ACURACY ASPECTS OF A THREE DIMENSIONAL PARTICLE TRACKING VELOCIMETRY BASED ON PHOTOGRAMMETRY

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1. Introduction.

Air movements in ventilated rooms are three-dimensional and there are often large regions of recirculating flow with velocities of the order 10 - 20 cm/s. Mapping of velocities in a whole room with a point measurement technique is a formidable task. Typically the size of the area one wants to cover is about 10 m^2. Therefore a measurement technique that instantaneously captures information from a whole field is an attractive alternative or complement to standard point measurement techniques (laser Doppler anemometry or hot-wire technique). Particle velocimetry (PV) offers the possibility to record velocities simultaneously over a large area. PV is "quantitative flow visualisation". The access to powerful image processing systems for computers has turned qualitative flow visualisation techniques into quantitative techniques. The fundamental principle of PV is the measurement of the displacements ($\Delta X,\Delta Y,\Delta Z$) of flow markers which travel with the air. The time interval Δt is fixed and the instantaneous velocity vector is obtained as: $U(X,t) = (\frac{\Delta X}{\Delta t}, \frac{\Delta Y}{\Delta t}, \frac{\Delta Z}{\Delta t})$

The displacement must be small enough for the above method to be a good approximation of the velocity. In order to be able to see the flow markers the room is "sliced" into subvolumes by a light sheet. The techniques used can be classified as being based on direct tracking of individual flow markers or techniques based on correlation/pattern tracking. Available commercial systems only measure the displacement ($\Delta X,\Delta Y$) in a plane [1]. Techniques for measurement of two-dimensional velocities are usually based on the use of correlation/pattern tracking. These techniques refer to measurement of displacements of whole groups of particles and requires a large amount of flow markers in the measuring volume. Correlation/pattern techniques produce measurements on a regular data grid and are suited for automatic processing. However, they are of course not well suited when there is a large out-of-plane component ΔZ of the displacement. The use of stereo-photogrammetry makes it possible to record the displacements in all directions. The three-dimensional co-ordinates of an object in the room are found from the corresponding two-dimensional image co-ordinates obtained by two (or more) cameras with different viewing directions. The tracks of *individual* flow-markers are registered. Therefore the velocities are recorded where the flow markers happen to be. To obtain the velocity field for the whole region, interpolation must be carried out. This method, where individual streaks of markers are identified is sometimes called *PSV* (particle streak velocimetry).

The time interval Δt can usually be recorded accurately and therefore the accuracy of the method depends on how accurately one can record the displacements [3], [4]. Most difficult it is to record the out of plane component ΔZ. The purpose of this report is to report on the estimation of the accuracy of stereo-photogrammetry at conditions typical for flows in rooms.

Two approaches to determine the accuracy have been used:

- analytically, by using the equations relating the co-ordinates on the image plane and the co-ordinates of the object in the room,
- simulation, by introducing random errors to the co-ordinates on the image plane.

2. Factors that affect choice of method.

There are practical restrictions (e.g. optical access) which hinder to place the cameras arbitrarily :

- if the difference in viewing angle is large it may be difficult to identify the same tracer on different images,
- synchronisation is required which can be more difficult when using many cameras,
- when having two pictures, measurements can be done stereoscopically by either using stereocomparators or analytical plotters.

The information available increases with the number of cameras which implies that:

- the risk of ambiguity reduces,
- coordinates can be determined with higher precision.

3. Description of a PV system based on stereo-photogrammetry.

Our system is based on the use of a stereo-model constructed from two overlapping pictures. The system consists of the following main components:

- two metric cameras with reference points (reseau marks) engraved into the camera body,
- lighting system consisting of 18 halogen lamps with a total power of 18x300 W=5,4 kW giving a light sheet of thickness 0.08 m,
- tracer particles (expanded microspheres or metaldehyde flakes),
- a reference system in the room consisting of 8 reference points whose coordinates have been determined with high precision.

4. Photogrammetry as a tool for determination of the air flow pattern in ventilated rooms.

A creation of a stereo model with two overlapping images belong to the basic principles of stereo photogrammetry. This process can be performed in many different ways. In this work we will present one solution which is very useful for determination of the air flow pattern in ventilated rooms. The method we use in this work is very suitable for reconstruction of narrow objects, in this case 3 m long, 3 m high but only 30 cm deep. The other advantages of this method are:

- the using all measured points for reconstruction of 3D object,
- easy and fast detecting of gross errors in every step of the process,
- full information about obtained accuracy after each stage.

All calculations are performed by the use of the least square method. The whole reconstruction process is shown below.

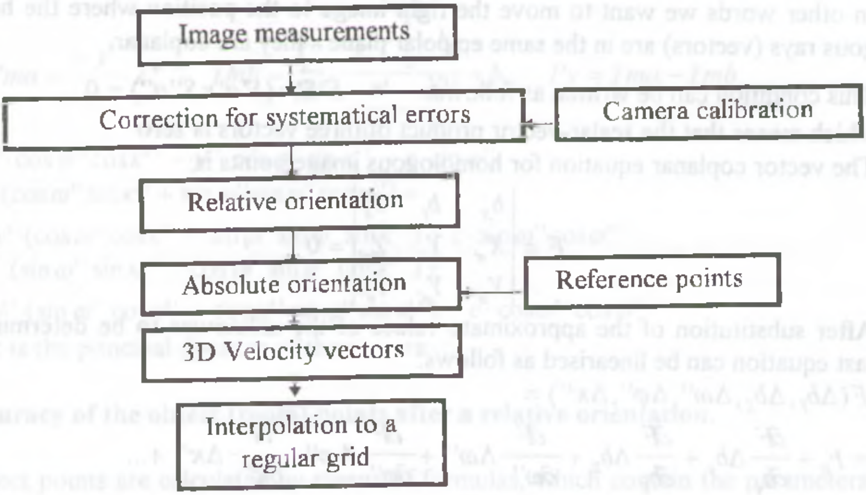

Figure 1 Reconstruction process

Relative orientation is the process by which a pair of overlapping photographs is related to one another in some arbitrary space to correspond with their co-orientation at the time of photography. In order to perform this process it is necessary to find five orientation elements, usually three rotations and two translation elements, between two individual bundles representing the photographs. The result is the formation of a 3D model in arbitrary space and at in arbitrary scale.

From the practical point of view it is very useful to assume that the elements of exterior orientation of the left-hand photo are fixed. The model coordinate system is shown in Fig. 2. Points S' and S'' being the projection centres, a' and a'' being the homologous image points and $S'a'$, $S''a''$ being the homologous rays.

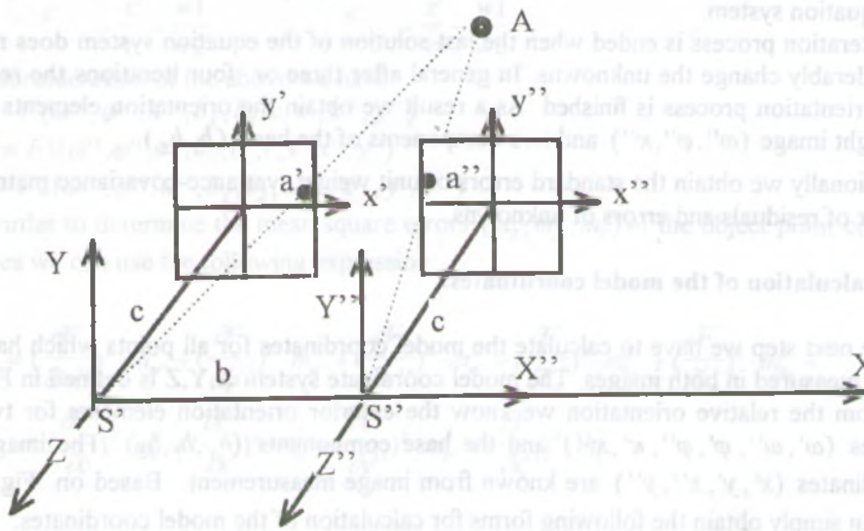

Figure 2. The relative orientation - start of the first iteration

In the process of relative orientation the left image will be fixed and the right image can make small rotations around X,Y,Z axes and translations along Y and Z axes [5].

In other words we want to move the right image to the position where the homologous rays (vectors) are in the same epipolar plane they are coplanar.

This condition can be written as follows: $\overrightarrow{S'S''} \bullet (\overrightarrow{S'a'} \times \overrightarrow{S''a''}) = 0$

which means that the scalar-vector product of three vectors is zero.

The vector coplanar equation for homologous image points is:

$$F = \begin{vmatrix} b_X & b_Y & b_Z \\ X_{a'} & Y_{a'} & Z_{a'} \\ X_{a''} & Y_{a''} & Y_{a''} \end{vmatrix} = 0$$

After substitution of the approximate values of the unknowns to be determined the last equation can be linearised as follows:

$$F(\Delta b_Y, \Delta b_Z, \Delta\omega'', \Delta\varphi'', \Delta\kappa'') =$$
$$= F_0 + \frac{\partial F}{\partial b_y}\Delta b_y + \frac{\partial F}{\partial b_Z}\Delta b_Z + \frac{\partial F}{\partial \omega''}\Delta\omega'' + \frac{\partial F}{\partial \varphi''}\Delta\varphi'' + \frac{\partial F}{\partial \kappa''}\Delta\kappa'' + \ldots$$

After calculation of coefficients and some modifications the following equation can be obtained:

$$(x'-x'')\Delta b_y + (x' \cdot y'' - x'' \cdot y')\Delta b_z - (1 + y' \cdot y'')\Delta\omega + (x'' \cdot y')\Delta\varphi - x''\Delta\kappa = y' - y''$$

assuming that $Z_{a'} = Z_{a''} = -1$ which is only changing the scale.

For each homologous point we can write one equation, which mean that we need at least five points in order to find five unknowns. In general the number of homologous points has to be more than five, so in this case the least square adjustment method is recommended to use in order to find unknowns.

The start values of the unknowns are determined only approximately. For this reason the solution of the system of equations (observation equations) $AX = L + V$ has to be done by the use of a few iterations. During this iteration process we accumulate values of each unknown, calculate a new system of equation $A'X = L + V$ and solve the equation system.

The iteration process is ended when the last solution of the equation system does not considerably change the unknowns. In general after three or four iterations the relative orientation process is finished. As a result we obtain the orientation elements of the right image $(\omega'', \varphi'', \kappa'')$ and two components of the base (b_Y, b_Z).

Additionally we obtain the standard errors of unit weight, variance-covariance matrix, vector of residuals and errors of unknowns.

4.1 Calculation of the model coordinates.

In the next step we have to calculate the model coordinates for all points which have been measured in both images. The model coordinate system X,Y,Z is defined in Fig. 2. From the relative orientation we know the exterior orientation elements for two images $(\omega', \omega'', \varphi', \varphi'', \kappa', \kappa'')$ and the base components (b_X, b_Y, b_Z). The images coordinates (x', y', x'', y'') are known from image measurement. Based on Fig. 2 we can simply obtain the following forms for calculation of the model coordinates:

$$Z = \frac{b_X - \frac{X''}{Z''} \cdot b_Z}{\frac{-x'}{c} - \frac{X''}{Z''}}, \qquad X = \frac{-x'}{c} \cdot Z, \qquad Y = \frac{Yma + Ymb}{2}$$

where $Yma = \frac{-y'}{c} \cdot Z$, $\quad Ymb = \frac{Y''}{Z''} \cdot \frac{b_x - \frac{x'}{c} \cdot b_z}{\frac{-x'}{c} - \frac{X''}{Z''}} + b_y$, $\quad Py = Yma - Ymb$

$$X'' = x'' \cdot \cos\varphi'' \cos\kappa'' - y'' \cdot \cos\varphi'' \sin\kappa'' - c \cdot \sin\varphi''$$
$$Y'' = x''(\cos\omega'' \sin\kappa'' + \sin\omega'' \sin\varphi'' \cos\kappa'') +$$
$$+ y''(\cos\omega'' \cos\kappa'' - \sin\omega'' \sin\varphi'' \sin\kappa'') + c \cdot \sin\omega'' \cos\varphi''$$
$$Z'' = x''(\sin\omega'' \sin\kappa'' - \cos\omega'' \sin\varphi'' \cos\kappa'') +$$
$$+ y''(\sin\omega'' \cos\kappa'' + \cos\omega'' \sin\varphi'' \sin\kappa'') - c \cdot \cos\omega'' \cos\varphi''$$

Where c is the principal distance of the camera.

4.2 Accuracy of the object (room) points after a relative orientation.

The object points are calculated by means of formulas, which contain the parameters we have obtained during the process of relative orientation. Let us write down once again the formulas in a different way.

$$Z = \frac{b_x - \frac{w1}{w2} b_z}{-\frac{x'}{c} - \frac{w1}{w2}}$$

where

$$w1 = x'' \cdot \cos\varphi'' \cos\kappa'' - y'' \cdot \cos\varphi'' \sin\kappa'' - c \cdot \sin\varphi''$$
$$w2 = x''(\sin\omega'' \sin\kappa'' - \cos\omega'' \sin\varphi'' \cos\kappa'') + y''(\sin\omega'' \cos\kappa'' + \cos\omega'' \sin\varphi'' \sin\kappa'')$$
$$- c \cdot \cos\omega'' \cos\varphi''$$

$$X = \frac{-x'}{c} \cdot \frac{b_x - \frac{w1}{w2} b_z}{-\frac{x'}{c} - \frac{w1}{w2}}, \qquad Y = \frac{-y'}{c} \cdot \frac{b_x - \frac{w1}{w2} b_z}{-\frac{x'}{c} - \frac{w1}{w2}}$$

In consideration of the above we have:

$$Z = F(\omega'', \varphi'', \kappa'', b_x, b_z, c, x', x'', y'')$$
$$X = F1(\omega'', \varphi'', \kappa'', b_x, b_z, c, x', x'', y'')$$
$$Y = F2(\omega'', \varphi'', \kappa'', b_x, b_z, c, x', x'', y', y'')$$

In order to determine the mean square errors (m_Z, m_X, m_Y) of the object point coordinates we can use the following expression:

$$m_Z^2 = (\frac{\partial F}{\partial \omega''})^2 \cdot m_{\omega''}^2 + (\frac{\partial F}{\partial \varphi''})^2 \cdot m_{\varphi''}^2 + (\frac{\partial F}{\partial \kappa''})^2 \cdot m_{\kappa''}^2 + (\frac{\partial F}{\partial b_X})^2 \cdot m_{b_X}^2 + (\frac{\partial F}{\partial b_Z})^2 \cdot m_{b_Z}^2 +$$
$$+ (\frac{\partial F}{\partial c})^2 \cdot m_c^2 + (\frac{\partial F}{\partial x'})^2 \cdot m_{x'}^2 + (\frac{\partial F}{\partial y''})^2 \cdot m_{y''}^2 + (\frac{\partial F}{\partial x''})^2 \cdot m_{x''}^2$$

$$m_X^2 = (\frac{\partial F1}{\partial \omega''})^2 \cdot m_{\omega''}^2 + (\frac{\partial F1}{\partial \varphi''})^2 \cdot m_{\varphi''}^2 + (\frac{\partial F1}{\partial \kappa''})^2 \cdot m_{\kappa''}^2 + (\frac{\partial F1}{\partial b_X})^2 \cdot m_{b_X}^2 + (\frac{\partial F1}{\partial b_Z})^2 \cdot m_{b_Z}^2$$
$$+ (\frac{\partial F1}{\partial c})^2 \cdot m_c^2 + (\frac{\partial F1}{\partial x'})^2 \cdot m_{x'}^2 + (\frac{\partial F1}{\partial y''})^2 \cdot m_{y''}^2 + (\frac{\partial F1}{\partial x''})^2 \cdot m_{x''}^2$$

$$m_y^2 = \left(\frac{\partial F2}{\partial \omega''}\right)^2 \cdot m_{\omega''}^2 + \left(\frac{\partial F2}{\partial \varphi''}\right)^2 \cdot m_{\varphi''}^2 + \left(\frac{\partial F2}{\partial \kappa''}\right)^2 \cdot m_{\kappa''}^2 + \left(\frac{\partial F2}{\partial b_X}\right)^2 \cdot m_{b_X}^2 + \left(\frac{\partial F2}{\partial b_Z}\right)^2 \cdot m_{b_Z}^2$$

$$+ \left(\frac{\partial F2}{\partial c}\right)^2 \cdot m_c^2 + \left(\frac{\partial F2}{\partial x'}\right)^2 \cdot m_{x'}^2 + \left(\frac{\partial F2}{\partial y''}\right)^2 \cdot m_{y''}^2 + \left(\frac{\partial F2}{\partial x''}\right)^2 \cdot m_{x''}^2 + \left(\frac{\partial F2}{\partial y'}\right)^2 \cdot m_{y'}^2 \quad (21c$$

The mean square error of a certain function can be determined in the subsequent steps [2]:

- the function existing between the quantity to be determined and the determining quantities is successively partially differentiated with respect to every determining quantity,
- the partial derivatives will be squared a end each of them multiplied by the square of the mean square error of the relevant determining quantity,
- the products are then summed up and the square root of the sum will be found.

Let's find the partial derivatives for the functions, which we are interested in:

$$\frac{\partial F}{\partial \omega''} = \frac{\frac{w1 \cdot b_z \cdot w3}{w2^2} \cdot \left(-\frac{x'}{c} - \frac{w1}{w2}\right) - \frac{w1 \cdot w3}{w2^2} \cdot \left(b_x - \frac{w1}{w2} \cdot b_z\right)}{\left(-\frac{x'}{c} - \frac{w1}{w2}\right)^2}$$

where

$w1 = x'' \cdot \cos\varphi'' \cos\kappa'' - y'' \cdot \cos\varphi'' \sin\kappa'' - c \cdot \sin\varphi''$

$w2 = x'' \cdot (\sin\omega'' \sin\kappa'' - \cos\omega'' \sin\varphi'' \cos\kappa'') + y'' \cdot (\sin\omega'' \cos\kappa'' + \cos\omega'' \sin\varphi'' \sin\kappa'')$
$\quad - c \cdot \cos\omega'' \cos\varphi''$

$w3 = x'' \cdot (\cos\omega'' \sin\kappa'' + \sin\omega'' \sin\varphi'' \cos\kappa'') + y'' \cdot (\cos\omega'' \cos\kappa'' - \sin\omega'' \sin\varphi'' \sin\kappa'')$
$\quad + c \cdot \sin\omega'' \cos\varphi''$

$$\frac{\partial F}{\partial \varphi'} = \frac{-\frac{w4 \cdot b_z \cdot w2 - w1 \cdot b_z \cdot w5}{w2^2} \cdot \left(-\frac{x'}{c} - \frac{w1}{w2}\right) + \frac{w4 \cdot w2 - w5 \cdot w1}{w2^2} \cdot \left(b_x - \frac{w1}{w2} \cdot b_z\right)}{\left(-\frac{x'}{c} - \frac{w1}{w2}\right)^2}$$

$w4 = -x'' \cdot \sin\varphi'' \cos\kappa'' + y'' \cdot \sin\varphi'' \sin\kappa'' - c \cdot \cos\varphi''$

$w5 = -x'' \cos\omega'' \cos\varphi'' \cos\kappa'' + y'' \cos\omega'' \cos\varphi'' \sin\kappa'' + c \cdot \cos\omega'' \sin\varphi''$

$$\frac{\partial F}{\partial \kappa''} = \frac{-\frac{w6 \cdot b_z \cdot w2 - w1 \cdot b_z \cdot w7}{w2^2} \cdot \left(-\frac{x'}{c} - \frac{w1}{w2}\right) + \frac{w6 \cdot w2 - w7 \cdot w1}{w2^2} \cdot \left(b_x - \frac{w1}{w2} \cdot b_z\right)}{\left(-\frac{x'}{c} - \frac{w1}{w2}\right)^2}$$

where

$w6 = -x'' \cdot \cos\varphi'' \sin\kappa'' - y'' \cdot \cos\varphi'' \cos\kappa''$

$w7 = x'' \cdot (\sin\omega'' \cos\kappa'' + \cos\omega'' \sin\varphi'' \sin\kappa'') + y'' \cdot (-\sin\omega'' \sin\kappa'' + \cos\omega'' \sin\varphi'' \cos\kappa'')$

$$\frac{\partial F}{\partial b_x} = \frac{1}{\left(-\frac{x'}{c} - \frac{w1}{w2}\right)}, \qquad \frac{\partial F}{\partial b_z} = \frac{-\frac{w1}{w2}}{\left(-\frac{x'}{c} - \frac{w1}{w2}\right)}$$

$$\frac{\partial F}{\partial x'} = \frac{-\frac{w8 \cdot b_z \cdot w2 - w1 \cdot b_z \cdot w9}{w2^2} \cdot \left(-\frac{x'}{c} - \frac{w1}{w2}\right) + \frac{w8 \cdot w2 - w9 \cdot w1}{w2^2} \cdot \left(b_x - \frac{w1}{w2} \cdot b_z\right)}{\left(-\frac{x'}{c} - \frac{w1}{w2}\right)^2}$$

$$\frac{\partial F}{\partial y''}=\frac{-\dfrac{w10\cdot b_z\cdot w2-w1\cdot b_z\cdot w11}{w2^2}\cdot\left(-\dfrac{x'}{c}-\dfrac{w1}{w2}\right)+\dfrac{w10\cdot w2-w11\cdot w1}{w2^2}\cdot\left(b_x-\dfrac{w1}{w2}\cdot b_z\right)}{\left(-\dfrac{x'}{c}-\dfrac{w1}{w2}\right)^2}$$

$$\frac{\partial F}{\partial c}=\frac{-\dfrac{w12\cdot b_z\cdot w2-w1\cdot b_z\cdot w13}{w2^2}\cdot\left(-\dfrac{x'}{c}-\dfrac{w1}{w2}\right)+\left(\dfrac{x'}{c^2}\;\dfrac{w12\cdot w2-w13\cdot w1}{w2^2}\right)\cdot\left(b_x-\dfrac{w1}{w2}\cdot b_z\right)}{\left(-\dfrac{x'}{c}-\dfrac{w1}{w2}\right)^2}$$

$$\frac{\partial F}{\partial x'}=\frac{1}{c}\cdot\frac{b_x-\dfrac{w1}{w2}b_z}{\left(-\dfrac{x'}{c}-\dfrac{w1}{w2}\right)^2}$$

where

$w8=\cos\varphi''\cos\kappa''$ $\quad w9=\sin\omega''\sin\kappa''-\cos\omega''\sin\varphi''\cos\kappa''$

$w10=-\cos\varphi''\sin\kappa''$ $\quad w11=\sin\omega''\cos\kappa''+\cos\omega''\sin\varphi''\sin\kappa''$

$w12=-\sin\varphi''$ $\quad w13=-\cos\omega''\cos\varphi''$

$$\frac{\partial F1}{\partial x'}=\frac{-1}{c}\cdot\frac{b_x-\dfrac{w1}{w2}b_z}{-\dfrac{x'}{c}-\dfrac{w1}{w2}}+\left(\frac{-x'}{c}\right)\cdot\frac{\partial F}{\partial x'}\qquad \frac{\partial F1}{\partial c}=\frac{x'}{c\cdot c}\cdot\frac{b_x-\dfrac{w1}{w2}b_z}{-\dfrac{x'}{c}-\dfrac{w1}{w2}}+\left(\frac{-x'}{c}\right)\cdot\frac{\partial F}{\partial c}$$

$$\frac{\partial F1}{\partial\omega''}=\frac{-x'}{c}\cdot\frac{\partial F}{\partial\omega''}\quad \frac{\partial F1}{\partial\varphi''}=\frac{-x'}{c}\cdot\frac{\partial F}{\partial\varphi''}\quad \frac{\partial F1}{\partial\kappa''}=\frac{-x'}{c}\cdot\frac{\partial F}{\partial\kappa''}\quad \frac{\partial F1}{\partial b_x}=\frac{-x'}{c}\cdot\frac{\partial F}{\partial b_x}$$

$$\frac{\partial F1}{\partial b_z}=\frac{-x'}{c}\cdot\frac{\partial F}{\partial b_z}\quad \frac{\partial F}{\partial y''}=\frac{-x'}{c}\cdot\frac{\partial F}{\partial y''}\quad \frac{\partial F}{\partial x''}=\frac{-x'}{c}\cdot\frac{\partial F}{\partial x''}$$

$$\frac{\partial F2}{\partial\omega''}=\frac{-y'}{c}\cdot\frac{\partial F}{\partial\omega''};\quad \frac{\partial F2}{\partial\varphi''}=\frac{-y'}{c}\cdot\frac{\partial F}{\partial\varphi''};\quad \frac{\partial F2}{\partial\kappa''}=\frac{-y'}{c}\cdot\frac{\partial F}{\partial\kappa''};\quad \frac{\partial F2}{\partial b_x}=\frac{-y'}{c}\cdot\frac{\partial F}{\partial b_x}$$

$$\frac{\partial F2}{\partial b_z}=\frac{-y'}{c}\cdot\frac{\partial F}{\partial b_z};\quad \frac{\partial F2}{\partial y''}=\frac{-y'}{c}\cdot\frac{\partial F}{\partial y''};\quad \frac{\partial F2}{\partial x''}=\frac{-y'}{c}\cdot\frac{\partial F}{\partial x''};\quad \frac{\partial F2}{\partial x'}=\left(\frac{-y'}{c}\right)\cdot\frac{\partial F}{\partial x'}$$

$$\frac{\partial F2}{\partial c}=\frac{y'}{c\cdot c}\cdot\frac{b_x-\dfrac{w1}{w2}b_z}{-\dfrac{x'}{c}-\dfrac{w1}{w2}}+\left(-\frac{y'}{c}\right)\cdot\frac{\partial F}{\partial c}\qquad \frac{\partial F2}{\partial y'}=\left(\frac{-1}{c}\right)\cdot\frac{b_x-\dfrac{w1}{w2}b_z}{-\dfrac{x'}{c}-\dfrac{w1}{w2}}$$

4.3 Calculating of the object point errors for the known object geometry.

Now we can perform the accuracy analysis of the object, we have used for determination of the air flow pattern at our laboratory. The analysis will be done on providing that:

- the geometry of the object is known (3m x 3m x 0.3m),
- distance between the cameras is 0.30m,

- distance between the base and the object is 3m,
- the number of measured points in two images is 40,
- the points are regularly distributed n the images.
- the distance between the images is approximately equal to the base in the image.

In order to calculate the value of the model point errors after the relative orientation we have to estimate the mean square error of the quantities: $m_{\omega'}, m_{\varphi''}, m_{\kappa''}, m_{b_y}, m_{b_z}, m_{x'}, m_c, m_{y''}, m_{x''}$. The five first errors we have got from the adjustment we have performed in the process of the relative orientation by the use of the least square method.

$m_{\omega'} = \pm s_0\sqrt{N^2_{\omega'}} = \pm 0.00004[rad]$ $\quad m_{\varphi''} = \pm s_0 \cdot \sqrt{N^2_{\varphi''}} = \pm 0.00003[rad]$

$m_{\kappa''} = \pm s_0 \cdot \sqrt{N^2_{\kappa''}} = \pm 0.00001[rad]$ $\quad m_{b_y} = \pm s_0 \cdot \sqrt{N^2_{b_y}} = \pm 0.0004[mm]$

$m_{b_z} = \pm s_0 \cdot \sqrt{N^2_{b_z}} = \pm 0.006[mm]$

s_0 - is the mean square error of the weight unit from the adjustment

For others errors we assume the following values:

$m_{x'} = m_{y'} = m_{y''} = m_{x''} = \pm 0.002$ mm $\quad m_c = \pm 0.01$ mm

Which is done providing that:

- the measurement of the point coordinates is done on an analytical plotter,
- the principal distance of the camera is obtained from the camera calibration.

Now we can calculate the errors of the object points, which are distributed as shown in Fig. 3.

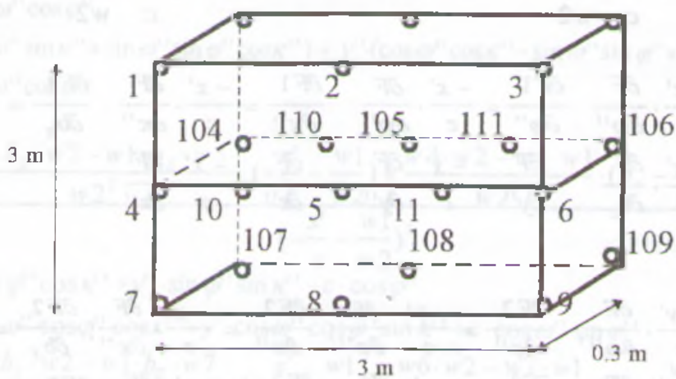

Figure 3. Distribution of the object points

The errors have been determined by both solving the above equations and carrying out simulations. In the latter case random number x_1 and x_2 were sampled from a uniform probability distribution in the interval $[0,1]$. They were transformed into random numbers, e, with a normal distribution by the use of the transform.

$\varepsilon = \sqrt{-2\ln X_2}\, Cos(2\pi X_1)$

If a point a in the image plane has true co-ordinates (x_0', y_0') its co-ordinates after introducing a random error, ε, becomes $(x_0' + \varepsilon_i^x, y_0' + \varepsilon_i^y)$.

Ten thousand iterations were carried out. The results are given in Table 1.

Table 1. The room point errors in mm in the room coordinate system

Point number	m_x [mm] Analytically	Simulation	m_y [mm] Analytically	Simulation	m_z [mm] Analytically	Simulation
1	± 1.05	± 1.05	± 1.09	± 1.07	± 2.26	± 2.23
2	± 0.10	± 0.14	± 0.99	± 0.73	± 2.06	± 1.34
3	± 1.09	± 0.73	± 1.06	± 0.76	± 2.19	± 1.37
4	± 1.04	± 0.85	± 0.12	± 0.12	± 2.23	± 1.84
5	± 0.10	± 0.14	± 0.12	± 0.13	± 2.05	± 1.43
6	± 1.08	± 0.83	± 0.12	± 0.13	± 2.17	± 1.55
7	± 1.05	± 0.86	± 1.09	± 0.88	± 2.26	± 1.89
8	± 0.10	± 0.14	± 0.99	± 0.72	± 2.06	± 1.54
9	± 1.09	± 0.95	±1.06	± 0.83	± 2.19	± 1.75
10	± 0.12	± 0.13	± 0.12	± 0.13	± 2.06	± 1.45
11	± 0.15	± 0.17	± 0.12	± 0.13	± 2.05	± 1.42
101	± 1.14	± 1.06	± 1.18	± 1.13	± 2.66	± 2.46
102	± 1.11	± 0.14	± 1.09	± 0.62	± 2.47	± 1.27
103	± 1.18	± 0.67	± 1.15	± 0.96	± 2.60	± 2.01
104	± 1.13	± 1.07	± 0.13	± 0.13	± 2.63	± 2.50
105	± 0.11	± 0.14	± 0.13	± 0.14	± 2.47	± 1.37
106	± 0.17	± 0.77	± 0.13	± 0.14	± 2.58	± 2.20
107	± 1.14	± 1.09	± 1.18	± 0.85	± 2.66	± 2.57
108	± 0.11	± 0.15	± 1.09	± 1.04	± 2.47	± 1.51
109	± 1.18	± 0.88	± 1.15	± 0.79	± 2.60	± 2.42
110	± 0.13	± 0.13	± 0.13	± 0.14	± 2.47	± 2.11
111	± 0.17	± 0.20	± 0.13	± 0.14	± 2.47	± 2.08

We see there is relatively close agreement between the errors obtained analytically and by simulation.

4.4 The error as a function of the length of the base.

Figure 4 displays the evolution of the error in point 5 (see Fig. 3) when the base is gradually changed from 30 cm to 120 cm. All other quantities are the same. Figure 5 displays the error in point 1.

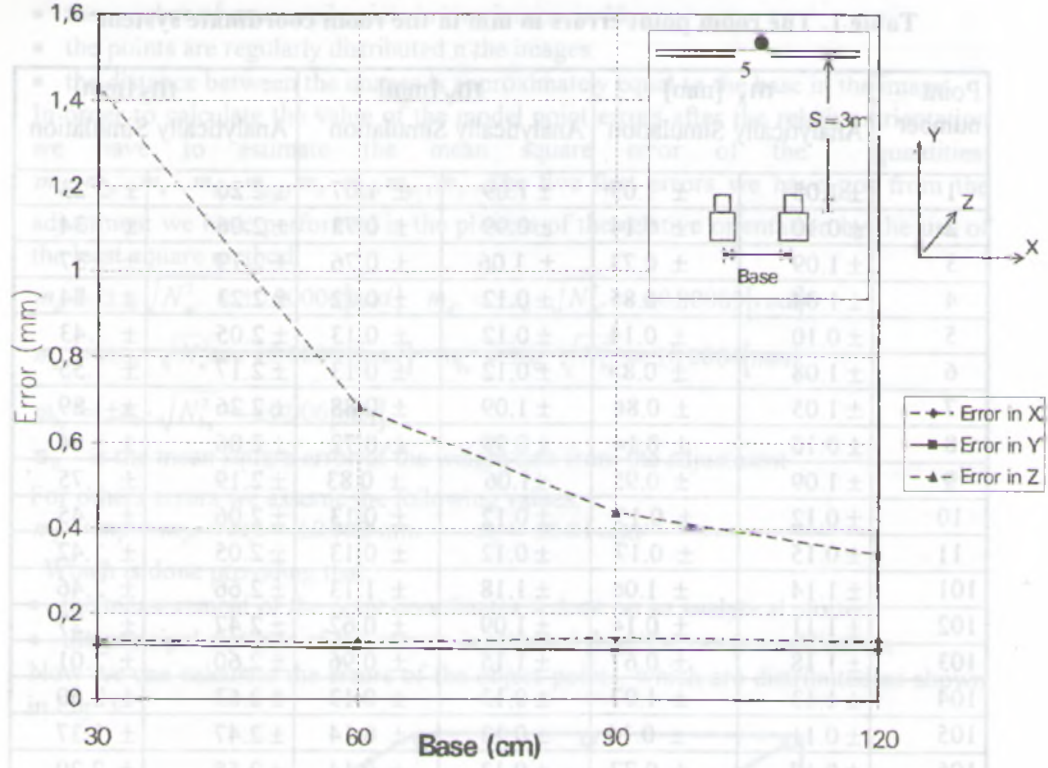

Figure 4. The error in point 5 as a function of the distance between the cameras.

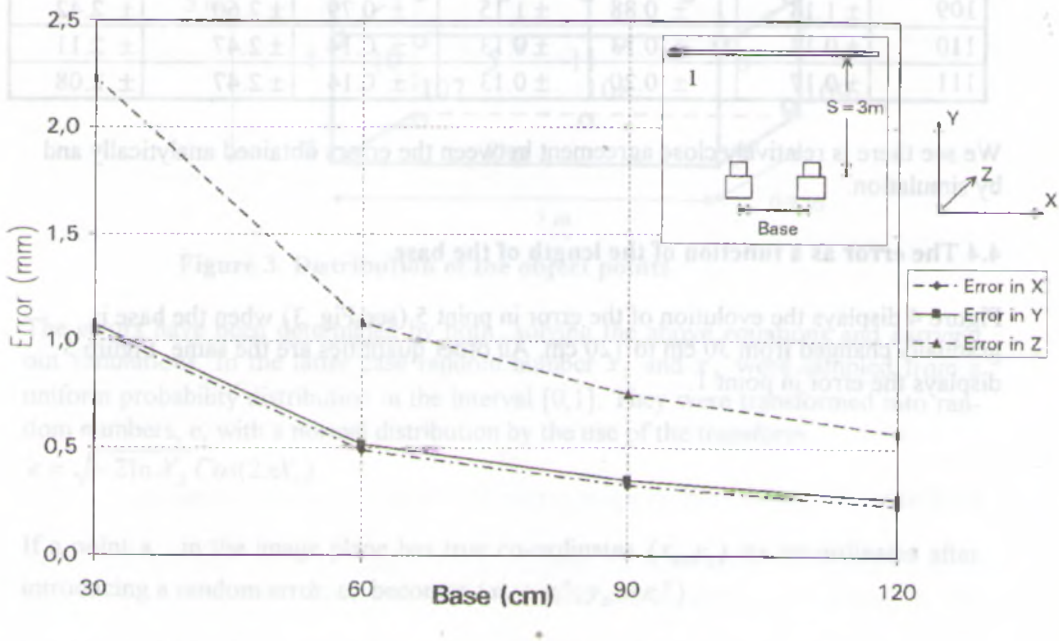

Figure 5. The error in point 1 as a function of the distance between the cameras.

5. Estimation of the error of the velocity components and the speed.

The velocity vector U(X,t) at point X is calculated as the change in position ΔX during a time interval Δt.

$$U(X,t)=(\frac{\Delta X}{\Delta t},\frac{\Delta Y}{\Delta t},\frac{\Delta Z}{\Delta t})=(\frac{X_2-X_1}{\Delta t},\frac{Y_2-Y_1}{\Delta t},\frac{Z_2-Z_1}{\Delta t})$$

Therefore the error in velocity is a function of the error in the distance between two points in the room and error in measurement of time.
Often in ventilation applications one is not interested in individual velocity components. For assessment of the risk of draught one is interested in the *speed V* defined as: V=S/t

where S is the distance: $S=|A_2-A_1|=\sqrt{(X_2-X_1)^2+(Y_2-Y_1)^2+(Z_2-Z_1)^2}$

The mean square error of the speed is: $m_V^2=(\frac{1}{t^2}m_s^2+\frac{S^2}{t^4}m_t^2)$

The mean square error m_s^2 in S can be expressed as:

$$m_s^2=\frac{1}{S^2}((\Delta X)^2m_X^2+(\Delta Y)^2m_Y^2+(\Delta Z)^2m_Z^2)=\frac{2}{S^2}((\Delta X)^2m_X^2+(\Delta Y)^2m_Y^2+(\Delta Z)^2m_Z^2)$$

We know m_X^2 and m_Y^2 to be of the same order of magnitude whereas the error in the out of plane component m_Z^2 is much greater. Therefore the error mean square error can be written as: $m_s^2=\frac{2}{S^2}(((\Delta X)^2+(\Delta Y)^2)m_X^2+(\Delta Z)^2m_Z^2)$

subsequently $m_s=\frac{\sqrt{2}}{S}\sqrt{(((\Delta X)^2+(\Delta Y)^2)m_X^2+(\Delta Z)^2m_Z^2)}$

If there is no out of plane component ΔZ =0 we obtain: $m_s=\sqrt{2}m_x$

When assessing the error of the speed the error due to measuring the time can usually be neglected and the relative error becomes: $\frac{m_v}{V}=\frac{1}{t}\frac{m_s}{V}=\frac{m_s}{S}$

When there is no out of plane component we obtain the relative error: $\frac{m_v}{V}=\sqrt{2}\frac{m_x}{S}$

and for the other extreme with only a component in the Z-direction we obtain:

$$\frac{m_v}{V}=\frac{\sqrt{2}}{S}m_z$$

The error for a velocity vector with components in all three directions lies in between the two curves. A typical streak has a length of about 10 cm. The relative error at this length is given in Table 2.

Table 2 The relative error for a 10 cm long streak.

Location	Relative error[%]	
	Components only in X-Y-plane	Components only in Z-direction
Point 5 (centre of image)	0.2%	2.0%
Point 1 (corner of image)	1.5%	3.1%

6. Recommendation for further research and development.

The research in the field determining air movements in ventilated rooms should be concentrated around the following problems:

- development of photogrammetrical methods optimised for streaks,
- improvement of the algorithms for determination of the coordintes of the end of the streaks,
- development of a suitable tracer for the low velocities occurring in ventilated rooms. To facilitate automatic analysis, tracers of uniform size are desired.
- development of portable light sources that can be used in field trials,
- integration of whole-field measurements methods for determination of three-dimensional velocities, temperatures and concentrations.

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Abstract

The purpose of this report is to investigate the accuracy of determining three-dimensional air movements in ventilated rooms by means of the photogrammetrical methods. Two approaches to determine the accuracy have been used. The first one is based on analytical calculation which analysis the relations between the image coordinates and the object coordinates. The second method is based on simulation by introducing random errors to the image coordinates and calculation of the object points. A relatively close agreement between the errors obtained analytically and by simulation has been achieved for the typical object of size 3x3x0.3 m. Some other aspects like optimisation of the network, a selection of a suitable mathematical model of the process are discussed in this paper. Finally recommendations for further research and development are presented.