COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 3, No. 1, 2013

SYMMETRY OF ENERGY

Vasyl Tchaban
Lviv Polytechnic National University, Ukraine, and Rzeszow University, Poland
vtchaban@polynet.lviv.ua

Abstract: Based on the principle of symmetry, we propose a universal expression of the energy of physical system. If we accept a generalized coordinate as an argument, we come to potential energy; if we accept a generalized velocity as an argument, we come to kinetic co-energy. Here, like in the variational principle, the concept of kinetic energy does not work. The examples of application relate to electromagnetic fields, electromagnetic circuits, and mechanics of concentrated masses.

Key words: symmetry, universal expression, po–tential energy, kinetic co-energy, electromagnetic field, electromagnetic circuit, mechanics.

1. Introduction

The basis of the energy approach to the analysis of physical systems is the variational principle of Hamilton-Ostrogradsky, which uses a concept of kinetic and po–tential energies. These concepts are often relative and depend on the choice of certain generalized coordinates and generalized velocities [1, 2]. But it would be at least unwisely to refuse the natural variables. The importance of a real physical value (characteristic) is connected to the concept of co-energy.

2. Formulation of the problem

An expression of energy stored in the volume *V* of a domain under consideration can be written down as

$$W_i = \int_V w_i dV, \qquad (1)$$

where $w_i \; (i = k, p, kc)$ are the densities of kinetic energy, potential energy, and kinetic co-energy (co-operative), correspondingly.

Since many different expressions of the energy densities may be obtained, from the perspective of field theory their expressions are postulated, but the right one has not been determined yet. That is why we have designated the simplest expressions as basic ones. We, according to the principle of symmetry, propose the following expression

$$w_i = \int_0^{\xi} \chi(\xi)\xi d\xi. \qquad (2)$$

The variable $\chi(\xi)$ always has a meaning of some static matrix that characterizes the medium or a finite element. If we choose to interpret the variable ξ as a generalized coordinate, the expression (2) corresponds to the density of potential energy. If we choose to interpret it as a generalized velosity, the expression (2) corres-ponds to the density of kinetic co-energy. We shall consider three different cases.

3. Electromagnetic field

It is a general practice to express the specific energies by means of vectors of electromagnetic field. But those vectors, as shown in electrodynamic of potentials, are derivatives of the vector potential **A** of electromagnetic field [1]

$$\vec{E} = -\partial\vec{A} / \partial t; \quad \vec{B} = \nabla \times \vec{A}, \qquad (3)$$

then

$$\vec{D} = \mathbf{E}(\vec{E})\vec{E}; \quad \vec{H} = \mathbf{N}(\vec{B})\vec{B}, \qquad (4)$$

where $\vec{E}, \vec{D}, \vec{B}, \vec{H}$ are the vectors of electromagnetic field. $\mathbf{E}, \mathbf{N}$ are the matrices of static permeabilities and static reluctivities of the environment.

As you can see, the vector **B** can be interpreted as a generalized coordinate and the vector **E** can be interpreted as a generalized velocity. Substituting (3) and (4) into (2), we obtain expressions of densities of potential energy and kinetic co-energy

$$w_p == \int_0^{\vec{B}} \mathbf{N}(\vec{B})\vec{B}d\vec{B}; \quad w_{kc} = \int_0^{\vec{E}} \mathbf{N}(\vec{E})\vec{E}d\vec{E}, \qquad (5)$$

It is evident that the magnetic field generates potential energy, and electric field generates kinetic co-energy, - their expressions are entirely consistent with those conventional in electrodynamics [2].

The expression of specific kinetic energy of the electromagnetic field does not go with the principle of symmetry, and its equation can not be obtained from (2). This expression can be obtained by a classical method, on the basis of the equations of electro–magnetic field in an immovable lossless environment written down in the vectors

$$-\int_V \left(\vec{E}\frac{\partial\vec{D}}{\partial t} + \vec{H}\frac{\partial\vec{B}}{\partial t} \right) dV = \int_S \left(\vec{E} \times \vec{H} \right) d\vec{S}, \qquad (6)$$

The left side of the equation (6) is treated as a decrease in the electromagnetic energy $\partial w / \partial t$ in some closed volume, and the right one details that it is spent on radiation. Therefore, according to (6)

$$w_k = \int_0^{\vec{D}} \Xi(\vec{D})\vec{D}d\vec{D}, \qquad (7)$$

where $\Xi(\vec{D})$ is the inverse matrix of static electric permeabilities ($\Xi = \mathbf{E}^{-1}$).

In case of a linear environment, the left side of the equation (6) gives us known expresions of electric (w_E) and magnetic (w_M) energy densities

$$w_E = \frac{\vec{E}\vec{D}}{2}; \qquad w_M = \frac{\vec{H}\vec{B}}{2}. \qquad (8)$$

According to the field theory, the expressions (8) are postulated. There are some known unsuccessful attempts to prove (8) [4], but they are wrong because their authors start from the results which imply the postulates (8). Among all the possible expressions, the expressions (8) are the simplest ones, therefore, they are generally used.

4. Electromagnetic circuit

Here the charge q is interpreted as a generalized coordinate, and the current $i = dq/dt$ is interpreted as a generalized velocity. In addition, the static characteristics of circuit elements have the form

$$\Psi = L(i)i; \quad u = \Sigma(q)q, \qquad (9)$$

where Ψ is the linkage of an inductance coil; u denotes the voltage of a capacitor; $L(i)$ stands for the static inductance of the coil; $\Sigma(q)$ is the inverse static capacitance of the capacitor.

Substituting (9) into (2), we obtain the expressions of densities of potential energy and kinetic co-energy

$$w_p = \int_0^q \Sigma(q)qdq; \quad w_{kc} = \int_0^i L(i)idi. \qquad (10)$$

As we can see, in electromagnetic circuits, in contrast to electromagnetic fields, the magnetic field generates kinetic co-energy and the electric one generates potential energy. Their expressions perfectly coincide with those generally accepted in the theory of electromagnetic circuits [5].

The expression of kinetic energy of electromagnetic circuit does not go with the principle of symmetry and its equation (2) can not be obtained. This expression can be obtained by the classical method

$$w_k = \int_0^\Psi \alpha(\Psi)\Psi d\Psi, \qquad (11)$$

where $\alpha(\Psi)$ is the inverse static inductance of the coil ($\alpha = L^{-1}$).

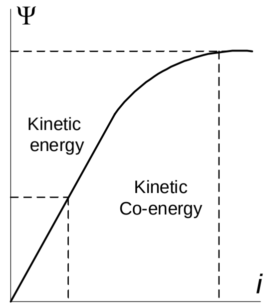

Fig. 1. To the interpretation of magnetic energy and magnetic co-energy in electromagnetic circuits

Fig. 1 depicts a geometrical structure of magnetic energy and co-energy in the axes of current – linkage. It is interesting that the energy and co-energy are connected by the well known integral formula

$$\int_0^i \Psi di = i\Psi - \int_0^\Psi id\Psi, \qquad (12)$$

which interprets the geometrical structure of Fig. 1.

5. Mechanics of concentrated masses

Here the distance x is interpreted as a generalized coordinate, and the velocity $v = dx/dt$ is interpreted as a generalized velocity. In addition, the static characte–ristics are of the form

$$p = m(v)v; \quad F = c(x)x, \qquad (13)$$

where p is the impulse; F represents the force; $m(v)$ is the static mass; $c(x)$ denotes the static stiffness of an elastic element.

Substituting (13) into (2), we obtain expressions of potentional energy and kinetic co-energy

$$w_p = \int_0^x c(x)xdx; \quad w_{kc} = \int_0^v m(v)vdv. \qquad (14)$$

As you can see, in mechanics, an elastic element generates potential energy, and moving masses generate kinetic co-energy.

The expression of kinetic energy of moving masses does not go with the principle of symmetry, and its expression (2) can not be obtained. It is known

$$w_k = \int_0^p s(p)pdp, \qquad (15)$$

where $s(p)$ is the inverse static mass ($s = m^{-1}$).

If we take under consideration, instead of separate masses, a system as a whole, the expressions (9)–(15) are given matrix content.

As it has been shown [1, 2], it is impossible to obtain an equation in electromagnetic field vectors on the basis of the energy approach. For this reason, we come only to vector potential equations, and afterwards, through (3) and (4), to vectors [1]:

$$\mathbf{E}^{\partial}\frac{\partial^2 \vec{A}}{\partial t^2}+\Gamma\frac{\partial \vec{A}}{\partial t}=-\left(\nabla\times\mathbf{N}\nabla\times\vec{A}\right), \qquad (16)$$

where $\vec{A}$ stands for the vector-potential of electromagnetic field; $\mathbf{E}^{\partial}$ is the matrix of differential electric permeabilities; Γ denots the matrix of static electric conductivities; $\mathbf{N}$ is the matrix of static reluctivities.

This approach becomes much more complicated in the case of nonlinear media. At first it was done in [1], where the author obtained the equation of vector potential in nonlinear nonhysteresis anisotropic medium, where the notion of electric co-energy was implemented instead of the notion of electric energy like it was done in the theory of electrical circuits [3], where the authors used the concept of magnetic co-energy.

So there is a reasonable question [2], whether the kinetic co-energy is just a calculated quantity or it implies a profound physical content. The first response suggests itself: such universal laws of physics as the law of conservation of energy and the principle of the least action, the basis of which is the principle of Hamilton-Ostrohradsky, can not lead to the physical laws of nature, based on energy transformations of non physical quantity. So we must admit that the kinetic co-energy is a physical characteristic too.

But then another idea suggests itself: we should refuse the kinetic energy in favor of co-energy as the kinetic energy does not work in variational principles, it is not fit to the equation of symmetry (2), shoudn’t we? Moreover, we come to the kinetic energy in accordance with equation (6), which encloses Maxwell’s postulate. But before the kinetic energy can be definitively sentenced, it has been proved [2] that it is possible to come to it through the experimental Coulomb's law. And only then by means of the kinetic energy, we have the possibility to come to the kinetic co-energy. It has been also shown that such measurable characteristic as field strength can be calculated either as gradient of energy or gradient of co-energy (both lead to the same results [2]):

$$\vec{F}_E=\nabla\int\limits_V\int\limits_0^{\vec{D}}\vec{E}(\vec{D})d\vec{D}dV=\nabla\int\limits_V\int\limits_0^{\vec{E}}\vec{D}(\vec{E})d\vec{E}dV, \qquad (16)$$

where $\mathbf{F}_E$ is the electric force.

It means that both energy and co-energy occupy their own positions. In the course of time we will see which one is dominant. In the meantime, the priority should be given to co-energy.

6. Conclusions

1. Since the kinetic co-energy in Hamilton-Ostro–gradskiy’s principle is the only one which satisfies the energy principle of the least action, and satisfies the proposed principle of symmetry, then it should be treated as a true energy characteristic.

2. The kinetic energy and co-energy in nonlinear media are not equal to each other, but their gradients are the same! So their utilization ensures identical power characteristics of the field. Futhermore, accrding to the practical results they are interchangeable.

СИМЕТРІЯ ЕНЕРҐІЇ

Василь Чабан

Виходячи з принципу симетрії, запропоновано універсальний вираз енергії фізичної системи. Якщо прийняти за арґумент узагальнену координату, то приходимо до потенціальної енергії, якщо узага–льнену швидкість, то – до кінетичної ко-енергії. Поняття кінетичної енергії тут, як і у варіаційних принципах, не діє. Приклади стосуються електро–маґнетного поля, електромаґнетних кіл і механіки зосереджених мас. Концепції ко-енерії надається фізичне значення (характеристика).

References

[1] A. Tchaban. *Mathematical modeling of oscilated processes in electromechanical systems,* Lviv, Ukraine: T. Soroka’s publishing house, 2007. (Ukrainian)

[2] V. Tchaban. *Mathematical modeling in electrical engineering,* Lviv, Ukraine: T. Soroka’s publishing house, 2010.

[3] D. White, H. Woodson. *Electromechanical trans–formation of energy.* Translated from English, Leningrad, Russia: Energiya, 1964.

[4] A. V. Netushil, K. M. Polivanov. *Theory of elect–romagnetic field,* Moscow, Russia: Gosenergoizdat, 1956.

Vasyl Tchaban - D.Sc., full professor at Lviv Polytechnic National University, Ukraine, as well as Rzeszow University, Poland. His Doctor of Engineering degree in Electrical Engineering he obtained at Moscow Energetic University, Russia, in 1987. His research interests are in the areas of mathematical modelling of electromechanical processes and theory of electromagnetic fields as well as surrealistic short-story writing. He is the autor of about 500 scentific publications and 500 surrealistic short stories including 33 books (8 monographs, 13 didactic, 2 humanistic and 10 artistic items).