

**MATHEMATICAL MODELING  
OF THE ENVIRONMENTAL RADIOACTIVE  
POLLUTION DURING OIL AND GAS WELLS DRILLING**

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**Abstract.** The aim of paper was development of an adequate mathematical model for predicting the level of environmental pollution by radionuclides emitted as a result of oil wells drilling. This problem was solved using Gaussian Plume Model which allowed to determine the concentration of radionuclides at a given point in space. Impurity flux density in Bq/(m<sup>2</sup> s) was been simulated.

**Key words:** Gaussian model, radioactive dispersion, air pollution, radionuclides distribution, wells drilling.

## **1. Introduction**

The process of development of oil and gas deposits, mainly drilling of wells, creates a number of environmental problems, among which radioactive contamination occupies a separate place. Significant source of Naturally Occurring Radioactive Materials (NORM) in this operation are rocks that actively contact with formation water, spent drilling mud, pass into drilling sludge and drilling wastewater, which determines the radioactivity of these wastes due to the presence of <sup>238</sup>U, <sup>232</sup>Th and <sup>40</sup>K [1, 2]. Besides of this aspect particles of drill cuttings containing super limit concentrations of the predominant radioactive elements isotopes are deposited on the drill pipes. In connection with this fact, drill pipes and cuttings in general, raised to the surface, are environmentally hazardous material and act as an object of radioactive contamination of the environment. Nowadays, modeling and forecasting the level of atmospheric air pollution with radioactive elements is a significant ecological problem, which are investigated by a lot of researchers, in order to prevent risks to life and health of the population.

Musayev [3] proves that the application of scientific and technical experiments for this task requires large financial costs. Also, the implementation of these experiments cannot be performed for technical reasons because of the complexity and laboriousness. Therefore, mathematical modeling is the most applicable scientific method to determine the level of atmospheric pollution with radionuclides. Quelo et al. [4] mentioned Gaussian Plume Model (GPM) is one of the most important and widespread atmospheric dispersion model in radioactive distribution assessments to predict the radiological consequences to the public

The spread of radionuclides and their decay products, i.e., their transfer between various components of the environment (in the atmosphere, water, soil) is due to different processes: chemical, mass transfer, external driving forces, transport within a given medium due to convection or diffusion, biological metabolism. According to Davidson Moreira et al. [5] the area of the radioactive release is determined by the following factors: amount of radiation released (source strength), wind direction and speed, weather conditions (particularly, atmospheric thermal stability states) and the physical characteristics of radioactive material released (half-life time and its deposition velocity).

Leelossy et al. [6] discussed Numerical Weather Prediction model (NWP) and computational fluid dynamics (CFD) models provide reliable wind data for all scales of atmospheric dispersion simulations, but their connection proved to be difficult. Recent developments of NWP's achieve more and more detailed resolution, CFD models with enhanced computational capacity, parallel computing and large eddy simulation (LES) simulation for anisotropic turbulence are becoming even better for planetary boundary layer simulations. Both the modification of an NWP model to

perform microscale simulations (the domain size is ~100 m) [7], and the extension of a CFD software for atmospheric studies hold promise, and these are current topics of research in both the meteorological and environmental engineering fields [8–10].

The results obtained by Najlaa D. Alharbi [11] suggest concentration values for any pollutant depend on, thermal stability class of the site under consideration and on the type of the radioactive pollutant and its lifetime. The normalized concentration distribution (NCD) of pollutants is calculated as a function of downwind horizontal distance up to 10 km from the release point. These estimates have been performed for Jeddah city (Kingdom of Saudi Arabia) using GPM.

Recent studies claim mathematical modeling of radioactive elements distribution in the atmosphere is topical task that requires solution. The aim of paper is development of an adequate mathematical model for predicting the level of environmental pollution by radionuclides emitted as a result of drilling oil wells.

## 2. Theoretical part

When radioactive elements are released into the atmosphere, radionuclides under the influence of turbulent diffusion are spread in the air and are transported sometimes in regional and even global scales.

The distribution of radioactive elements in the geosphere is associated with a sorption mechanism, which has three varieties:

- physical, caused by the attraction between the sorbent and the particles that bind radionuclides to the sorbent surface in the form of a series of layers;
- chemical, caused by chemical interactions between the sorbent and radionuclides; this process depends on the concentration of radioactive elements and can be slow and irreversible;
- electrostatic (for example, in ion-exchange processes); the process depends on the concentration of the radioactive substance.

Further propagation in the atmosphere occurs by scattering due to turbulent diffusion and wind transfer.

Wind transfer leads to the fact that with the continuous release of impurities into the atmosphere, an ejection jet is formed. With a weak wind or with its complete absence (calm), diffuse transport in air can predominate over the wind transfer, and then a calm cloud of impurity forms around the source of continuous emissions.

Turbulence is caused by the presence in the atmosphere of random turbulence, in which a certain mass of air is involved. They have their sizes, speed of movement and in a complex way interact with each

other and with the earth's surface, decaying and forming smaller whirlpools or merging into large ones. Usually in the atmosphere there are vortices of various sizes and shapes. The sources of their origin are frictional forces in the interaction of the wind flow with the earth and vertical air flows over the heated surface. The vertical dimensions of the vortices in the atmosphere are usually limited to several hundred meters.

The ejected radioactive elements propagate and dissipate in the atmosphere by constantly existing turbulent vortices of different scales in it, and the intensity of turbulent diffusion is determined by the parameters of the wind flow and the vertical temperature gradient, which depend on the properties of the underlying surface, heat balance and ground surface, the dynamic and temperature characteristics of the air masses, containing and transporting radioactive impurities. When the earth's surface warms up, some of the thermal energy passes from it to the adjacent layers of air, promoting vertical mixing of the air masses.

If the vertical temperature gradient is close to zero or even negative, the vertical movement of the air masses rapidly damps, creating stable, inversion conditions characterized by a weak turbulent exchange. Pasquill's stability classification method [12] is used to determine atmospheric stability classes (Table 1). This method defines six stability classes ranging from A (extremely unstable) to F (moderately stable) on the basis of wind speed at 10 m level, amount of incoming solar radiation during the day and cloud cover at night.

Steady stratification of the atmosphere is unfavorable for discharges from sources located at the surface of the earth, and in conditions of a weak rise in the ejected elements, as well as in cases where high sources are located near the source or the source is located in a deep narrow valley. In case of ejection to a higher altitude, a stable stratification even favors the removal of the maximum of the surface concentration from the source and the reduction of its absolute value (due to meteorological dilution). With unstable stratification, the maximum surface concentration of radioactive impurities is reached near the source, and its absolute value is much larger than in the first case, but it falls rapidly (by a distance from the source in the direction of the wind).

The intensity of atmospheric diffusion depends on the active spectrum of turbulent vortices and the size of the ejection cloud. The spectrum of vortices in the atmosphere is determined mainly by two factors: the vertical distribution of temperature in the atmosphere and the speed of the wind. The vertical component of the air temperature gradient is of great importance.

The gradient of temperature in the air primarily depends on the ratio of the temperature of the ambient

air and the temperature of the earth's surface. As a rule, during the day it changes greatly. Accordingly, the impurity scattering conditions also change. In practical calculations, the impurity diffusion conditions are distributed according to the categories of stability.

Determination of the stability category is carried out in accordance with the data of Table 2. Pasquill's stability classes: very unstable (A), unstable (B), slightly unstable (C), neutral (D), slightly stable (E) and stable (F) boundary layer. Note: neutral (D) class has to be used for overcast conditions and within one hour after sunrise / before sunset.

Table 1

**Atmospheric stability classes**

Qualitative characteristics of sustainability	Wind speed, m/s	Description	Vertical temperature gradient, °C/100m	Stability class		
				Pasquill	Pasquill-Turner	Willig
Very unstable state, highly developed convection	1	Very sunny summer quiet weather	<-1.9	A	1	VII
Unstable state, moderate convection	2	Sunny and warm	-1.9...-1.7	B	2	VII
Slightly unstable state, weak convection	5	Partly cloudy during the day	-1.6...-1.5	C	3	V
Neutral state, neutral stratification	5	A cloudy day or a cloudy night	-1.4...-0.5	D	4	IV
Slightly stable condition, weak stability	3	Partly cloudy over night	-0.4...+1.5	E	5	III
Stable state, moderate resistance	2	Clear night	1.6...4.0	F	6	II

Table 2

**Categories of the stability of the atmosphere according to Pasquill**

Daytime insolation				Night time	
Surface wind speed	Strong	Moderate	Slight	Thin overcast or Min. 4 octas low cloud	max. 3 octas low cloud
< 1 m/s	A	A	B	E	F
1-3 m/s	A	B	C	E	F
3-5 m/s	B	C	C	D	E
5-8 m/s	C	D	D	D	D
> 8 m/s	C	D	D	D	D

The scale of the distances from the emission source is divided into three classes:

- 1) within a few tens of kilometers – a local scale;
- 2) from tens to hundreds of kilometers – a regional scale;
- 3) more than a thousand kilometers – a global scale.

According to talk about local, regional and global models of impurity dispersion.

For practical calculations, local scattering models are used. The volumetric activity of radionuclides in air

for continuous and short-term emissions is calculated by the formula common to all:

$$C = Q' \cdot G, \tag{1}$$

where  $C$  is concentration of radionuclides in air, Bq/m<sup>3</sup>;  $Q'$  is emission intensity, Bq/s;  $G$  is the meteorological dilution factor, s/m<sup>3</sup>.

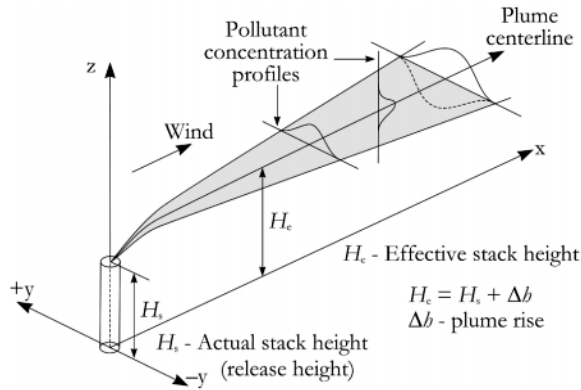
Nowadays, the most popular model for describing the local dispersion of radioactive impurities in the atmosphere is the Gaussian model:

$$\bar{C}(x, y, z) = \frac{Q}{2\pi\sigma_y\sigma_z\bar{u}} \exp\left(\frac{-y^2}{2\sigma_y^2}\right) \left( \exp\left(\frac{-(z-h)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+h)^2}{2\sigma_z^2}\right) \right), \tag{2}$$

where  $\bar{C}$  is the time-averaged concentration at a given position,  $Q$  is the source term,  $x$  is the downwind,  $y$  is

the crosswind,  $z$  is the vertical direction and  $\bar{u}$  is the time-averaged wind speed at the height of the release  $h$ .

The standard deviations  $\sigma_y$  and  $\sigma_z$  describe the crosswind and vertical mixing of the pollutant. Eq. (2) describes a mixing process that results in a Gaussian concentration distribution both in crosswind and in vertical directions, centered at the line downwind from the source (Fig. 1).



**Fig. 1.** Schematic figure of a Gaussian plume. The effective stack height  $H_e$  and the crosswind and vertical deviation of the profile are the key parameters of the model

The last term of (2) expresses a total reflection from the ground. Gaussian models have an extremely fast response time, because they only calculate a single formula (Eq. (2) or similar) for each receptor point instead of solving differential equations. This calculation is almost immediate even on common computers, however, meteorological data preprocessing and sophisticated turbulence parameterizations can increase the computational cost. Gaussian models are widely applied in decision support software where robust model setup and fast response time is a key priority [13]. Gaussian models provide poor results in situations with low wind speeds, where the three-dimensional diffusion is significant. Unfortunately, these situations have proven to be the most dangerous ones in real-life atmospheric dispersion problems as they are often connected to a stably stratified atmosphere or low-level inversions.

### 3. Results

For a short-term point release for concentrations at ground level and at the axis of the jet, the dilution factor is described by the expression:

$$G(x) = \frac{F(x)}{n\sigma_y\sigma_z u} \exp\left(-\frac{h^2}{2\sigma_z^2}\right), \quad (3)$$

where  $G(x)$  is dilution factor,  $s/m^3$ ;  $F(x)$  is the cloud exhaustion function;  $\sigma_y$ ,  $\sigma_z$  is standard deviations of the distribution in the direction of the corresponding

coordinate axes, m;  $u$  is wind speed, m/s;  $h$  is the height of the jet, m.

With continuous release, the average dilution factor at the ground level over a long period of time (usually one year) is given by:

$$G(x) = \frac{2\eta}{(2\pi)^{2/3} x} \sum_j \frac{\omega_j F_j}{\sigma_{z,j} \bar{u}_j} \exp\left(-\frac{h^2}{2\sigma_{z,j}^2}\right), \quad (4)$$

where the subscript  $j$  denotes the quantities characteristic of the  $j$ -th category of atmospheric stability;  $\omega_j$  is repeatability of the  $j$ -th category during the time of emission;  $\bar{u}_j$  is the average wind speed in the  $j$ -th category;  $\eta$  is elongation of the wind rose in the given direction.

In the general form,  $\sigma_y$ ;  $\sigma_z$  depend on the class of stability of the atmosphere, the distance from the source to the observation point, and the surface roughness. For a wide range of parameters, they are tabulated.

In the Gaussian model, a part of the initial data is directly measured or estimated based on the processing of measurement data ( $Q$ ,  $u$ ,  $H$ ,  $y$ ), and the variance parameters are calculated using tables (or graphs) and a number of empirical formulas that take into account the atmospheric stability class, in turn, is determined very arbitrarily and mainly on the basis of indirect data. Thus, the International Atomic Energy Agency (IAEA) recommends the following formulas for the calculation of  $\sigma_y$  and  $\sigma_z$ , respectively, for stable ( $s$ ) and unstable ( $u$ ) conditions:

$$\sigma_y^s = 0.15\sigma_\ominus x^{0.71}; \quad \sigma_y^u = 0.045\sigma_\ominus x^{0.86};$$

$$\sigma_z^s = 0.15\sigma_\ominus x^{0.71}; \quad \sigma_z^u = 0.045\sigma_\ominus x^{0.86}, \quad (5)$$

where  $\sigma$  is lateral and vertical wind fluctuations obtained by averaging the wind direction at the intervals  $x / u\beta$  for the time  $\tau$  (the duration of the ejection or the averaging period of the data);  $\beta$  is the ratio of Lagrangian and Euler length scales, usually taken equal to  $\beta = 4$ , and the dependence  $\sigma_0$  for the different Pausquill-Turner stability classes of the atmosphere is assumed to be as follows [14]:

$\sigma$	25°	20°	15°	10°	5°	2,5°
stability class	1	2	3	4	5	6

An approximate estimate of the average annual coefficient of meteorological dilution near and at a distance of up to 10 km behind the zone of maximum surface concentration can be made by the envelope method:

$$G = \frac{2}{(2\pi)^{2/3}} \frac{\eta F(x)}{\sqrt{eh\bar{u}x}}, \quad (6)$$

where  $e = 2,73$ ;  $\bar{u}$  is average wind speed, m/s.

Formula (5) gives the maximum estimates in the sense that for any law of variation of  $\sigma_z$ , any recurrence of weather conditions (sustainability categories) large and concentration values cannot be obtained.

Behind the zone of maximum surface concentration, where the height distribution of the impurity stabilizes, the impurity flux density at the earth's surface is described by the expression:

$$I = (v_g + \wedge H_{\max}) \cdot C, \tag{7}$$

where  $I$  is impurity flux density, Bq/(m<sup>2</sup> s);  $v_g$  is speed of dry sludge, m/s;  $H_{\max}$  is height of the lower boundary of the cloud of the source of atmospheric precipitation, m;  $C$  is concentration of radionuclides above the precipitation point, Bq/m;  $\wedge$  is the washout constant, s<sup>-1</sup>, is calculated according to the formula:

$$\wedge = k_r k_0 J, \tag{8}$$

where  $J$  is the intensity of precipitation, mm/h;  $k_r$  is the standard value of the absolute leaching ability of the rain, for all nuclides it is assumed equal to 10<sup>-5</sup> hours/(mm s);  $k_0$  is the relative leaching ability of precipitation of other types.

The value of  $k_0$  for different precipitation types is given in Table 3.

Table 3

Relative leaching ability for different types of precipitation

Precipitation type	$K$	Precipitation type	$K$
Rain	1.0	Snow	3.0
Rain with a thunderstorm	1.1	Mist	4.5
Snow with rain	2.4	Fog	5.0
Shower	2.8		

As already mentioned above, the Gaussian model considers the propagation of radioactive impurities under conditions of constant velocity and direction of the wind, as well as of one particular class of stability, which is characterized by a straight monotonically expanding trail, as shown in Fig. 2. This train can be considered as continuous, and in the form of a simple superposition of a large number of individual clubs. In practice, meteorological conditions are very variable, and the train changes its shape and direction, and when turning the wind, three cases are sometimes considered. With a narrow plume, its cross-section at the turning point can be taken as a new quasi-point source (Fig. 2, b). With a

large blur of the loop, consider the initial rectilinear segment of the plume as a spatial source (Fig. 2, b). Also, the train can be viewed as an ensemble of clubs, which, when the wind changes, move independently of each other (Fig. 2, d). For a real (fluctuating) plume, the clubs' centers (or, if disregarding them, the disks) are randomly distributed relative to their average position.

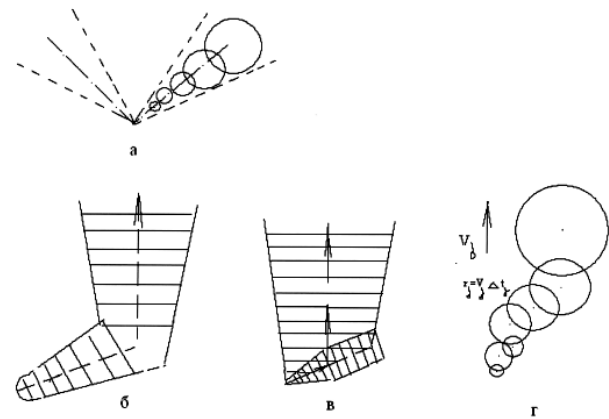


Fig. 2. Loops with unchanged (a) and variable (b) – (d) weather conditions

The coefficient of depletion of the plume due to the radioactive decay of the  $i$ -th radionuclide has the form

$$f(\lambda_i) = \exp\left(-\frac{\lambda_i x}{u}\right), \tag{9}$$

where  $\lambda_i$  is the half-life of the  $i$ -th radionuclide, hours;  $x$  is the distance from the source of radioactive contamination to the point of interest along the wind axis, m;  $u$  is wind speed, m/s.

The depletion of the plume due to precipitation is described by the expression

$$f(\wedge) = \exp\left(-\wedge \frac{x}{u}\right). \tag{10}$$

#### 4. Conclusion

Concentration values for any pollutant depend on, thermal stability class of the site under consideration and on the type of the radioactive pollutant and its life-time. Consideration of the parameters of the Gaussian model of transport and distribution of impurities allows to conclude that the accuracy of simulation results decreases with decreasing averaging time, complication of meteorological conditions and relief, as well as with increasing distance from the emission source. A model is proposed that allows to increase the accuracy of prediction of carcinogenic radioactive aerosol admixtures in the atmosphere.

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