UDC 519.711; 681.5; 621.382

A. Holovatyy[1], V. Teslyuk[2], R. Panchak[2], S. Koshyrets[1]
[1]The National Forestry and Wood-Technology University of Ukraine,
Department of information technologies
[2]Lviv Polytechnic National University,
Department of computer-aided design systems

MATHEMATICAL MODELLING AND SIMULATION OF THE MECHANICAL COMPONENT OF THE FULLY DIFFERENTIAL CAPACITIVE MEMS ACCELEROMETER USING MATLAB/SIMULINK ENVIRONMENT

Розроблено імітаційну модель механічної компоненти повністю диференційного ємнісного МЕМС акселерометра в середовищі MATLAB/Simulink. Модель дає змогу моделювати переміщення робочої маси, зміну ємностей вимірювальних конденсаторів чутливого елемента, чутливість мікродавача від прикладеного прискорення, а також проводити часовий аналіз інтегрального пристрою на системному рівні проектування.

Ключові слова: Мікроелектромеханічні системи (МЕМС), повністю диференційний ємнісний МЕМС акселерометер, прискорення, математичне моделювання, імітаційна модель, поведінковий аналіз, САПР, MATLAB/Simulink.

Simulation model of the mechanical component of the fully differential capacitive MEMS accelerometer has been developed using MATLAB/Simulink environment. The model allows to simulate movement of the proof mass, capacitance changes of the sense capacitors of the sensitive element, sensitivity of the sensor depending on the applied force of acceleration, and to perform the transient analysis of the integrated device at the system level of computer-aided design.

Key words: Microelectromechanical systems (MEMS), fully differential capacitive MEMS accelerometer, acceleration, mathematical modeling, simulation model, transient analysis, CAD system, MATLAB/Simulink.

Introduction

Micro-Electro-Mechanical systems (MEMS) are miniaturized integrated devices or systems which combine electrical and mechanical components fabricated by using micromachining technologies. Mechanical components are typically divided into sensors and actuators. The sensors convert a physical action into an electrical signal. In the case of the actuators, they convert the electrical signal into some physical actions. Nowadays, MEMS technologies are being developed very rapidly because of the high demand for such devices in the different engineering areas. Some of such devices are acceleration sensors (integrated microaccelerometers (MEMS accelerometers)). MEMS accelerometers play an important role in the state-of-the-art technology. They are widely used in automobiles (air bag deployment systems, anti-lock systems ABS, traction control, active suspension systems, anti-theft systems); consumer electronics (inertial navigation, smartphones, tablets, laptops (free-fall protection systems for hard disk drives); sports equipment (simulators, pedometers); geophysical application (earthquake monitoring systems); defense industry (high-tech military gear and gadgets, airplanes, helicopters, unmanned aerial vehicles, ground-based robotic systems), etc. [1].

The important role in the design of such heterogeneous systems as MEMS accelerometers play computer-aided design systems which allow to reduce the development time of integrated devices and their cost. The analysis of the literature in the field of MEMS design and modeling, allows to say that the creation of qualitatively new mathematical and computer models of the integrated devices to optimize their design parameters and the technical characteristics is an actual problem [2–4].

I. Construction and mathematical model of the mechanical component of the fully differential capacitive MEMS accelerometer

In Fig. 1 construction and mechanical parameters of the sensitive element (SE) of the fully differential capacitive MEMS accelerometer are schematically shown. The core of the SE of the MEMS accelerometer is a proof mass *M*, which is suspended by the spring elements with a spring coefficient *K* to the frame.

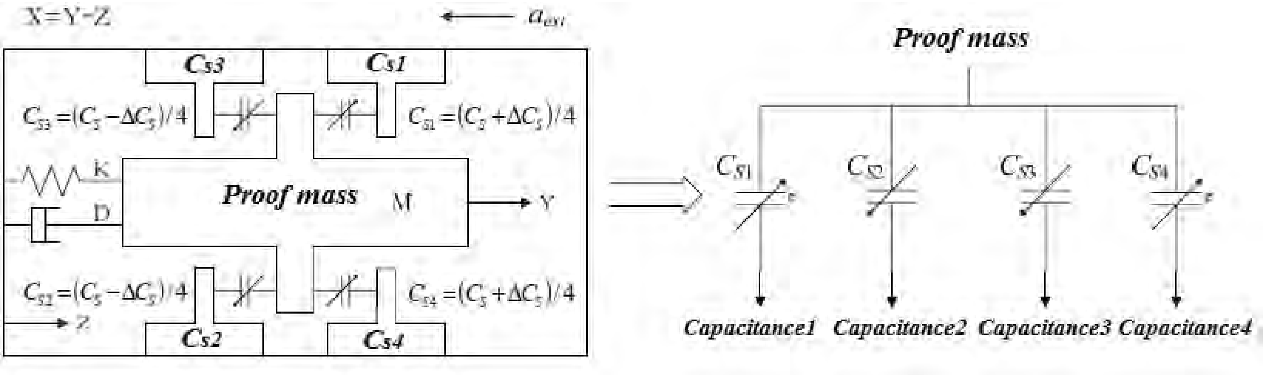

Fig. 1. Schematic view of the SE of the fully differential capacitive MEMS accelerometer

Motion of the SE of the working mass can be described using Newton's second law by the following differential second order equation, as a system mass-spring-damper:

$$M\frac{d^2x}{dt^2}+D\frac{dx}{dt}+K_{eff}\,x(t)=F_{ext}(t)=Ma_{ext}(t)\,, \qquad (1)$$

where *M* – proof mass, *D* and K_{eff} – damping and spring coefficients, F_{ext} – external inertia force, which acts on the proof mass when the external acceleration appears a_{ext}.

The analytical solution of such inhomogeneous differential equation (1) will be the sum of the solutions of the generic solution of its homogeneous differential equation $x_c(t)$ and particular solution of the differential equation (1) $X_p(t)$: $x(t)=x_c(t)+X_p(t)$.

Having the appropriate characteristic equation $\left(M\lambda^2+D\lambda+K_{eff}\right)e^{\lambda t}=0$ for a homogeneous differential equation $x_c(t)$ and solving it we obtain the following roots: $\lambda_{1,2}=\frac{-D\pm\sqrt{D^2-4MK_{eff}}}{2M}$.

If $D^2-4MK_{eff}>0$, then the solution is a function of the form $y=Ae^{\alpha t}+Be^{\beta t}$ (where *A*, *B*, *α*, *β* – constants, *α*, *β* – negative), which reflects the short-fading process (duration depends on the damping time constant *α*, *β*). This so-called inert review and meets great value *D* >> *M* ($D^2>4MK_{eff}$), presents itself a heavily damped system. If $D^2=4MK_{eff}$, then the solution is a function of the form $y=(A+Bt)e^{\alpha t}$ (with constants *A*, *B*, K_{eff}, *M*), having regard to the critically damped system.

For our oscillation system $D^2-4MK_{eff}<0$, then $\lambda_{1,2}=\frac{-D}{2M}\pm\frac{\sqrt{4MK_{eff}-D^2}}{2M}i=\alpha\pm\beta i$. The general solution will be: $x_c(t)=c_1e^{\alpha t}\cos(\beta t)+c_2e^{\alpha t}\sin(\beta t)=e^{\alpha t}\left(c_1\cos(\beta t)+c_2\sin(\beta t)\right)$, or

$$x(t)=ae^{\alpha t}\cos(\beta t-\varphi)\,, \qquad (2)$$

where $a=\sqrt{A^2+B^2}$ and φ – amplitude and phase shift of the movement of the SE. If $\omega \neq \beta$, then the particular solution of the differential equation (1) we search using the following formula: $X_p(t)=A\cos(\omega t)+B\sin(\omega t)$.

$$\left(-M\omega^2 A+K_{eff}A\right)\cos(\omega t)+\left(-M\omega^2 B+K_{eff}B\right)\sin(\omega t)=F_0\cos(\omega t)$$

$$\cos(\omega t):\ \left(-M\omega^2+K_{eff}\right)A=F_0,\ A=\frac{F_0}{K_{eff}-M\omega^2};\ \sin(\omega t):\left(-M\omega^2+K_{eff}\right)B=0,\ B=0$$

$$X_p(t)=\frac{F_0}{K_{eff}-M\omega^2}\cos(\omega t)=\frac{F_0}{M\left(\frac{K_{eff}}{M}-\omega^2\right)}\cos(\omega t)=\frac{F_0}{M\left(\omega_0^2-\omega^2\right)}\cos(\omega t). \tag{3}$$

Thus, the final solution of the differential equation (1) at $\omega \neq \beta$ will be the following:

$$x(t)=ae^{\alpha t}\cos(\beta t-\varphi)+\frac{F_0}{M\left(\omega_0^2-\omega^2\right)}\cos(\omega t). \tag{4}$$

If $\omega = \beta$, then the particular solution of the differential equation (1) has the following form:

$$X_p(t)=At\cos(\omega_0 t)+Bt\sin(\omega_0 t)$$

$$\left(-M\omega_0^2+K_{eff}\right)At\cos(\omega t)+\left(-M\omega_0^2+K_{eff}\right)Bt\sin(\omega t)+2M\omega_0 B\cos(\omega t)-2M\omega_0 A\sin(\omega t)=F_0\cos(\omega t)$$

$$-M\omega_0^2+K_{eff}=-M\left(\sqrt{\frac{K_{eff}}{M}}\right)^2+K_{eff}=-M\left(\frac{K_{eff}}{M}\right)+K_{eff}=0$$

$$2M\omega_0 B\cos(\omega t)-2M\omega_0 A\sin(\omega t)=F_0\cos(\omega t)$$

$$\cos(\omega t):\ 2M\omega_0 B=F_0,\ B=\frac{F_0}{2M\omega_0};\ \sin(\omega t):\ 2M\omega_0 A=0,\ A=0$$

$$X_p(t)=\frac{F_0}{2M\omega_0}t\sin(\omega_0 t). \tag{5}$$

So, the final solution of the differential equation (1) at $\omega = \beta$ has the following form:

$$x(t)=ae^{\alpha t}\cos(\beta t-\varphi)+\frac{F_0}{2M\omega_0}t\sin(\omega_0 t). \tag{6}$$

The effective spring coefficient of the spring suspension system of the accelerometer SE (K_{eff}) can be calculated by the formula:

$$K_{eff}=K_{mechanical}-K_{electrical}. \tag{7}$$

The mechanical and electrical spring coefficients can be calculated by the formulas:

$$K_{mechanical}=6E_x h\left(\frac{w}{l}\right)^3,\quad K_{electrical}=C_s\frac{V^2}{2d^2}, \tag{8}$$

where E_x – Young's modulus; h, w and l – thickness, width and length of the spring element, respectively; C_s – nominal rest capacitance between moving and fixed electrodes of the proof mass at the initial distance between them d; V – voltage applied to the measuring capacitor. MEMS accelerometer is designed taking into account that $K_{electrical} << K_{mechanical}$.

The air-damping coefficient D can be calculated by the following formula [4]:

$$D=n\mu_{eff}l_e\left(\frac{h_e}{d}\right)^3, \tag{9}$$

where n – total number of the comb-drive electrodes; μ_{eff} – effective air-viscosity coefficient (1,839×10^{-5} Pa·s); h_e and l_e – height and length of the comb-drive electrode, respectively.

The MEMS accelerometer, which is designed, has the fully differential topology, it means, that there are four sense capacitors C_{s1}, C_{s2}, C_{s3}, C_{s4} in the single node of the proof mass.

$$C_{s1,2} = \left(C_s \pm \Delta C_s\right)/4\,, \tag{10}$$

where C_s – nominal rest capacitance and ΔC_s – the change of the nominal capacitance of the microsensor. Under the action of the external acceleration, the proof mass begins to move along the sensitivity axis with respect to the moving frame of reference ($X = Y - Z$) which leads to the change of the distance between its electrodes and the neighboring fixed comb-drive electrodes of the SE. This movement of the proof mass can be measured as small changes of the capacitances between the movable and fixed electrodes of the integrated device.

If $a_{ext} = 0$, then capacitances of the sense capacitors are equal and calculate them by the formula:

$$C_{s1} = C_{s2} = C_{s3} = C_{s4} = A_c\varepsilon_r\varepsilon_0 / d = C_s / 4\,, \tag{11}$$

where A_c – area of the sense electrode formed by comb-drive electrodes; d – distance between comb-drive electrodes (plates) of the capacitor; ε_r – dielectric permeability of the environment between the capacitor electrodes; ε_0 – dielectric permeability of vacuum (8,8541×10^{-12} F/m).

If $a_{ext} \neq 0$ and $\Delta d << d$, then the capacitance change of the sense capacitor can be obtained from the formula:

$$\Delta C_s = C_{s1} + C_{s4} - C_{s2} - C_{s3} = 2A_c\varepsilon_r\varepsilon_0\left(\frac{1}{d-\Delta d} - \frac{1}{d+\Delta d}\right) \approx C_s\frac{\Delta d}{d} \tag{12}$$

and the capacitances of the capacitors from the formulas:

$$C_{s1} = C_{s4} = \left(C_s + \Delta C_s\right)/4 \qquad\qquad C_{s2} = C_{s3} = \left(C_s - \Delta C_s\right)/4\,, \tag{13}$$

where $\omega_0 = \sqrt{\dfrac{K_{eff}}{M}} = 2\pi f_0$ and quality factor $Q = \sqrt{\dfrac{K_{eff}\cdot M}{D}}$. When the frequencies are much lower then the resonant frequency ($\omega << \omega_0$), movement $x \approx a_{ext} / \omega_0^2$, which leads to the sensitivity S_e:

$$\frac{x}{a_{ext}} \approx \frac{1}{\omega_0^2}\,. \tag{14}$$

This ratio determines the sensitivity dependence on the bandwidth of the microsensor: the low resonance frequency leads to large displacements, and thereby the high resolution limit the bandwidth of the microsensor. But usually, the lower border of the resonant frequency is limited by many factors, such as resistance to the mechanical shock, the lowest coefficient of elasticity, the most effective mass, manufacturing technology.

From (12) and (14) the capacitance sensitivity of the accelerometer can be calculated by the formula:

$$S_c = \frac{\Delta C_s}{a_{ext}} = \frac{C_s}{d}\cdot\frac{M}{K_{eff}} = \frac{C_s}{d}\cdot\frac{1}{\omega_0^2}\,. \tag{15}$$

Brownian noise equivalent of acceleration (*BNEA*) can be obtained by the formula:

$$BNEA = \sqrt{\frac{a_{ext}^2}{\Delta f}} = \frac{\sqrt{4k_BTD}}{M} = \sqrt{\frac{4k_BT\omega_0}{MQ}}\,. \tag{16}$$

Another limiting factor is the electrical circuit noise equivalent acceleration (CNEA), which depends on the capacitive resolution of the interface circuit (ΔC_{min}) and the capacitive sensitivity of the accelerometer S_c ($S_c = \Delta C_s/a_{ext}$), and which can be defined by the formula:

$$CNEA = \frac{\Delta C_{\min}}{S}\,. \tag{17}$$

For a_{max} = 5g and maximum displacement x_{max} = 50 nm (10 % of capacitive gap in 0,5 мкм), we find the resonant frequency f_r = 5 kHz respectively to (4). Respectively, values of M, and K_{eff} we calculate at the constant f_0. M = 9,011×10^{-10} kg, K_{eff} = 0,889 N/m, i D = 28,287×10^{-6} Ns/m. Finally, total noise equivalent of the acceleration (TNEA) is:

$$TNEA = \sqrt{BNEA^2 + CNEA^2}\ . \tag{18}$$

The sensitivity of the system we can calculate from the following formula:

$$S_e = \frac{V_{out}}{a_{ext}} = \frac{K_v}{\omega_0^2}, \tag{19}$$

where V_{out} – output voltage which we obtain from the formula $V_{out} = V_m \cdot \left(\frac{C_{s1} - C_{s2}}{C_{s1} + C_{s2}} - \frac{C_{s3} - C_{s4}}{C_{s3} + C_{s4}} \right)$, V_m – modulation voltage.

The minimum sense acceleration a_{min} can be obtained from the total incoming accelerometer noise, including noise from mechanical microsensor from (16) and electronic noise from the interface circuit from (17). The maximum sense acceleration a_{max} we can obtain from Δd_{max}.

$$a_{\max} = \Delta d_{\max} \cdot \omega_0^2\ . \tag{20}$$

II. MATLAB-Simulink modelling of the mechanical component of the fully differential capacitive MEMS accelerometer

In Fig. 2 the developed MATLAB-Simulink model of the mechanical component of the fully differential capacitive MEMS accelerometer is shown.

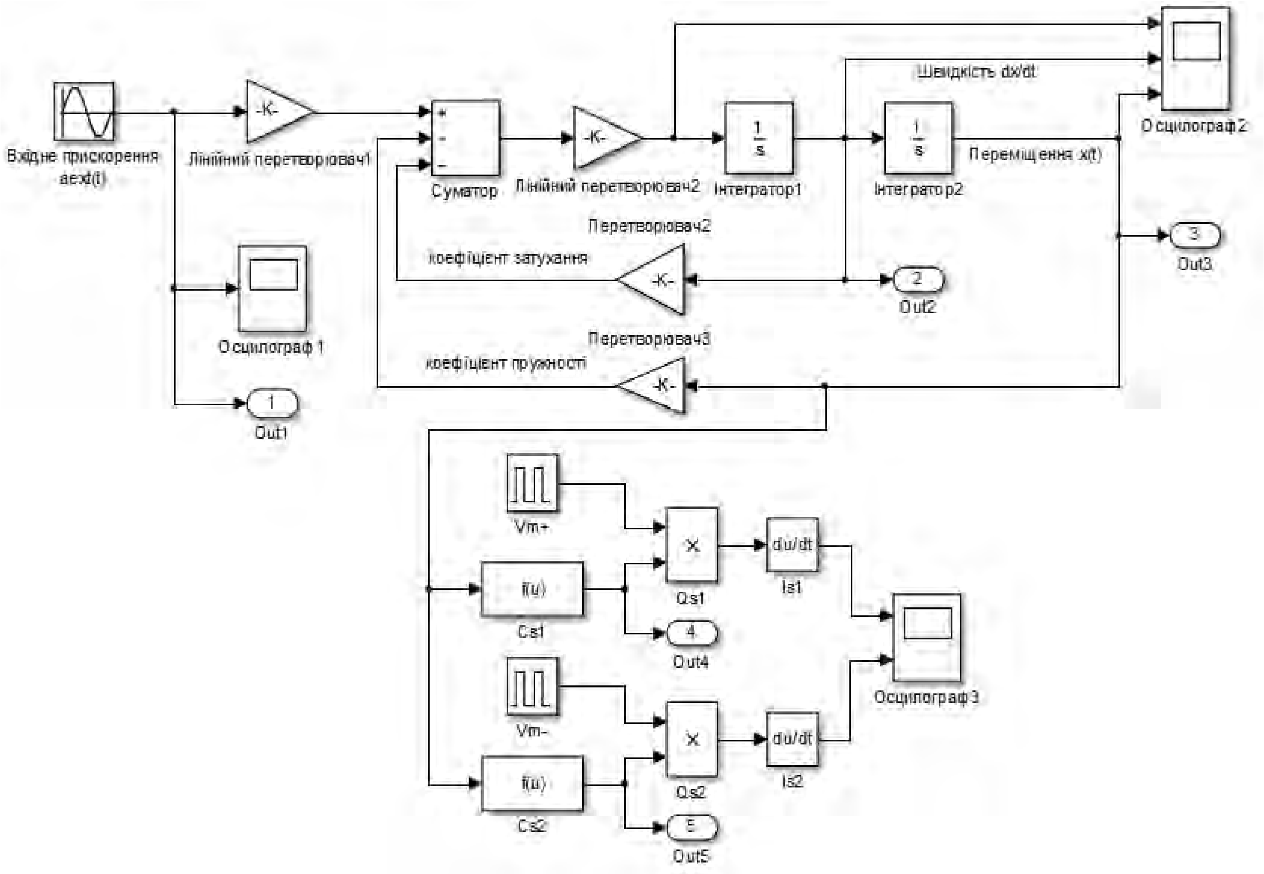

Fig. 2. Simulink –model of the mechanical component of the fully differential capacitive MEMS accelerometer

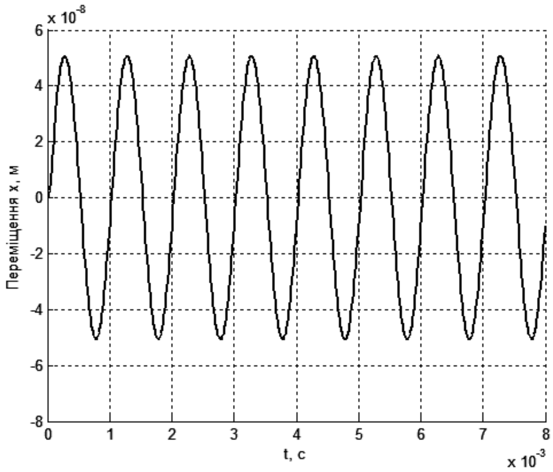

Fig. 3. Motion of the SE x(t) caused by the action of the applied acceleration in the sinusoidal form with an amplitude of 5g

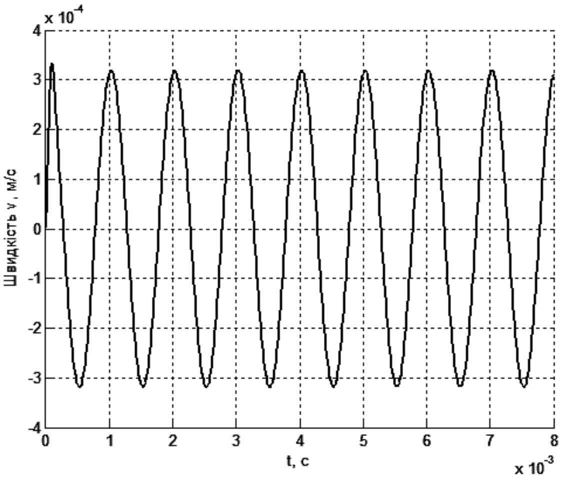

Fig. 4. Velocity of the SE v(t) when the external acceleration is applied with an amplitude of 5g

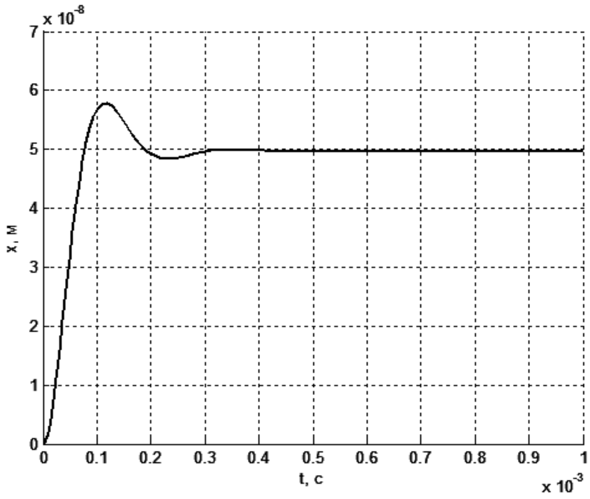

Fig. 5. Movement of the SE when the external acceleration is applied with an amplitude of 5g

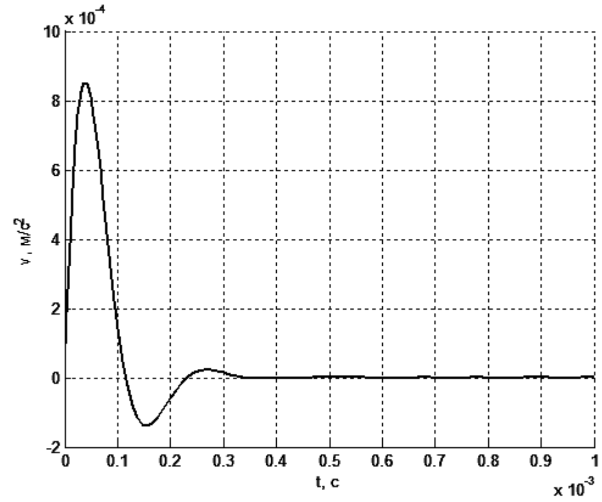

Fig. 6. Response of the SE to the action of the applied acceleration 5g

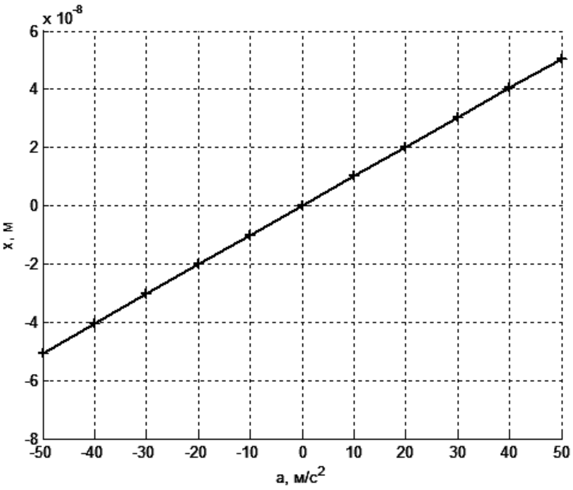

Fig. 7. Dependence of the movement of the SE on the applied acceleration in the range ±5g

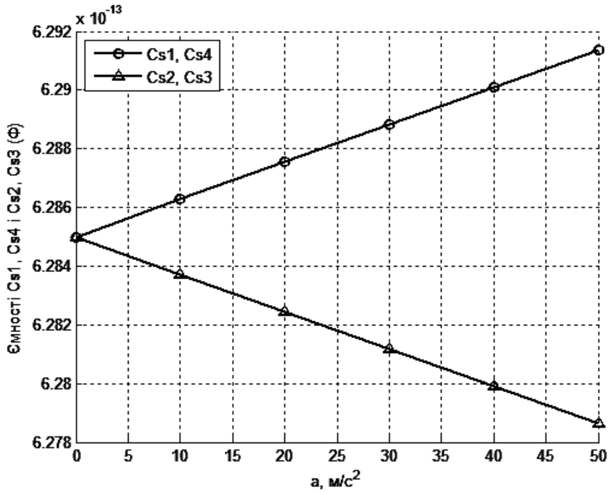

Fig. 8. Change of the capacitances C_{s1}, C_{s4} and C_{s2}, C_{s3} from acceleration a(t)

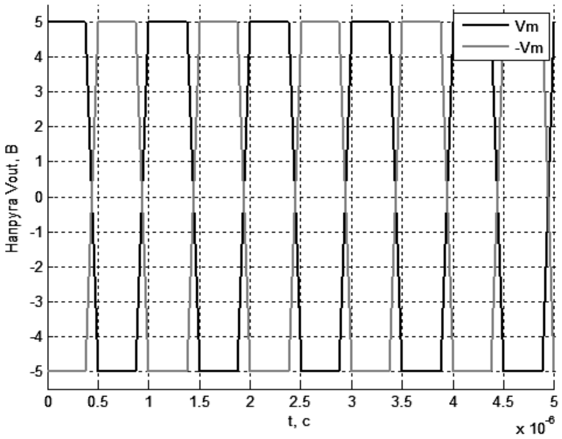

Fig. 9. Change of the output voltage under the action of acceleration with the amplitude in 5g

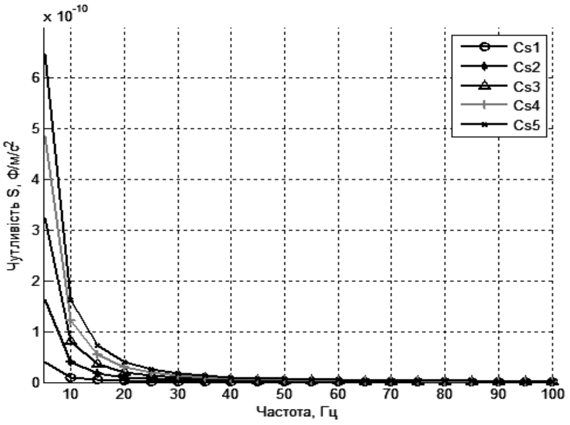

Fig. 10. Dependence of the sensitivity on the resonant frequency S_e, F/m/s^2 for various values of the nominal capacitance

In Fig. 3, 4 the simulation results of the motion and response of the SE of the proof mass under the action of the sinusoidal acceleration with the amplitude of 5 g are shown. In the chart it can been seen that the movement of the proof mass is in the range from -50 nm to 50 nm. In Fig. 5 the transient movement of the proof mass under the influence sustainable acceleration 5 g is shown. The graph in Fig. 6 shows the speed of response and SE operating mass transition to a new equilibrium under the the action of acceleration 5 g. Fig. 7 shows the dependence of the movement of the proof mass of the SE of applied acceleration in the range of ± 5 g. The graphs in Fig. 8 show the capacitance change of the sense capacitors at the acceleration 5 g, which is within 627.8 ... 629.2 fF. In Fig. 9 the simulation of the output voltages of the microsensor at the sinusoidal change of the acceleration with the amplitude of 5 g. In Fig. 10 dependence of the capacitive sensitivity of the microsensor at the change of the resonant frequency of the SE in the range of 5 to 100 Hz at the different values of the nominal capacity. From the obtained results it can been seen that for such defined design parameters of the mechanical component of the fully differential capacitive MEMS accelerometer high-precise circuits for processing such small changes of the output signals are needed.

Conclusion

The simulation model of the mechanical component of the fully differential capacitive MEMS accelerometer has been developed using MATLAB/Simulink environment. On the base of the developed model the behavioral simulation of the SE of the fully differential capacitive MEMS accelerometer has been conducted. From the obtained simulation results the relationship graphs are depicted and analysis of the static and dynamic characteristics of the SE of the MEMS accelerometer such as: movement parameters of the proof mass, change of the output sense capacitances of the SE, sensitivity of the SE depending on its construction parameters has been conducted.

References

1. Yazdi N. Micromachined Inertial Sensors / Yazdi N., Ayazi F., Najafi K. // Proc IEEE, Aug. 1998, Vol. 86, № 8. 2. Kanchan Sharma, Isaac G. Macwan, Linfeng Zhang, Lawrence Hmurcik, Xingguo Xiong. Design Optimization of MEMS Comb Accelerometer, Department of Electrical and Computer Engineering, pp. 10. 3. Tolga Kaya, Behrouz Shiari, Kevin Petsch and David Yates. Design of a MEMS Capacitive Comb-drive Accelerometer. Central Michigan University, 2012, pp. 6. 4. Akila Kannan. Design and Modeling of a MEMS-Based Accelerometer with Pull In Analysis. Thesis, University of British Columbia, 149, pp. 149. 5. Dr. Lynn Fuller, Motorola Professor. MEMS Capacitor Sensors and Actuators. / Dr. Fuller Lynn. – Rochester Institute of Technology, Microelectronic Eng. – May, 2007. – 34 p. 6. Yun W. A Surface Micromachined Accelerometer with Integrated CMOS Detection Circuitry : Doctoral Dissertation. – Univ. of California at Berkeley, 1992. 7. Fedder G. K. Simulation of Microelectromechanical Systems : Doctoral Dissertation / G. K. Fedder. – Univ. of California at Berkeley, 1994. 8. Pelesko John A. Modeling MEMS and NEMS / John A. Pelesko and David H. Bernstein. Chapman & Hall/CRC, 2002, 384 p. 9. Frank W. Pietryga. Solving Differential Equations Using MATLAB/Simulink. University of Pittsburgh at Johnstown, 2005, 11 p. 10. Houcque David. Introduction to Matlab for engineering students. Northwestern University. 2005, 74 p. 11. Matlab Programming Fundamentals R2013b. The MathWorks, Inc. 2013, 985 p. 12. Matlab – The Language of Technical Computing. The MathWorks, Inc. 2002, 602 p.