COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 3, No. 2, 2013

MODIFICATION OF INDIRECT METHODS OF ELECTRICAL PARAMETERS DETERMINATION IN LIVE CIRCUITS

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Abstract: The existing indirect methods for electrical parameters' measurement of two-terminal elements exhibit a substantial limitation. Such parameters as resistance, inductance and capacitance are determined by means of current, voltage or power measurement and calculation using relevant formulas. Accuracy of these procedures is lowered by internal impedance of the applied meters. In the paper, there are proposed a few modifications of the well-known indirect methods aimed at eliminating their errors.

Key words: parameters of electrical two-terminal elements, indirect methods, errors.

1. Introduction

Engineering employs the methods of so-called "indirect" determination the methods of so-called "indirect" determination of electrical parameters of two-terminal elements, i.e. separate elements or circuits.

These procedures for the determination of two-terminal elements' impedance parameters in a live circuit (in normal operating conditions or fed by an auxiliary test source) mainly utilize typical multimeters and/or voltmeters, amperemeters or wattmeters. Sought values of the parameters (resistance, capacitance and inductance) are determined by the measurement of voltages, currents or powers, and their calculation by relevant formulas, e.g. Ohm's law. The most commonly used procedures are an amperemeter-voltmeter method and a method of three-voltmeter readings [1], [2]. However, all these procedures exhibit errors caused not only by insufficient accuracy of measuring devices, but also by their internal impedances. There have not been presented simple ways of raising accuracy of these methods so far. The paper provides a description of the modification of a few selected methods for the indirect determination of electrical parameters of two-terminal elements. The methods are aimed at eliminating the errors caused by the internal impedances of the elements.

2. Selected existing methods of indirect determination of electrical parameters of two-terminal elements

In the method of three-voltmeter readings, this meter is connected in parallel successively with a supply source, a resistor of the known value R_0 and with a tested two-terminal element of the unknown parameters R_i , X_i (*Fig. 1*).

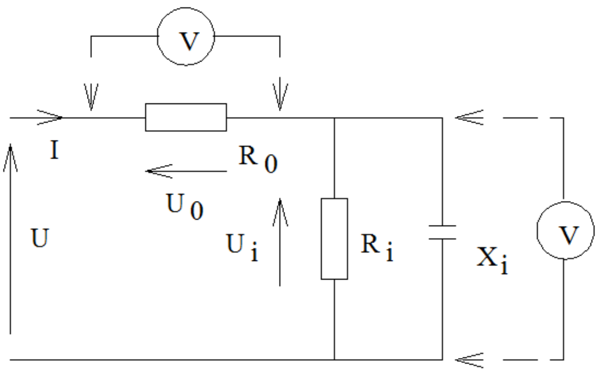

Fig. 1. Explanation of a three-voltmeter readings method.

The sought parameters are given by the formulas below

$$R_i = R_0 \cdot \frac{2 \cdot U_i^2}{U^2 - U_0^2 - U_i^2} \quad (1)$$

$$X_i = R_0 \cdot \frac{2 \cdot U_i^2}{\sqrt{4 \cdot U_0^2 \cdot U_i^2 - (U^2 - U_0^2 - U_i^2)^2}} \quad (2)$$

With the help of the amperemeter-voltmeter method, it is possible to determine an impedance module (when using AC test voltage U) or resistance (when using DC test voltage) of the two-terminal element. Two different test circuits are available for the application of this simplest indirect measurement method (*Fig. 2 a, b*).

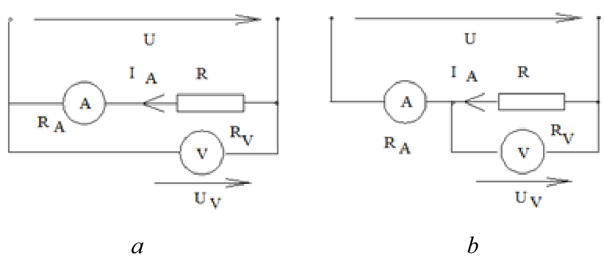

Fig. 2a,b. Determination of resistance R with the help of the amperemeter-voltmeter method using a DC test voltage U.

For both measuring circuits, the sought parameter R is given by the following formula

$$R = \frac{U_V}{I_A} \tag{3}$$

The three voltmeter readings and amperemeter-voltmeter procedures require a constant RMS value of supply voltage U. For derivation of formulas (1), (2), (3), internal impedances of measuring devices were not taken into account. Therefore, the results obtained from these expressions may substantially differ from actual values of two-terminal element's parameters if $Z_V \neq \infty$ and $R_A \neq 0$. Dependence of a relative error of the determination of R_i, X_i parameters on R_i level for the fixed R_V and R_0 values in the method of three-voltmeter readings is shown in Fig. 3.

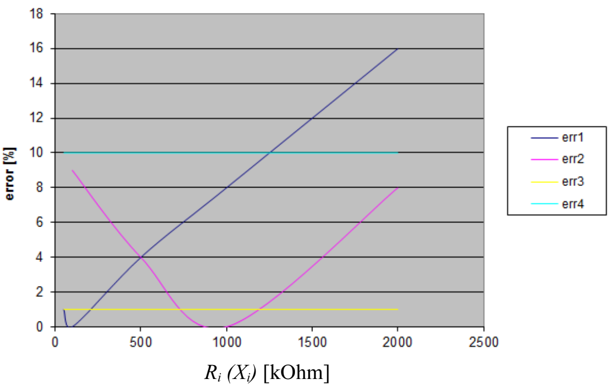

Fig. 3. Relative errors of R_i, X_i parameters determination for fixed R_V=10 MOhm. Curves:
1. determination of R_i for R_0=100 kOhm, X_i=100 kOhm,
2. determination of R_i for R_0=1000 kOhm, X_i=100 kOhm,
3. determination of X_i for R_0=1000 kOhm, R_i=100 kOhm,
4. determination of X_i for R_0=100 kOhm, R_i=1000 kOhm.

Due to these inaccuracies, the indirect methods have got only a limited application for elements parameters determination. In fact, internal resistance of multimeters is of the order of magnitude of 10 MOhm (voltage input) and 10-100 Ohm (current input) which may be comparable to the measured elements resistances. There is, however, a simple way to eliminate this drawback with the help of the modification presented below.

3. Modified methods of determining electrical parameters of two-terminal elements

The above presented indirect methods exhibit the errors caused by internal impedances of measuring devices $Z_V \neq \infty$ and $Z_A \neq 0$. For the amperemeter-voltmeter method, this drawback can be eliminated when the voltmeter is connected in parallel with an amperemeter (Fig. 4b) [3]. In this way the current, flowing through the element when measuring the voltage across its terminals, is accurately determined.

U_V voltage is equal to (Fig. 4a)

$$U_V = \left| U \cdot \frac{\frac{\underline{Z} \cdot \underline{Z}_V}{\underline{Z} + \underline{Z}_V}}{R_A + \frac{\underline{Z} \cdot \underline{Z}_V}{\underline{Z} + \underline{Z}_V}} \right| = \tag{4}$$
$$= U \cdot \left| \frac{\underline{Z} \cdot \underline{Z}_V}{R_A \cdot \left(\underline{Z} + \underline{Z}_V\right) + \underline{Z} \cdot \underline{Z}_V} \right|$$

whereas I_A current is (Fig.4b)

$$I_A = \left| U \cdot \frac{\frac{\underline{Z}_V}{\underline{Z}_V + R_A}}{\underline{Z} + \frac{\underline{Z}_V \cdot R_A}{\underline{Z}_V + R_A}} \right| = \tag{5}$$
$$= U \cdot \left| \frac{\underline{Z}_V}{R_A \cdot \left(\underline{Z} + \underline{Z}_V\right) + \underline{Z} \cdot \underline{Z}_V} \right|$$

It turns out that the ratio of U_V voltage to I_A current is equal to impedance module Z of the element.

$$\frac{U_V}{I_A} = \left| \frac{\underline{Z} \cdot \underline{Z}_V}{R_A \cdot \left(\underline{Z} + \underline{Z}_V\right) + \underline{Z} \cdot \underline{Z}_V} \cdot \frac{\left[R_A \cdot \left(\underline{Z} + \underline{Z}_V\right) + \underline{Z} \cdot \underline{Z}_V\right]}{\underline{Z}_V} \right| = \left|\underline{Z}\right| = Z \tag{6}$$

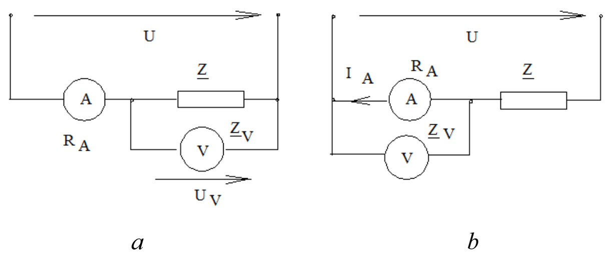

Fig. 4. Method of accurate determination of impedance module Z:
a) measurement of U_V voltage across the element,
b) measurement of I_A current flowing through the element when measuring voltage across its terminals.

It follows from formula (6) that this procedure of the impedance module determination is not affected by internal impedances of measuring devices.

Using the above presented method of accurate measurement of the current flowing through the element when measuring the voltage across its terminals, other procedures of indirect determination of two-terminal elements parameters can be modified [4]. This improvement is explained below in relation to the three-voltmeter readings method. At first, impedance modules $\left|R_0 + \underline{Z}_i\right|$ and Z_i should be determined as shown in Fig.5a, b.

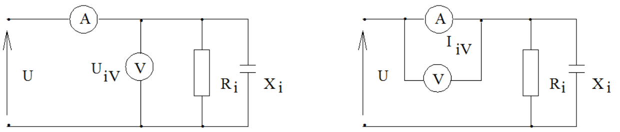

Fig. 5a. Impedance module Z_i determination.

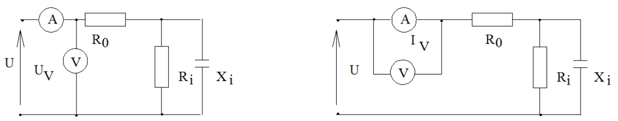

Fig. 5b. Impedance module $\left|R_0 + \underline{Z}_i\right|$ determination.

For derivation of the sought formulas, the circuit shown in Fig. 1 **without a voltmeter** should be considered. Let the current flowing through resistors R_0 and R_i without a voltmeter be marked as I. With the use of the above determined modules of impedances $\left|R_0 + \underline{Z}_i\right|$ and Z_i voltages U, U_0, U_i (RMS values) can be expressed as

$$U_0 = R_0 \cdot I \tag{7}$$

$$U_i = Z_i \cdot I = \frac{U_{iV}}{I_{iV}} \cdot I \tag{8}$$

$$U = \left|R_0 + \underline{Z}_i\right| \cdot I = \frac{U_V}{I_V} \cdot I \tag{9}$$

On substituting these expressions into (1) and (2), the following formulas for impedance parameters are obtained

$$R_i = R_0 \cdot \frac{2 \cdot \left(\frac{U_{iV}}{I_{iV}}\right)^2}{\left(\frac{U_V}{I_V}\right)^2 - R_0^2 - \left(\frac{U_{iV}}{I_{iV}}\right)^2} \tag{10}$$

$$X_i = R_0 \cdot \frac{2 \cdot \left(\frac{U_{iV}}{I_{iV}}\right)^2}{\sqrt{4 \cdot R_0^2 \cdot \left(\frac{U_{iV}}{I_{iV}}\right)^2 - \left[\left(\frac{U_V}{I_V}\right)^2 - R_0^2 - \left(\frac{U_{iV}}{I_{iV}}\right)^2\right]^2}} \tag{11}$$

It should be noticed that the modified three-voltmeter readings method has been transformed into a procedure employing one voltmeter and one amperemeter.

4. Other methods of indirect determination of electrical parameters of two-terminal elements

Besides the methods described above, there can be developed a few other procedures, which are free of errors caused by internal impedances of measuring devices. An example of the procedure using a voltmeter and a wattmeter is shown in Fig. 6.

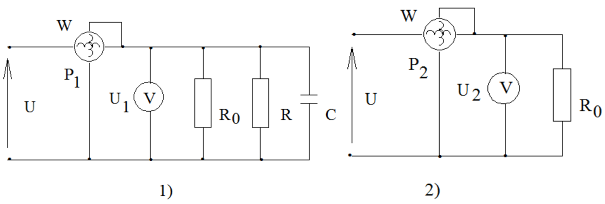

Fig. 6. Two-stage procedure of indirect determination of R resistance with the help of a voltmeter and a wattmeter.

The wattmeter indication in the first stage is

$$P_1 = U_1^2 \cdot \left(G_W + G_V + G_0 + G\right) \tag{12}$$

whereas in the second one it is

$$P_2 = U_2^2 \cdot \left(G_W + G_V + G_0\right) \tag{13}$$

where G, G_W, G_V, G_0 are the conductances of a two-terminal element, a wattmeter, a voltmeter, and an additional resistor respectively. It is not necessary to know values G_W, G_V, G_0. Besides, supply voltage U does not have to be the same at both stages. From equations (12) and (13), the sought value of R is calculated as follows:

$$R = \frac{1}{G} = \frac{U_1^2 \cdot U_2^2}{P_1 \cdot U_2^2 - P_2 \cdot U_1^2} \tag{14}$$

For the determination of B susceptance with the known value of G and constant U voltage, one can use the modified amperemeter-voltmeter method as shown in Fig. 4.

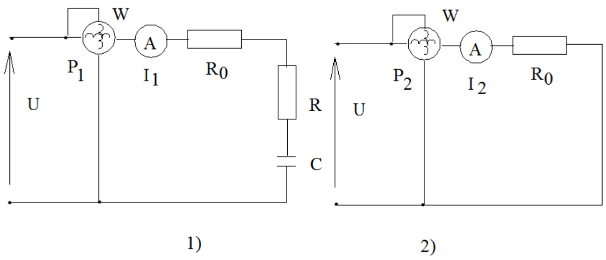

Fig. 7. Two-stage procedure of indirect determination of R resistance with the help of an amperemeter and a wattmeter.

From (15)

$$I_A^2 = U_V^2 \cdot \left(G^2 + B^2\right) \qquad (15)$$

the sought value of B is obtained.

For a two-terminal element consisting of a resistor connected in series with a reactive element, e.g. a capacitor, an alternative circuit (Fig.7) can be used.

The sought value of R is given by the formula below

$$R = \frac{P_1 \cdot I_2^2 - P_2 \cdot I_1^2}{I_1^2 \cdot I_2^2} \qquad (16)$$

5. Conlusion

1. The methods of “indirect” determination of two-terminal elements parameters may exhibit considerable errors caused by their internal impedances.
2. The modified indirect methods of parameters determination by formulas (6) (10), (11), (14), (16) (and other similar procedures as well) provide accurate results which are not influenced by internal impedances of the applied measuring devices.
3. The modified methods can be used for the circuits not only supplied from test sources but also under normal operation. One of the possible areas of application may be the determination of insulation parameters, i.e. resistance and capacitance.

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МОДИФІКАЦІЯ НЕПРЯМИХ МЕТОДІВ ВИЗНАЧЕННЯ ЕЛЕКТРИЧНИХ ПАРАМЕТРІВ У СХЕМАХ ПІД НАПРУГОЮ

Пійотр Ольшовєц

Існуючі непрямі методи для визначення електричних параметрів двоклемових елементів мають суттєве обмеження. Такі параметри як опір, індуктивність і ємність визначаються через вимірювання струму, напруги і потужності, а також розраховуючи їх за допомогою відповідних формул. Точність цих методів знижується через власний імпеданс вимірювальних приладів, що використовуються. У цій статті запропоновано декілька модифікацій добре відомих непрямих методів. Завдання цих модифікацій полягає в усуненні похибок, що виникають у разі застосування існуючих методів.

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