ANALYSIS OF A DISTRIBUTED SLOT DISCONTINUITY IN THE GROUND PLANE OF A MICROSTRIP LINE

Yulia Rassokhina, Vladimir Krizhanovski
Donetsk National University, Ukraine
yu.rassokhina@donnu.edu.ua, v.krizhanovski@donnu.edu.ua

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Abstract: This paper presents a method of analysis of distributed slot discontinuity in a microstrip line ground plane based on transverse resonance technique. The scattering characteristics of symmetrical and nonsymmetrical slot resonators of a complex shape have been calculated by Galerkin’s procedure and taking into account the harmonics of current density in the microstrip line and waveguide modes in the slot aperture. As a computational example the analysis of a finite cell periodic structure on the basis of Π- and O-shaped slot resonators in the ground plane of a microstrip line are presented. The effectiveness and accuracy of the proposed algorithm has been verified by the comparison with measurements conducted on experimental prototypes.

Key words: microstrip line, slot resonator, transverse resonance technique, eigenfrequency, scattering matrix, periodic structure.

1. Introduction

Planar structures in the form of microstrip transmission lines with a slotted ground plane are widely used for designing various microwave devices, for example, periodic structures, wideband filters [1-4] and antenna elements [5-7]. The analysis of discontinuities in planar transmission lines with borders shaped parallel to rectangular coordinate axes can be performed using strict numerical techniques, a transverse resonance technique in particular.

The transverse resonance technique was originally developed for the calculation of characteristics of ridged waveguides [8] and quasi-planar transmission lines [9]. In [10] the transverse resonance technique was extended to the calculation of the characteristics of open, conductor-backed or shielded structures. Noticeably, the transverse resonance algorithm can be applied even to open structures owing to the representation of boundary conditions by reflection coefficient matrices.

Later on, the transverse resonance technique was further extended to the analysis of step discontinuities in transmission lines, for example, a width step in a finline [11] and a microstrip line [12]. In [13] the theory of long lines combined with transverse resonance techniques was presented for calculation of scattering parameters of a microstrip line with step discontinuity printed on a biaxialotropic substrate.

In [14] while solving boundary problem for a resonator with discontinuity in the form of two microstrip lines crossed and positioned on the different sides of a dielectric substrate, the current density components in the lines were represented as double series with basis functions of two types: a harmonic and a singular one, thus taking into account the peculiarity of the field behavior on a thin edge. The analysis of more complex discontinuity in the form of a stripline resonator in a unilateral finline was performed in [15]. In that work a finline discontinuity, which consists of a rectangular conducting strip placed on the opposite face of dielectric substrate transversely to the slot, was analyzed. Appropriate basis functions were chosen to model a field in the slot and a current in the strip. Herewith in the expression for cross-slot electric field the harmonics of the electromagnetic field caused by the resonator length were taken into account. Since the discontinuities considered in [15] had small sizes compared with the wavelength, the parameters of an equivalent circuit were calculated from the condition of transverse resonance. Full wave solutions in the frequency domain for crossed microstrip line and slotline were implemented [16].

In [17] the method of the calculation of amplitude characteristics and phase shift characteristics for one- and multi-stage 1-D periodic structures comprising narrow-slot resonators in the ground plane of a microstrip line was presented.

2. Transverse resonance method for irregular slot resonators in the ground plane of a microstrip line

The transverse resonance technique is based on the solution of a boundary problem for a resonator on a feed line with discontinuity, and serves for calculating scattering matrix elements [17, 18]. The scattering matrix \( S \) of the two-port network consisting of the microstrip line with a complex shape slot resonator in its ground plane (Fig. 1) has the following form:
It is typically assumed that the electromagnetic boundaries of the resonator (electric or magnetic walls) are situated at such a distance from the discontinuity that the higher wave modes of the microstrip line may be considered as already attenuated. Under the condition of existence of electric walls in $z = -L_z$ and $z = L_z$ planes (e.w.-e.w. condition or so-called «electric» boundary problem, Fig. 1a), the characteristic equation of the transverse resonance has the following form \[18\]:
\[
(S_{11} + \Gamma_1)(S_{22} + \Gamma_2) - S_{12}^2 = 0,
\]
(1)
where $\Gamma_1 = \exp(2j\beta_xL_z)$, $\Gamma_2 = \exp(2j\beta_yL_z)$, $\beta_z$ is the phase constant of the fundamental wave of the microstrip line. For structures nonsymmetric with respect to $z = 0$ plane, Fig. 1a, the scattering matrix elements in $A$–$A'$ planes ($S_{11} \neq S_{22}, S_{21} = S_{12}$) are calculated from the three pairs of solutions of the “electric” boundary problem for the microstrip resonator with respect to its dimensions ($l_k, l_{k+1}$), where $k = 1, 2, 3$ is the solution number.

For the analysis of the structure which is symmetric with respect to $z = 0$ plane, for example, a symmetric $\Pi$-shaped slot resonator, Fig.1b, it is sufficient to solve two boundary problems: one under electric wall condition at the region's boundary $z = L$ and at the plane of symmetry $z = 0$, and another with electric wall condition at $z = L$ plane and magnetic wall (m.w.) condition at $z = 0$ plane \[17, 18\]. Then the elements of the scattering matrix $S_{11} = S_{22}$, $S_{12} = S_{21}$ are calculated from the solutions of the “electric” (e.w.-e.w.) and “magnetic” (m.w.-e.w.) boundary problems with respect to the dimensions of the resonator $l_k$ ($k=1, 2$ is the number of solution) according to the formulae obtained from the solution of two equations (1):
\[
S_{11} = -\frac{(\Gamma_1 + \Gamma_1')}{2}, S_{12} = \frac{(\Gamma_1 - \Gamma_2')}{2},
\]
(2)
where $\Gamma_1$, $\Gamma_2$ are defined above. From (2) we conclude that the frequencies of resonant interaction are determined by the intersection points of the spectral curves (when $\Gamma_1 = \Gamma_2'$) obtained from the solutions of two boundary problems. For the calculation of the scattering matrix of a single distributed discontinuity, it is preferable to solve the “magnetic” boundary problem under magnetic wall conditions in the symmetry plane and at the longitudinal boundary of the resonator (m.w.–m.w.) \[11\].

In filtering or matching networks periodic structures of finite size are considered to consist of several discontinuities in the transmission line positioned at some distance from one another. If the interrelation between the series-connected discontinuities can be neglected, the overall scattering matrix of the finite periodic structure can be calculated using formulae for the cascade connection of the matrices of each discontinuity. However, for multi-cell periodic structures with the interrelation between discontinuities it is necessary to solve the boundary problem for the structure as a whole.
Fig. 2 shows a resonant structure non-symmetric in the transverse direction with of a Π-shaped slot resonator in the microstrip line's ground plane (the cross-section and the top view are shown). The structure is symmetric with respect to x=0 plane.

The first layer (i=1) represents a dielectric substrate with relative dielectric permeability \(\varepsilon_{r1}\) and thickness \(h\), the second and third layers are the air ones, \(\varepsilon_{r2}\) = \(\varepsilon_{r3}\) = 1. The field components have to satisfy the conditions of an ideally conductive electric wall at \(x = \pm A\), \(y = B\), \(y = -b\) and electric wall conditions at the longitudinal boundaries \(z = 0\) and \(z = L\) for the “electric” boundary problem.

The boundary problem for the resonator is solved by dividing the original region into 3 partial subregions (Fig. 2(a)). Then for each subregion the Helmholtz equation is solved for electric (denoted by \(e\) subscript) and magnetic (denoted by \(h\) subscript) Hertz vector potentials \((0, A_{(e)(y)}, 0)\):

\[
\Delta A_{(e)(y)}, + k_0^2 e_i A_{(e)(y)}, = 0,
\]

where \(k_0 = \omega_0/c\) is the wavenumber and \(\varepsilon_{ri}\) is the relative dielectric permittivity of the \(i\)-th layer.

Electric and magnetic Hertz vector potentials are represented in the chosen coordinate system as double Fourier series expansions of the following form:

\[
A_{y,j} = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} P_{mn} \cos k_{mn} x \cos k_{zn} z F_{yj, mn} (y),
\]

\[
A_{y,j} = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} P_{mn} \sin k_{mn} x \cos k_{zn} z F_{yj, mn} (y),
\]

where \(i = 1, 2, 3\) is the partial region number. The following notations are introduced here:

\[
P_{mn} = \sqrt{2/A} \sqrt{(2 - \delta_{\varepsilon0})/Lx_{mn}^3}, \quad x_{mn}^3 = k_{mn}^2 + k_{zn}^2,
\]

\[
k_{mn} = \pi (2m-1)/2A, \quad k_{zn} = \pi n/L,
\]

\(\delta_{\varepsilon0}\) is the Kronecker symbol. The functions \(F_{yj, mn} (y)\) are written as follows (\(y_0 = h/2\)):

\[
F_{y1, mn} = R_{y1, mn} \cos k_{y1, mn} (y - y_0) + R_{y2, mn} \cos k_{y1, mn} y_0,
\]

\[
F_{y2, mn} = R_{y2, mn} \cos k_{y2, mn} (B - y) + R_{y3, mn} \cos k_{y2, mn} b_2,
\]

\[
F_{y3, mn} = R_{y3, mn} \cos k_{y2, mn} (b + y) + R_{y4, mn} \cos k_{y2, mn} y_0,
\]

\[
F_{y1, mn} = R_{y1, mn} \sin k_{y1, mn} (y - y_0) + R_{y2, mn} \sin k_{y1, mn} y_0,
\]

\[
F_{y2, mn} = R_{y2, mn} \sin k_{y2, mn} (B - y) + R_{y3, mn} \sin k_{y2, mn} b_2,
\]

\[
F_{y3, mn} = R_{y3, mn} \sin k_{y2, mn} (b + y) + R_{y4, mn} \sin k_{y2, mn} y_0,
\]

where \(k_{mn}^2 = k_0^2 \varepsilon_{r1} - k_{zn}^2 - k_{mn}^2\) and \(R_{y1, mn}\), \(R_{y2, mn}\), \(R_{y3, mn}\), \(R_{y4, mn}\) are unknown coefficients of the Fourier series expansions for the partial regions.

For the scattering matrix calculation of a single discontinuity being symmetrical in transversal direction, the two boundary problems, “electric” and “magnetic” ones, are solved for a half of the structure shown in Fig. 3. In addition, as far as the problem is nonsymmetrical with respect to \(x=0\) plane, the electric and magnetic Hertz vector potentials are expanded into the double Fourier series of the following form:

\[
A_{y,j} = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} P_{mn} \cos k_{mn} (x-A) \left\{ \sin k_{mn} z \frac{\sin k_{zn} z}{\cos k_{mn} z} \right\} F_{yj, mn} (y)
\]

\[
A_{y,j} = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} P_{mn} \cos k_{mn} (x-A) \left\{ \sin k_{mn} z \frac{\sin k_{zn} z}{\cos k_{mn} z} \right\} F_{yj, mn} (y)
\]

“electric” and “magnetic” boundary problems, correspondingly, where \(k_{mn} = \pi n/2A\): the upper expressions in the braces correspond to the “electric” boundary problem and the lower expressions in the braces correspond to the “magnetic” boundary problem; other notations are the same as previously introduced.

The boundary problem is solved by Galerkin’s method. In order to do so, the field in the slot resonator...
Yulia Rassokhina, Vladimir Krizhanovski

is written as the series of eigenfunctions \( T_{n(E),k} \) of the \( H \)- and \( E \)-waves of a II-shaped waveguide [19, 20] (odd modes for the resonator in Fig. 2).

\[
\mathbf{E}_n = \sum_{k=1}^{\infty} V_{nk} \left( \nabla T_{n,y,k} \times \mathbf{e}_y \right) + \sum_{k=1}^{\infty} V_{nk} \nabla T_{n,x,k},
\]

\[
\mathbf{H}_n = \sum_{k=1}^{\infty} I_{nk} \nabla T_{n,y,k} - \sum_{k=1}^{\infty} I_{nk} \left( \nabla T_{n,x,k} \times \mathbf{e}_x \right),
\]

where \( V_{nk} \) are unknown coefficients of the expansion.

Current density in the microstrip line defined through the difference of tangential components of the magnetic field in the \( y = h \) plane \( \mathbf{H}_{1,1} - \mathbf{H}_{1,2} = J \times \mathbf{e}_y \), is written in the form of double series of Chebyshev polynomials (along the \( x \) axis) and eigenfunctions of the resonator (along the \( z \) axis) with the unknown coefficients of the expansions \( c_{n,k}, d_{n,k} \):

\[
J_z(x,z) = \sum_{q=0}^{N_q} \left( \frac{2-\delta_{q0}}{L} \cos k_{q,y} z \right) \left( \sum_{k=0}^{\infty} C_{q,k} \psi_k \left( \frac{x}{w/2} \right) \right),
\]

\[
J_x(x,z) = \sum_{q=0}^{N_q} \left( \frac{2-\delta_{q0}}{L} \sin k_{q,x} z \right) \left( \sum_{k=0}^{\infty} d_{q,k} \psi_k \left( \frac{x}{w/2} \right) \right),
\]

where \( N_q \) is the order of trigonometric series truncation (the number of spatial harmonics in the current density spectrum) and \( \psi_q \left( 2x/w \right) \), \( \psi_q \left( 2y/h \right) \) are functions normalized with weight taking into account the field’s behavior on a thin edge [16, 17, 19].

From the formal point of view the summation in trigonometric series (5) starts from a zero index, however in nonsymmetrical structures with absence of external charge sources the zero component of longitudinal current density \( J_{z,0} \) (the DC component) does not have physical sense. Due to this, while solving the boundary problem for a nonsymmetrical structure, the current density in the microstrip line is approximated with trigonometric series with summation running from index one.

By applying Galerkin’s procedure to continuity equations of the field’s tangential components on the partial regions’ boundaries \( y = 0 \) and \( y = h \), we obtain the homogeneous linear system of algebraic equations with an unknown parameter (longitudinal size \( L \) of the resonator or its eigenfrequency \( f_{\text{res}} \)) with respect to unknown coefficients of the field expansion on the II-shaped slot \( V_{n,k}, V_{e,k} \) and current density \( c_{n,k}, d_{n,k} \) in the microstrip line. By equating the determinant of the linear system of equations to zero the characteristic equation for determining that unknown parameter is obtained.

3. Results of numerical analysis

After investigating the convergence of the algorithms, the numerical computation of eigenfrequencies of the microstrip resonator with distributed discontinuity in its ground plane has been performed by applying double Fourier series truncated to 300 terms. In the series of the complex-shape slot resonator’s eigenwaves the two waves of both \( H \)- and \( E \)-types were taken into account, while in the double series (5) the summations of spatial harmonics and the Chebyshev polynomials were limited to five terms \((N_q = 5 \text{ and } k = 0, 5)\).

Fig. 4. The characteristics of the microstrip resonator with nonsymmetric II-shaped slot, Fig 1a: (a) eigenfrequency spectrum, and (b) insertion losses of 3-cells periodic structure. The resonator parameters (in mm): substrate \( h=1.0, \varepsilon_r=9.8 \), the microstrip line width \( w=1.0 \) (characteristic impedance \( Z_0=50 \) Ohm), screen dimensions \( b_1=9.0, b_2=5.0, A=24.0 \), slot resonator dimensions: \( a=8.0, b=8.0, s_1=0.5, s_2=0.75 \).

Fig. 4a shows the eigenfrequency spectrum \( f_{\text{res}} \) of the microstrip line resonator with the II-shaped slot in the groundplane (Fig. 1a) versus the resonator size \( L \) and its length ratio \( K_p = L_p/L_1 \), \( L_1 = l_1 + h/2, i = 1, 2 \).

The eigenfrequency spectrum in the range from 1.5 GHz to 5.5 GHz consists of two branches, therefore, while solving the boundary problem with respect to the resonator’s sizes \( l_1, l_2 \) it is necessary to make a transition
Analysis of a Distributed Slot Discontinuity in the Ground Plane of a Microstrip Line

to the second spectrum branch at the frequency approximately equal to 2.6 GHz.

Fig. 4b shows the characteristic of the transmission coefficient of the microstrip line’s fundamental wave for the periodic structure consisting of three \((n=3)\) series-connected \(\Pi\)-shaped slot resonators in the ground plane.

The scattering matrix of the periodic structure with finite number of cells is obtained as a cascade connection of the scattering matrices of the single slot resonator through the sections of microstrip line with the length \(d_m\) [17]. Fig. 4 also shows the measurement results of the structure prototype.

Fig. 5a shows the results of eigenfrequency spectrum calculation for the microstrip resonator with symmetric \(\Pi\)-shaped slot, Fig 1b: (a) eigenfrequency spectrum, and (b) insertion losses of 5-cell periodic structure. The resonator parameters (in mm): screen dimensions \(b_1=b_2=12.0\), \(A=24.0\); slot resonator dimensions: \(2a=4.4\), \(b=2.2\), \(s_1=s_2=0.5\).

Fig. 5a shows the results of eigenfrequency spectrum calculation for the microstrip resonator with symmetrically located distributed discontinuity (Fig. 3). The calculation has been performed taking into account the spatial zero harmonic of current density. In this case the eigenfrequency spectrum obtained from the solution of the “electric” boundary problem contains one solution branch in the frequency range from 2 GHz to 8 GHz, while the eigenfrequency spectrum of the “magnetic” boundary problem contains two solution branches. This is explained by the fact that the symmetry of the structure with the magnetic wall in \(z=0\) plane corresponds to the fundamental wave of the \(\Pi\)-shaped waveguide. Fig. 5b shows the characteristic of the transmission coefficient of a 5-cell periodic structure comprising \(\Pi\)-shaped slot resonators located with the period of \(d = 5.4 + a = 9.8\) mm.

It can be seen that the characteristics calculated for both nonsymmetric and symmetric resonant structure correspond well to the measurement results. The characteristics of transmission coefficient were measured using Agilent N5230A vector network analyzer.

4. Analysis of coupled distributed slot resonators in microstrip line ground plane

Fig. 6 shows a structure topology consisting of three series-connected \(O\)-shaped (i.e. shaped as a coaxial rectangular transmission line) slot resonators in the groundplane of a microstrip line. As a performed numerical calculation shows, a resulting scattering matrix obtained from elementary scattering matrices for the cascade connection of the single discontinuities does not correspond to the measurement results of the transmission coefficient for the experimental prototype. This is due to the fact that, as it was shown in [20], distributed discontinuities interact with each other on distances comparable with the size of the discontinuities themselves, giving rise to additional frequencies of the resonant interaction of the feed line with several series-connected discontinuities.

For taking into account the mutual interaction between discontinuities, the 3-cell periodic structure is analyzed as a whole. Just as it has been done above, due to the symmetry of the structure the scattering matrix elements are determined by solving two boundary problems, the “electric” \((e.w.–e.w.)\) and “magnetic” \((m.w.–m.w.)\) one, see Fig. 6.
In that case in the expansion of the field in the O-shaped slot the TEM-wave of the coaxial waveguide is taken into account in addition to even \((e)\) and odd \((o)\) wave modes of the O-waveguide

\[
E_{ei} = \sum_{k=1}^{\infty} V_{he,k} \left( \nabla T_{xy,k} \times \mathbf{e}_y \right) + \sum_{k=1}^{\infty} V_{oe,k} \nabla T_{xy,k} + \sum_{k=1}^{\infty} V_{eo,k} \left( \nabla T_{xy,k} \times \mathbf{e}_y \right) + V \nabla T_{TEM},
\]

where \(V_{he,k}\) and \(V_{oe,k}\) are the amplitudes of even \(H\)- and \(E\)-waves of the O-waveguide, \(V_{eo,k}\) is the amplitude of odd \(H\)-waves of O-waveguide, \(V\) is the amplitude of the TEM-wave of the coaxial waveguide.

Fig. 7. The characteristics of the 3-cell periodic structure with symmetric O-shaped slot resonators in the microstrip line’s ground plane: (a) eigenfrequency spectrum, and (b) insertion losses of the structure. Dimensions of the O-shaped slot resonator (in mm): \(a=12.9, b=2.15, s_1=s_2=0.5\).

Fig. 7a shows the eigenfrequency spectrum of the microstrip resonator with three serially connected slot resonators in the ground plane of the microstrip line. The spectrum is obtained from the solutions of “electric” and “magnetic” boundary problems. The parameters of the microstrip line are the same as mentioned above, the dimensions of the O-shaped slot resonator are marked at the figure. In the frequency range from 2 GHz to 6 GHz the resonance frequency spectrum consists of four branches.

Fig. 7b shows calculated and measured transmission coefficient characteristics of the periodic structure, which consists of three O-shaped slot resonators. The calculated characteristic is obtained from the solutions of “electric” and “magnetic” boundary problems. Just like in [20, 21], the stopband of the fundamental wave of the microstrip line in the system of interacting slot resonators is wider than that of the system of isolated discontinuities.

5. Conclusion

A new type of discontinuity in the microstrip line ground plane has been analyzed using the transverse resonance technique. Appropriate basis functions have been chosen to model the field in the microstrip resonators with \(\Pi\)- and O-shaped slot resonators and the current density on the discontinuity strip. Herewith in order to account for the mutual interaction between discontinuities, the spatial zero harmonic of the current density is taken into account in solving the boundary problems for the symmetric resonator. The spectral curves and characteristics of the transmission coefficient for the distributed discontinuities are also presented. It is demonstrated that the calculated results match the measured ones reasonably well, what confirms the choice of basis functions.

References


Analysis of a Distributed Slot Discontinuity in the Ground Plane of a Microstrip Line


АНАЛІЗ РОЗПОДІЛЕННИХ ЩІЛИНИХ НЕОДНОРІДНІСТЕЙ В УЗЕМЛЮЮЧОМУ ШАРИ МІКРОСТРИЧКОВОЇ ЛІНИЇ ПЕРЕСИЛАННЯ

Юлія Рассохіна, Владмір Кріжановський

У роботі представлено метод аналізу розподілених щілинних неоднорідностей в узелюючему шарі мікросмужкової лінії пересилання методом поперечного резонансу. Характеристики розсіювання симетричних та несиметричних щілинних резонаторів складної форми розраховано за методом Гальоркіна з урахуванням гармоній густини струму в мікросмужковій лінії та хвилевих мод на апертурі щілини. Як приклад проектування представлено аналіз періодичних структур на базі П- та О-подібних щілинних резонаторів в узелюючому шарі. Ефективність та точність запропонованого методу демонструється шляхом порівняння розрахованих характеристик із даними, отриманими в результаті вимірювань на експериментальних макетах.
Yulia Rassokhina, Vladimir Krizhanovski

Vladimir G. Krizhanovski (M’96–SM’01) received his M.S. degree in Radio physics from Donetsk State University, Ukraine in 1974, his Ph.D. degree in Physical electronics from Kharkiv State University, Ukraine in 1987, and his Dr. of Science degree in Technical sciences from Kharkiv National University of Radioelectronics, Ukraine in 2010.

He is currently a Full Professor of the Radio Physics Department, Donetsk National University, Ukraine. His research interests include UHF electrodynamics and electronics, traveling wave tubes, and transistor high-efficiency power amplifiers. He is the author of the book *Transistor Amplifiers with High Efficiency* (Apex, Donetsk, Ukraine, 2004) and the coauthor of the book “Advanced Design Techniques for RF power Amplifiers” (Springer, Germany, 2006). He has published over 120 technical papers and holds 6 patents.

Yulia V. Rassokhina (M’02) was born in Ukraine, in 1966. She received her M.S. degree in Physics from Donetsk State University, Ukraine in 1988 and her Ph. D. degree in Radio physics from the Institute of Radio Physics & Electronics of the National Academy of Sciences of Ukraine, in 1997.

Now she is working as a senior researcher of the Radio Physics Department, Donetsk National University, Ukraine. Her current research interests include integrated circuits, systems based on waveguides and multilayered planar structures, and modeling the microwave devices. She has published over 35 technical papers.