$; $J_0(\cdot)$ is the Bessel function of the first kind of the zero-order; z and ρ are local cylindrical coordinates of the point Q (bounded to a current element); ω is angular frequency; i is imagenary unit.

If we consider $\dot{I}(i\omega)$ to be a frequency spectrum of a nonsinusoidal current, the expressions (3) i (4) give us frequency spectra of the vector potential and the magnetic field, correspondingly. The first integration terms describe the magtetic field of the current loop only, i. e. without taking into account the eddy currents in the conducting body. The second integration terms describe the field of induced currents and correspond to the current $\dot{I}_1$ that flows in the mirrored loop. The values of the currents $\dot{I}_1$ and $\dot{I}$ are equal, and their directions are described by the relation $\dot{I}d\mathbf{l} = -\dot{I}_1 d\mathbf{l}_1$, which means that $\dot{I}_1$ and $\dot{I}$ flow in opposite direction for horizontal proections of the element $\boldsymbol{dl}$ and do in the same direction for vertical proections. Since the current frequency does not show up in the first and second terms, their frequency spectra do not differ from the frequency spectrum of the current $\dot{I}(i\omega)$. The first and second terms describe the solution of the problem of field distribution of an arbitrary current loop for high-speed or high-frequency processes when the field penetration $\delta = \sqrt{2/(\omega\mu_0\mu_i\gamma_i)}$ is considerably less than system dimensions [9].

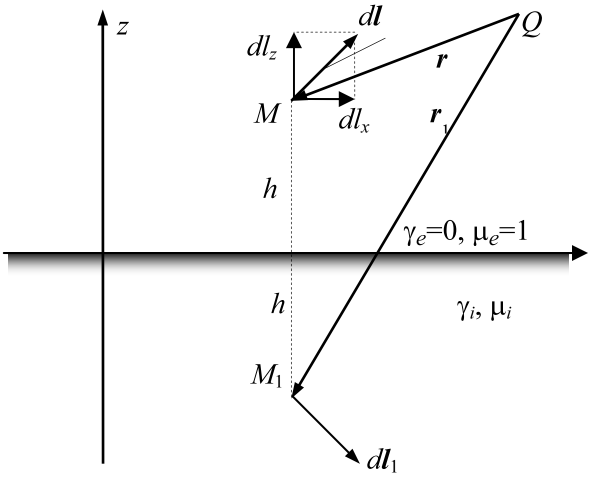

Fig. 1. Current element over the interface of the media.

For smaller frequencies the impact of electro-physical properties of the medium is taken into account by the third integration term. Namely, this term is responsible for the difference between the frequency spectrum of the current and the frequency spectrum of the field. That is why, to find the frequency spectrum of the vector potential and the magnetic field it is necessary to obtain the frequency spectrum of the function $\dot{G}(i\omega)$ as a part of the expressions (4) and (5) under integration sign to receive its spatial derivatives and to perform loop integration.

Utilizing the formula of Fourrier transformation for a unit pulse $u(t) \div U(i\omega) = 1/i\omega$, we receive the expression of the frequemcy spectrum component caused by the third term in (4) i (5) in the form of the product:

$$V_{1G}(i\omega) = U(i\omega) \cdot G(i\omega). \qquad (6)$$

In general case the representation of $\dot{G}(i\omega)$ in the form of improper integral (6) from the expression containing special functions is fairly complicated for the analysis and computation. However, it was shown in [8] that under the condition $\varepsilon = \mu_i\delta/\sqrt{2}r_1 < 1$ the function $\dot{G}$ may be approximated by an asymptotic series with limited number N of its terms as follows:

$$\dot{G}_N \approx \sum_{n=0}^{N} \dot{G}_n\ , \qquad (7)$$

where each series term has its own frequency dependency:

$$\dot{G}_n = (-1)^n \frac{2a_n(\mu_i)}{\left(\frac{1}{\mu_i}\sqrt{i\omega\mu_0\mu_i\gamma_i}\right)^{n+1}} \frac{\partial^{(n)}}{\partial z^n}\left(\frac{1}{r_1}\right) = \frac{g_n}{(i\omega)^{(n+1)/2}}\ . \quad (8)$$

In the latter expression $a_n(\mu_i)$ denotes the coefficients of the Taylor series of such function:

$$w^{-1}=\frac{1}{\vartheta+\frac{1}{\mu_i}\sqrt{\vartheta^2+i\omega\mu_0\mu_i\gamma_i}}=\frac{\mu_i\delta}{\sqrt{2i}}\sum_{n=0}^{\infty}a_n(\mu_i)\left(\frac{\chi}{\sqrt{i\omega}}\right)^n, \quad (9)$$

де $\frac{\chi}{\sqrt{\omega}}=\frac{\mu_i\vartheta}{\sqrt{\omega\mu_0\mu_i\gamma_i}}=\frac{1}{\sqrt{2}}\mu_i\delta\vartheta$.

Each term of the series (8) depends only on one coordinate $\frac{1}{r_1}$, what allows us to utilize the integration in accordance with the Biot-Savart law only along the mirrored loop [8].

Taking into account (6)-(8), the component of the frequency spectrum $\dot{V}_{1G}(i\omega)=\dot{U}(i\omega)\cdot\dot{G}(i\omega)$ may be represented in the form of the following asymptotic series:

$$\dot{V}_{1G}=\sum_{n=0}^{N}\frac{g_n}{(i\omega)^{(n+3)/2}}. \quad (10)$$

The reverse Fourrier transformation for each term of the series $\dot{V}_1(i\omega)$ is known [10]. As a result, the time dependency for magnetic field computation in case of unit pulse current $u(t)$ takes the form

$$v_{1G}(t)=\sum_{n=0}^{N}\frac{g_n}{\Gamma\left(\frac{n+3}{2}\right)}t^{(n+1)/2}, \quad (11)$$

where $\Gamma(\cdot)$ is gamma-function.

Putting the expression (11) into (2), one receives the dependency $v_G(t)$ for the case of an arbitrary current $i(t)$:

$$v_G(t)=\sum_{n=0}^{N}\frac{n+1}{2}\frac{g_n}{\Gamma\left(\frac{n+3}{2}\right)}\int_0^t(t-\tau)^{(n-1)/2}i(\tau)d\tau \quad (12)$$

3. Conditions for the application of the asymptotic method

Despite the relatively simple form of the approximate analytical solution of the general problem of three-dimensional pulse field distribution with consideration of eddy currents in the lower conducting half-space, the possibility of its application requires an analysis of the conditions under which the calculation error does not exceed a certain preset value. In particular, the requirement of smallness of ε parameter in the series (7) - (9) implies that the representation of the results in the form of asymptotic series is justified not in the entire frequency range. The condition is violated for small frequencies and expansion is possible if $\omega>\frac{\mu_i}{\mu_0\gamma r_1^2}$.

The asymptotic expansion (8) - (10) is featured be the fact that its every term is calculated with a certain error which depends on the small parameter ε and increases along with increasing the series term number n. This fact urges us to confine to smaller number N of the series terms, so the error of the last term included does not exceed a certain value. The lower the value ε, the greater number of the series terms may be considered. For this reason, as shown in [8], there is an optimal number of the series term when the error is minimal.

Let us analyze how the influence of the series terms' number on the approximate value of the function modulus $|\dot{G}_N|$ in comparison with the exact value $|\dot{G}|$ depends on the value of the small parameter ε. In the points located over a mirrored current loop element with $\rho=0$ ($r_1=z+h$), when $J_0(\vartheta\rho)=J_0(0)=1$ takes its maximum value, the improper integral (5) may be received in analytical form for nonmagnetic medium ($\mu_i=1$) [11]. In such case the exact expression of the function $\dot{G}$ may be written as follows:

$$\dot{G}=\frac{\pi\varepsilon}{2\sqrt{i}}\left[H_1\left(\frac{\sqrt{i}}{\varepsilon}\right)-N_1\left(\frac{\sqrt{i}}{\varepsilon}\right)\right]-\frac{\varepsilon^2}{i}, \quad (13)$$

where H_1 i N_1 are the Struve function and Neumann function, correspondingly.

Fig. 2 shows the dependencies of the moduli of the exact function $|\dot{G}|$ and the approximate functions $|\dot{G}_N|$ from the ε parameter for different number of series terms taken into consideration. As one can see, by small ε the most exact results are obtained when greater number of the terms of the asymptotic series is taken into consideration. With increasing the value of ε the deviation from the exact values increases, and the deviation may reach its maximum when senior series terms are taken into consideration. The latter is more evident while relative calculation errors of the modulus of the approximate functions $\Delta_N=\left||\dot{G}_N|-|\dot{G}|\right|/|\dot{G}|$ are compared (Fig. 3).

For the dependencies shown in the case of small ε the most accurate approximation includes four series terms ($N=4$). When the value of ε parameter increases, at some moment the calculation error starts to be greater for $N=4$ that for $N=2$. Further, for even greater values of ε parameter the error increases, and the most accurate approximation includes less asymptotic series terms, namely $N=1$. At the same time the biggest error in this range of ε value appears by the approximation with the biggest number of series terms. While computing the magnetic field of a pulse current applying the asymptotic expansion, the limitation by certain values of $\varepsilon = \dfrac{\mu_i}{r_1\sqrt{\omega\mu_i\mu_0\gamma}}$ parameter is manifested by requirement to apply not the full spectrum of pulse frequencies but only the frequencies that are greater than appropriate values. Moreover, for each series term its own cutoff frequency exists. The application of frequencies that are less than the cutoff frequency leads to the calculation error of the series term that exceeds a given tolerable level.

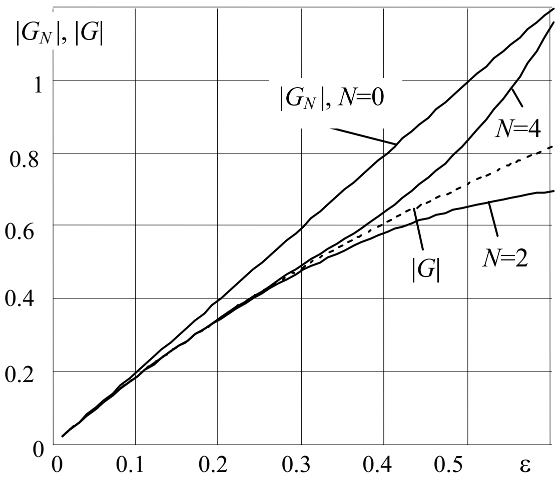

Fig. 2. The approximate and exact values of the modulus of $\dot{G}$ function for different number N of series terms by $\mu_i = 1$ in the observation point.

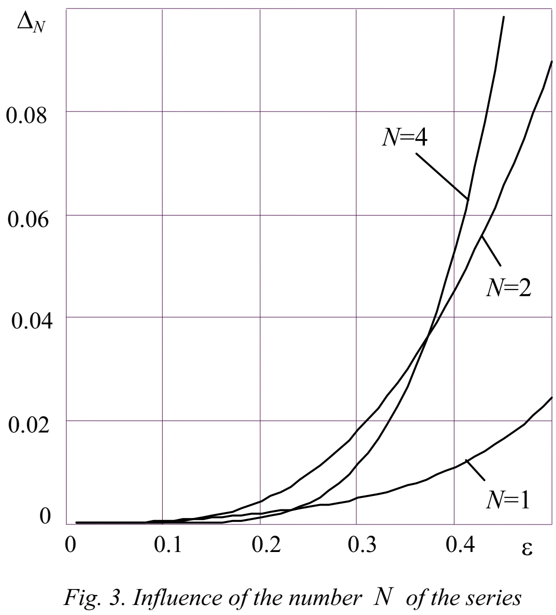

Fig. 3. Influence of the number N of the series termson the relative error of the approximate value of $\dot{G}$ function calculation by $\mu_i = 1$ in the observation point $\rho = 0,\ z = 0$.

To estimate a relative calculation error of each term $\dot{G}_n$ of the asymptotic series the formula $\Delta_n = \left|\dot{G}_n - \dot{G}_{0n}\right| / \left|\dot{G}_{0n}\right|$ was used in [8], where $\dot{G}_{0n}$ is its exact value; this error depends on the number of a series term and on the value of ε parameter in the form:

$$\Delta_n = \frac{e^{-1/\varepsilon}\sum\limits_{k=0}^{n}\dfrac{1}{k!\varepsilon^k}}{1 - e^{-1/\varepsilon}\sum\limits_{k=0}^{n}\dfrac{1}{k!\varepsilon^k}}. \tag{14}$$

Since the parameter ε depends on the frequency, to obtain its cutoff value $f_n = 1/2\pi\omega_n = = \dfrac{\mu_i}{2\pi\left(z+h\right)^2\varepsilon_n^2\mu_0\gamma}$ it is necessary to solve the equation (15) for a given tolerable calculation error Δ_n of a series term and find the frequency f_n. We used normalized cutoff frequencies with the basic frequency $f_b = \left(\pi h^2\mu_i\mu_0\gamma\right)^{-1}$ as normalization basis; the basic frequency is the frequency when the field penetration into the conducting body is equal to the vertical coordinate $\delta = h$ of a current loop element. In that case, for computation of the normalized frequency $f_n^* = \dfrac{f_n}{f_b} = \dfrac{\mu_i^2}{2\varepsilon_n^2\left(1+z/h\right)^2}$ it is necessary to find from (14) corresponding values of ε_n

Fig. 4 shows the normalized values of cutoff frequencies as dependences from the number n of a series term for three permissible relative calculation errors for the determination of the asymptotic series terms. The cutoff frequencies have been obtained by $\mu_i = 1$ for the point $\rho = 0,\ z = 0$, i.e. for the point with the minimum value of the distance $r_1 = h$ and, therefore, with the maximum value of the parameter ε and, respectively, with maximum values of cutoff frequencies. For this point the part of the pulse frequency spectrum that is not taken into account is the greatest one.

The conducted analysis shows that application of the asymptotic expansion leads to the fact that the high-frquency part of a spectrum is taken into consideration the most. When the spectrum frequency decreases, it is necessary to decrease the number of series terms up to that complete elimination for the frequencies $f^* < f_0^*$. It is worth to emphasize that the extent of the signal spectrum consideration or, on the other hand, the excluded part of the spectrum depends on the frequency spectrum of the signal, i.e. on the location of corresponding cutoff frequencies f_n^* in the spectrum, and the values of the latters depend, inter alia, on the given accuracy Δ_n for calculation of the series terms.

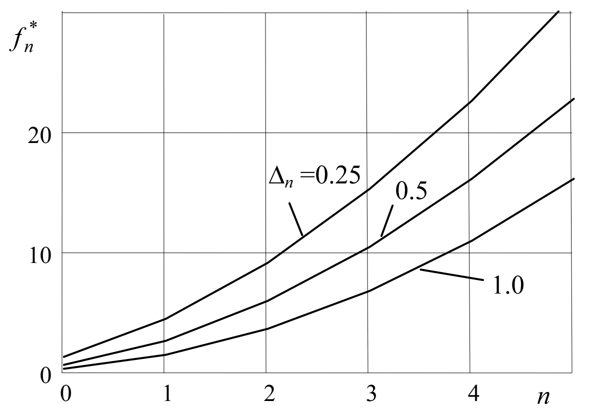

Fig. 4. Normalized cutoff frequencies for different terms of the asymptotic series in the point $\rho = 0, z = 0$.

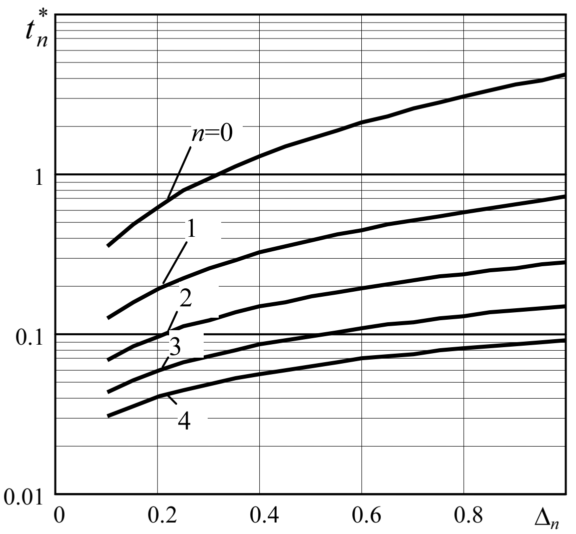

Fig. 5. Normalized maximum instants for different terms of the asymptotic series depending on their calculation error Δ_n *in the point* $\rho = 0, z = 0$.

For a pulse current changing in time the confinement of the spectrum by certain cutoff frequencies is manifested by the fact that computation is performed not during the entire time span but only until some instants and taking into account only certain number of series terms. Such relative maximum time intervals may be calculated using the obtained frequencies f_n^* by the formula $t_n^* = 1/f_n^*$, where $t_n^* = t_n f_b$.

Fig. 5 shows normalized values of cutoff instants as dependencies from the given accuracy Δ_n for the first five series terms.

It should be mentioned that the permissible time span for the calculations strongly depends both on the number of the last series term and on the chosen accuracy Δ_n . It mens that, the closer to the pulse start the calculation instant is, the more accuratly the calculation (with application of the proposed asymptotic method) of three-dimensional electromagnetic field is carried on.

4. Conclusion

The proposed approach to the calculation of the three-dimensional electromagnetic field induced by a pulse current flowing in an arbitrary loop with taking into account eddy currents in an external conductive body can significantly simplify the problem and allows us to present the solution in the form of asymptotic series, whose terms are functions of the current loop field.

The peculiarity of this method is that each series term is calculated with a certain error that increases with increasing of the number of a series term and depends on the signal frequency. As a result, in the calculation not the entire range of the frequency spectrum of the pulse is used. Therewith, concerning the time domain, it is possible to obtain the time dependency of the field not for the entire time span but only for its part.

Found for each series term, the lower boundaries of the frequency spectrum and the corresponding maximum values of time intervals (counted from the pulse start) allow us, depending on the chosen permissible calculation error, to get the permissible number of the series terms which determines the accuracy of the calculation.

Since the lower cutoff frequencies increase with increasing of the series term number, the most accurate calculation may be carried out for the initial period. Usually a current pulse varies very rapidly and reaches its highest values during a relatively small period of time, so for this very period the electromagnetic field is caculated.

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УМОВИ ЗАСТОСУВАННЯ АСИМПТОТИЧНОГО МЕТОДА РОЗРАХУНКУ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ В СИСТЕМІ "СТРУМОВИЙ КОНТУР–ЕЛЕКТРОПРОВІДНЕ СЕРЕДОВИЩЕ"

Юрій Васецький, Ірина Мазуренко,
Костянтин Дзюба

Проведено дослідження умов застосування наближеного методу розрахунку тривимірного імпульсного електромагнітного поля довільного контура зі струмом, розташованого поблизу плоскої поверхні електропровідного тіла. Показано, що кожний член асимтотичного розкладання обчислюється з певною похибкою, у зв'язку з чим він має власну мінімальну граничну частоту. Отримано оцінку гранічних частот та встановлено відповідні проміжки часу від початку імпульса в залежності від номера члена ряду і обраної припустимої його похибки.

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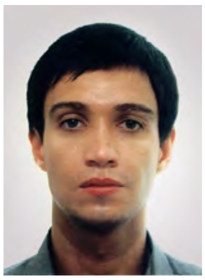

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