MATHEMATICAL MODELS OF PRECISE PHOTOGRAMMETRY SURVEY.

Alexander Dorozhynsky, Natalia Moskal

Chair of Photogrammetry and Geoinformatic, State University "L'vivska Politechnika" L'viv-Ukraine

ABSTRACT

A great variety of different method and techniques are used for the processing of images that are got by photogrammetry survey. The methods of the digital photogrammetry on the stage of geometrical model constructing and analyzing results of measuring this model completely use the the apparatus of analytical photogrammetry. Thus the elaboration of different mathematical models is of great importance. The article presents the analysis of the photogrammetry data and methods of their adjustment. The authors formulate the generalized mathematical model that is universal concerning photogrammetry tasks. There is the theoretical solution for joint adjustment of the measured quantities and their functions and control data given in the article. The authors supply with the examples of partial cases that come from generalized model.

INTRODUCTION

The fundamental task of the photogrammetry is to reconstruct the object and to define its metric parameter on the base of their photoimages. This task is still very important. As it is known the methods of the digital photogrammetry on the stage of geometrical construction of object model used completely the apparatus of analytical photogrammetry. From this point of view it is very important to elaborate mathematical models that were not enough investigated in the sphere of analytical photogrammetry.

The analysis of photogrammetry data and methods of adjustment permitted to make a scheme containing main characteristics of measured quantities series, extra data and geometrical conditions that appear in different photogrammetry tasks (see Fig.1).

1. MODELS OF PHOTOGRAMMETRY SURVEY AND METHODS FOR THEIR MATHEMATICAL PROCESSING

We are going to concentrate on some explanation of this scheme, putting aside the classical points that do not need discussion.

1.1 Photogrammetry measuring suggests the direct measuring quantities on photos. Usually they are flat right-angled coordinate of points. When the mathematical processing is carried on the results are usually adjusted according to the classical scheme.

1.2 The space photogrammetry coordinates of point of model are considered to be the functions of 1.1-type quantities. If one includes them in the adjustment he should use well-known theorem [2.3] to get strong theoretical solution. The theorem [2.3] states that if there is adjustment of F - functions of correlated measuringY and the matrix is introduced

$$Q = \alpha \, \Sigma_Y \, \alpha', \qquad (1)$$

then the result gained will be identical with the adjustment of values measured directly.

Here $\alpha = \dfrac{\partial F}{\partial Y}$, $F = F(Y)$,

$\sum_Y$ - codispersive matrix of measured values.

1.3 Bearing data include the following data classes: elements of projection; elements of geodetic orientation of photography (linear and angular); coordinates of the control points; geodetic survey (angles, directions, length of lines, exceeding and so on); size and forms of objects and some others.

Each type of data requires an equation to be put down. This equation later is introduced into the general system of equations when making adjustment.

4.1-4.2 The notion of the function of losses as well as notion of minimization of regressive remains is widely used in the regressive analysis [1]. For the linear model of regressive function the function of losses is as follows:

$$\rho(\varepsilon) = |\varepsilon|^{2+d} \qquad (2)$$

here ε - - regressive remains, d - parameter of unsquareness $(-1 < d \le 0)$.

If d = -1 there is so called robust-method of evaluation. If d = 0 there is a square function of losses that is classical method of least squares.

The use of the notion of the function of losses provides with the chance to process the series of surveys with non-Gaussian division of the errors.

5.1-5.2. The use of the hypothesis about dependence (independence) of measuring equitation requires the awareness (or neglecting) of codispersive matrix of errors of measuring. Thus introducing codispersive matrix $\sum_Y$ into mathematical model.

The authors elaborated a great variety of mathematical models. These models are presented in [4]. To reconstruct the objects with precise ground-camera survey The class of mathematical models in operation [5] was proposed. to reconstruct the objects with precise ground-camera survey.

1. Type of values subjected to adjustment	1.1 Photogrammetrical servey
	1.2 Function of Photogram metrical survey
	1.3 Control data
⇓	
2. Errors data nature	2.1 Just occasional
	2.2 Occasional and systematical
	2.3 No errors or they are disdainfully small
⇓	
3. The possibility of including the systematic errors to adjustment	3.1 YES
	3.2 NO
⇓	
4. Function of losses	4.1 Square
	4.2 Unsquare
	4.3 Mixed
⇓	
5. Is the codispersive matrix known	5.1 YES
	5.2 NO
⇓	
6. The matrix weight of photogrammetrycs data known	6.1 YES
	6.2 NO
⇓	
7. The existance of additional geometrical conditions	7.1 YES
	7.2 NO

PHOTOGRAMMETRIC PROBLEMS

1. Partial calibrating of the photographs
2. Complete calibrating
3. Self calibrating
4. Reverse photogrammetric intersection
5. Double photogrammetric intersection
6. Direct repeated intersection
7. Relative orientation of a couple of the photographs
8. Relative orientation of three photographs
9. Phototriangulation as to the Join-method
10. Phototriangulation as to the method of the models
11. Formation of quasiphotograph
12. Geodetic orientation of the route model
13. Geodetic orientation of the model (the couple of photographs)
14. Research stereophotogrammetrical devises

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Pic.1. The choice of the model for mathematical processing photogrammetry survey

2.GENERALIZED MATHEMATICAL MODEL OF PHOTOGRAMMETRY SURVEY

In this article we are discussing the general problem of joint adjustment of the measured quantity functions, control data and direct measuring in the following statement:

Let us admit that there are set the following data: n-dimensional vector Y of measured quantities, which are free from systematical errors; its codispersive matrix $\sum_{Y_1}$;

r - values of functions are calculated.

$$T = F(Y_1) \tag{3}$$

It is necessary to make the adjustment with respect to r-conditions.

$$\Phi = \Phi(T', \overline{Y}' U') = 0 \tag{4}$$

here $T', \overline{Y}', U'$ – correspondingly means adjusted values of

functions $T' = T + \Delta T,$ (5)

control data $\overline{Y'} = \overline{Y} + \gamma$ (6)

additional unknown quantities

$$U' = U + \Delta U \tag{7}$$

Besides the measured quantities Y_2 with the codispersive matrix $\sum_{Y_2}$ are connected by the equations of corrections with the vectors $Y', U,$, so that

$$\varepsilon_2 = -B_2\gamma - S\Delta U - W_2 \tag{8}$$

The codispersive matrix $\sum_{\overline{Y}}$ of vector $\overline{Y}$ is known.

We consider the errors for Y_1 and Y_2 to be divided according to the Gauss law, and for $\overline{Y}$ they differ a bit from the Gauss.

dimensional of all the vectors are such that there is not the indetermination of the linear equations system, that is The matrix possess the complete rang. Let us consider as well that the vectors $Y_1, Y_2, \overline{Y}$ are correlated with one another.

As far as

$$T' = T + \Delta T = F(Y_1 + \varepsilon_1 = F(Y_1) + \frac{\partial F}{\partial Y_1}\varepsilon_1 = T + \alpha\varepsilon_1, \qquad (9)$$

than $\Delta T = \alpha\varepsilon_1$ (10)

When carrying out the linearization (4) we get

$$\frac{\partial \Phi}{\partial T}\Delta T + \frac{\partial \Phi}{\partial Y'}\gamma + \frac{\partial \Phi}{\partial U}\Delta U + \Phi(T',Y',U') = 0,$$

or

$$A\Delta T + B_1\gamma + C\Delta U + \omega_1 = 0. \qquad (11)$$

Substituting expression (10) instead ΔT taking into account (8), we get the initial system if equations

$$\begin{aligned} D\varepsilon_1 + B_1 + C\Delta U + \omega_1 = 0 \\ \varepsilon_2 + B_2\gamma + S\Delta U + \omega_2 = 0 \end{aligned} \qquad (12)$$

where $D = A\alpha$ (13)

Let us complete The adjustment for (12) with the condition of the minimization of function of losses, with

$$\rho(\varepsilon_1) = \varepsilon_1^2, \ \rho(\varepsilon_2) = \varepsilon_2^2, \rho(\gamma) = |\gamma|^{2+d}, \qquad (14)$$

where d - parameter of unsquareness.

Let us compose the function of Lagrang, for conditional equations (12) where instead $\gamma' \sum_{\bar{Y}}^{-1} \gamma$ we introduce the sum marked with Gauss symbols:

$$\Psi'_\gamma = [p\gamma^{2+d}] = p_1\gamma_1^{2+d} + p_2\gamma_2^{2+d} + \ldots + p_s^{2-d}, \qquad (15)$$

$$\Psi = \varepsilon_1' \Sigma_{y_1}^{-1} + \varepsilon_2' \Sigma_{y_2}^{-1} + \Psi_\gamma - 2k_1(D\varepsilon_1 + B_1\gamma + C\Delta U + \omega_1) - 2k_2(D\varepsilon_1 + B_2\gamma + C\Delta U + \omega_2). \qquad (16)$$

Thus we can get partial derivatives:

$$\frac{\partial \Psi}{\partial \varepsilon_1} = -2\varepsilon_1' \Sigma_{y_1}^{-1} \ 2k_1 D = 0,$$

$$\frac{\partial \Psi}{\partial \varepsilon_2} = -2\varepsilon_2' \Sigma_{Y_2}^{-1} - 2k_2' = 0$$

$$\frac{\partial \Psi}{\partial U} = -2k_1C - 2k_2S = 0 \qquad (17)$$

$$\frac{\partial \Psi}{\partial U} = (2+d)\mathrm{B}\gamma\,'\Sigma_{y}^{-1} + (2+d)\delta_{\gamma}\Sigma_{y}^{-1} - 2k_1B_1 - 2k_2B_2 = 0,$$

$$\frac{\partial \Psi}{\partial \gamma} = (2+d)\mathrm{B}\gamma\,'\Sigma_{\bar{y}}^{-1} + (2+d)\delta_{\gamma}\Sigma_{\bar{y}}^{-1} - 2k_1B_1 - 2k_2B_2 = 0,$$

where $\delta_{\gamma} = \left[\gamma_1\Delta_1 - \gamma_2\Delta_2 - \ldots - \gamma_s\Delta_s\right]$ (18)

$$\Delta = |\gamma|^{d} - 1 \quad or \quad \Delta = \sum_{j=1}^{p}\frac{\left(d\ln|\gamma|\right)^{j}}{j!} \qquad (19)$$

When solving (17) concerning $\varepsilon'_1, \varepsilon_2, \gamma$, we get

$$\varepsilon_1 = \Sigma_{y_1}D'k_1,$$

$$\varepsilon_2 = \Sigma_{y_2}k_2,$$

$$\gamma = \frac{2}{2+d}\sum\nolimits_{\bar{y}}B_1'k_1 + \frac{2}{2+d}\sum\nolimits_{\bar{Y}}B_2k_2 + \delta_{\gamma}. \qquad (20)$$

On the base of (12) and (17) we get the system of the equations correlate

$$(D\Sigma_{Y_1}D' + \frac{2}{2+d}\sum\nolimits_{\bar{y}}B_1')k_1 + \frac{2}{2+d}B_1\sum\nolimits_{\bar{Y}}B_2'k_2 + C\Delta U + \omega_1 + B_1\delta_{\gamma} = 0,$$

$$\frac{2}{2+d}B_2\sum\nolimits_{\bar{Y}}B'_1k_1 + (\sum\nolimits_{y_2} + \frac{2}{2+d}B_2')k_2 + C\Delta U + \omega_2 + B_2\delta_{\gamma} = 0. \qquad (21)$$

The development completed by means of consecutive exclusion of unknown quantities leads to finding out the vector ΔU and correlate

$$\Delta U = -\left(C'M_{11} - S'R_{11}^{-1}R_{12}\right)^{-1}\left(C'M_{12} - S'R_{11}^{-1}R_{13}\right), \qquad (22)$$

$$k_2 = -R_{11}^{-1}\left(R_{12}\Delta U + R_{13}\right). \qquad (23)$$

$$k_1 = -N_{11}^{-1}N_{12}k_2 - N_{11}^{-1}C\Delta U - N_{11}^{-1}\omega_1 \qquad (24)$$

Here are introduced the following symbols

$$N_{11} = D\Sigma_{Y_1} D' + \frac{2}{q} B_1 \Sigma_{\bar{Y}} B_1', \; q = 2 + d$$

$$N_{12} = N_{21}' = \frac{2}{q} B_1 \Sigma_{\bar{Y}} B_2',$$

$$N_{22} = \left(\Sigma_{Y_2} + \frac{2}{q} \Sigma_{Y_2} B_2' \right),$$

$$\overline{\omega_1} = \omega_1 + B_1 \delta_y \;, \; \overline{\omega_2} = \omega_2 + B_2 \delta_Y, \tag{25}$$

$$R_{12} = S - N_{21} N_{11}^{-2} C, \;\; R_{13} = \overline{\omega_2} - N_{21} N_{11}^{-1} \overline{\omega_1},$$

$$M_{11} = N_{11}^{-1} N_{12} R_{11}^{-1} R_{13} - N_{11}^{-1} \omega_1,$$

$$R_{12} = -N_{21} N_{11}^{-1} N_{12} + N_{22}.$$

The codisprsive matrix of adjusted vector ΔU is equal to

$$\sum\nolimits_{\Delta U} = \left(C' M_{11} - S' R_{11}^{-1} R_{12} \right). \tag{26}$$

Basing on the task described we can formulate new variants of aerophototriangulation. For example when one part of the net is constructed by the method of models and another part is constructed by the method of connections. Such interpretation makes it possible to fulfill in a new way the densibleness of the net limited by the couple of the photographs. Besides it is possible to construct models of locality and relief.

3. PARTIAL PROBLEMS AND MODELS

Let us discuss some partial tasks that proceed from generalized model (12) and which are of practical interest for photogrammetry.

1.Measuring Y was not completed. It leads the general statement to the task of joint adjustment of measured quantities functions and the control data with the mixed division of errors. Instead the system (12) we get

$$D\varepsilon_1 + B\gamma + C\Delta U + \omega_1 = 0, \tag{27}$$

$$\left(\varepsilon_2 = B_2 = S = \omega_2 = \Sigma_{Y_2} = 0 \right).$$

Then $\Delta U = -\left(C'N_{11}^{-1}C\right)^{-1} C'N_{11}^{-1}\overline{\omega_1}$, (28)

$$\sum\nolimits_{\Delta u} = \left(C'N_{11}^{-1}C\right)^{-1} \qquad (29)$$

2.The task is analogous to the previous but with the condition that for the errors ε_2 and γ the law of the division is normal. Then d = 0 and

$$\begin{gathered} \Delta U = -\left(C'N_{11}^{-1}C\right)^{-1} C'N_{11}^{-1}\overline{\omega_1}, \\ N_{11} = D\Sigma_{\varepsilon}D' + B_1\Sigma_{\gamma}B_1', \\ \overline{\omega_1} = \omega_1. \end{gathered} \qquad (30)$$

$\sum_{\Delta U}$ is expressed by the equation (29).

3. The initial equations are analogous to (12) but $B_2 = 0$. It means that the measuring for the control data was not done $(Y_2 = 0)$.

$$\begin{gathered} \Delta U = \left(C'N_{11}^{-1}C + S'\Sigma_{Y_2}^{-1}S\right)^{-1}\left(C'N_{11}^{-1}\overline{\omega_1} - S'\Sigma_{Y_2}^{-1}\omega_2\right), \\ \Sigma_{\Delta U} = \left(C'N_{11}^{-1}C + S'\Sigma_{Y_2}^{-1}S\right)^{-1}. \end{gathered} \qquad (31)$$

4.The task is analogous to the third, but the functions of losses for ε_1, ε_2 and γ are square. The solution will be as follows:

$$\Delta U = \left(C'N_{11}^{-1}C + S'\Sigma_{Y_2}^{-1}S\right)^{-1}\left(C'N_{11}^{-1}\overline{\omega_1} - S'\Sigma_{Y_2}^{-1}\omega_2\right). \qquad (32)$$

The matrix $\sum_{\Delta U}$ is analogous to (29).

5. It is necessary to fulfill the joint adjustment of the measured quantities functions under condition when the errors of control data can be neglected. The initial system (12) Will be as follows:

$$\begin{gathered} D\varepsilon_1 + C\Delta U + \omega_1 = 0, \\ \varepsilon_2 + S\Delta U + \omega_2 = 0. \end{gathered} \qquad (33)$$

Here is $B_1 = B_2 = 0$, $\sum_{\bar{\gamma}} = 0$ and

$$\Delta U = \left(C'N_{11}^{-1}C + S'\Sigma_{Y_2}^{-1}S\right)^{-1}(C'N_{11}^{-1}\overline{\omega_1} - S'\Sigma_{Y_2}^{-1}\omega_2),$$
$$\Sigma_{\Delta U} = \left(C'N_{11}^{-1}C + S'\Sigma_{Y_2}^{-1}S\right)^{-1} \qquad (34)$$

Where $N_{11} = D\Sigma_{Y_1}D'$.

6. If there is no measuring Y_2 in task five we shall have the adjustment of the measured quantities functions with the additional unknown quantities U [3].

Then

$$\varepsilon_1 = S = \omega_2 = \Sigma_{Y_2} = 0,$$
$$\Delta U = -(C'(D\Sigma_{Y_1}D')^{-1}C)^{-1}(C'(D\Sigma_{Y_1}D')^{-1}\omega_1), \qquad (35)$$
$$\Sigma_{\Delta U} = (C'(D\Sigma_{Y_1}D')^{-1}C)^{-1}.$$

CONCLUSIONS

The cases (1 - 6) described above do not restrict the list of the partial problems that can be gained by the change of coefficients A, D, B_2, B_1, C, S, correlation matrix $\sum_{\bar{\gamma}}, \sum_{Y_1}, \sum_{Y2}$ and quantity d. It should be noticed that it is not difficult to get formulas when the functions of losses for the vectors ε_1 and ε_2 are unsquare and are analogous to $\rho(\gamma)$ from (14).

Thus the generalized task of the joint adjustment of the measured quantities, their functions and the control data covers the wide range of photogrammetry tasks, including both classical solution and new adjusting tasks of photogrammetry.

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