ON THE COMPUTATION OF GEODETIC LATITUDE

O.A.Abrikosov

(National University "Lviv Polytechnic")

***Summary**. The iterative formula based on Newton method is proposed for the computation of geodetic latitude.*

In spite of existence of a number of various methods for computation of geodetic latitude (see, for instance, Morozov, 1979), this task remains as interesting for geodesists. For illustration, it is enough remind that the method developed in recent paper (Fukushima, 1999) became the basis for computation of geodetic coordinates in accordance with IERS Conventions (2000). This method provides stable computation of geodetic latitude in very wide space. As it was mentioned in (Fukushima, 1999), the method was tested for geocentric distances from small neighborhood of the Earth's center to those near Moon's orbit. Nevertheless, the space near the Earth's surface remains most important for practical purposes. Therefore, our paper deals with deriving of simple stable iterative formula for computation of geodetic latitude near the Earth's surface.

We will start from the well-known relations between 3D rectangular coordinates X, Y, Z and geodetic latitude B, longitude L, and ellipsoidal height H:

$$\left.\begin{aligned} X &= (N+H)\cos B\cos L \\ Y &= (N+H)\cos B\sin L \\ Z &= (N+H)\sin B - e^2 N\sin B \end{aligned}\right\} \quad (1)$$

where N is the radius of curvature of prime vertical

$$N = \frac{a}{\sqrt{1-e^2\sin^2 B}} \quad (2)$$

a is semimajor axis of the Earth's ellipsoid of revolution, e is first eccentricity of the ellipsoid

The basic equation for computation of geodetic latitude follows immediately from (1):

$$\sqrt{X^2+Y^2}\sin B - Z\cos B = e^2 N\sin B\cos B \quad (3)$$

With the notations

$$\eta = e\sin B,\quad s = \frac{Z}{ae},\quad p = \frac{\sqrt{X^2+Y^2}}{ae} \quad (4)$$

the equation (3) become

$$f(\eta) = \left(s\sqrt{1-\eta^2}+\eta\right)\sqrt{e^2-\eta^2} - p\eta\sqrt{1-\eta^2} = 0 \quad (5)$$

This equation may be solved for the unknown η iteratively by means of Newton method:

$$\eta_{n+1} = \eta_n - \frac{f(\eta_n)}{f'(\eta_n)} \quad (6)$$

where

$$f'(\eta)=\left(1-s\frac{\eta}{\sqrt{1-\eta^2}}\right)\sqrt{e^2-\eta^2} \qquad (7)$$
$$-\left(s\sqrt{1-\eta^2}+\eta\right)\frac{\eta}{\sqrt{e^2-\eta^2}}-p\frac{1-2\eta^2}{\sqrt{1-\eta^2}}$$

By substituting (5) and (7) into (6) we get the recursive formula

$$\eta_{n+1}=\eta_n+u_n v_n\frac{A_n}{C_n} \qquad (8)$$

where

$$u_n=\sqrt{1-\eta_n^2} \qquad (9)$$

$$v_n=\sqrt{e^2-\eta_n^2} \qquad (10)$$

$$A_n=(pu_n-v_n)\eta_n-su_nv_n \qquad (11)$$

$$C_n=2s\eta_n^3+(pv_n-u_n)\eta_n^2 \qquad (12)$$
$$-s(1+e^2)\eta_n-u_nv_n(pu_n-v_n)$$

The iterative process starts from the initial value

$$\eta_0=e\frac{Z}{\sqrt{X^2+Y^2+Z^2}} \qquad (13)$$

and should stop if a necessary accuracy ε is reached

$$|\eta_{n+1}-\eta_n|<\varepsilon \qquad (14)$$

The derived formulas were tested for the space bounded by geocentric latitude −90° ≤ φ ≤ +90° and by ellipsoidal height −1000 km ≤ H ≤ +1000 km. In all computations the convergence of the iterative process to $\varepsilon=10^{-15}$ was riched after no more 4 iterations. Since the main benefit of this method consists of no any (explicit or implicit) usage of tangents (which are singular at poles), we can propose this method for practical applications.

References

IERS Conventions (2000).

Fukushima T. (1999) Fast transform from geocentric to geodetic coordinates. *Journal of Geodesy*, 73, p.603-610.

Morozov V.P. *The course of spheroidal geodesy*. Nedra, Moscow, 1979 (in Russian)

О.А.Абрикосов

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Резюме

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