OPTIMUM POINT MODEL OF THE EARTH GRAVITY FIELD COMPILED ON THE BASIS OF ABSOLUTE GRAVITY VALUES

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Abstract. *An optimum point model of the global gravity field was compiled on the basis of data for absolute values of gravity. The model was composed of point masses. Optimum parameters of the masses were determined by the least squares method This model is unique and stable for the used set of data and provides the best description of the field with least number of parameters of the sources. It approximates the used absolute values of the gravity with a mean square error of about 40 mGal, and only for data from the USA territory - 30 mGal. The field of the optimum point model is close in values to the field of the reference ellipsoid (the normal gravity field), and that of the residual anomalies - to the free-air anomalies. An advantage of the point model of the earth's global field, in comparison to the ellipsoidal one, is its flexibility and higher precision in data processing and its application in geophysics and geodesy.*

Introduction

The idea of presenting the earth's gravity field and its sources by an optimal model of point masses was proposed in 1962 by the outstanding Bulgarian scientist Dr. Dimitar Zidarov (Zidarov,1992). J. A. Weightman (1967) suggested that the point masses should be used in geodesy to create a mathematical model of gravity field Due to the advantages of this class of models to the traditional decomposition of gravity potential to spherical functions the idea got a wide development We are not going to mention all of the numerous publications on the above problem, since they refer mainly to the formal description of the gravity field. We would recommend the works of Meshcheryakov, Marchenko and others to those to those, interested in the multipoint models of the geopotential. Worth mentioning are the papers of J. Jelev.

In our paper we offer an approach of creating an optimal point model of the Earth and its gravity field using non-reduced gravity data.

As an approximate construction the point model is universal (it approximates arbitrary, smooth enough functions of mass distribution), expressive (it allows the description of both the primary and secondary elements of complex mass distributions, using finite number of parameters) and flexible (it gives the possibility to use models of different complexity). The model construction reduces to the solution of the inverse gravity problem.

When the experiment is successful, i e a unique and stable model of point masses for the given set of data is constructed, the obtained physical and mathematical model gives the possibility for a more radical way of solving all gravity and geodetic problems.

In order to avoid non-uniqueness and instability of the problem and to find concrete models, two important questions should be answered: how to define the maximum number of point masses and how to find a unique and stable solution within the chosen class of models.

It is obvious that the bigger the number of the point masses is, the more precise the approximation will be. It is also clear that the information, on which the density cross-section of the Earth is modelled, is finite and the complexity of the model is limited. The approximation is made by gradually making the model more elaborate, until reaching a model class, for which a unique and stable solution does not exist. When that degree of model complexity is reached that would mean that the whole information has been exploited out of data. At the second stage of the problem solution, the uniquely defined point masses should be redistributed to bodies of given density by applying the method of the "sweeping" of the masses, proposed by D. Zidarov (1968, 1984, 1990).

Unique and stable point models can only be found by selection, based on minimisation. The square method has an advantage in this case. The convergence of the iteration procedure in the point of global minimum is proved.

Mathematical formulation and solution of the problem

The mathematical formulation of the inverse gravity problem is: the absolute values of earth acceleration $\mathbf{g(P_i)}$ are given at points $\mathbf{P_i}$, $\mathbf{i = 1, ..., n}$ of the earth's surface and also the relation between $\mathbf{g_m(P_i)}$ and the parameters **m** of the model.

A unique and stable model **m** is to be found, whose field differs from the real geologic body with a minimum value.

The given values of the earth's acceleration form a number of data **y** and define a **n** - dimensional vector **Y**. The function $\mathbf{g(P_i)}$ is the operator solving the direct problem $\mathbf{\Phi}$. The model **m** is a function of parameters $\mathbf{q_j}$, $\mathbf{j = 1,..., s}$. In order to find the unique and stable model **m**, we have to determine the operator $\mathbf{\Phi}^{-1}$, the inverse operator of $\mathbf{\Phi}$. To solve this problem we apply the automatic selection where the inverse operator will be the minimisation procedure on the following objective function.

$$U = \sum_{i=1}^{n} [g(P_i) - g_m(P_i)]^2$$

The minimisation of the objective function U will be done by the least squares method (Gauss-Newton method of minimisation of the corrections developed by us, which is much similar to the methods of Levenberg and Marquardt.

The optimal multipoint model found by unreduced data will be the simplest model possible which describes the gravity field with high precision. For example the normal field calculated by the Helmert formula was approximated with precision of 0.007 mGal with 9 point model with the parameters shown in the Table 1.

It is obvious that the construction of such an optimal multipoint model of real data would create possibilities for quick and precise solution of a variety of geodetic problems related to the shape of the geoid and the trajectories of the artificial satellites of the Earth.

Table 1

N	X_i [km]	Y_i [km]	Z_i [km]	f.M_i
1	92.9333	-347.0725	-0.0001	34019.88378
2	-92.9336	347.0724	-0.0001	34019.88368
3	347.0724	92.9333	-0.0001	34019.88363
4	-347.0724	-92.9335	-0.0001	34019.88385
5	311.1312	-179.7036	0.0000	34019.88382
6	-311.1312	179.7033	0.0000	34019.88381
7	179.7034	311.1311	0.0000	34019.88363
8	179.7035	-311.1311	0.0000	34019.88388
9	0.0000	0.0000	0.0000	126444.13071

The proposed method for solutions of the inverse gravity problem was applied on 701 absolute values of gravity, measured with a gravity pendulum. The basic part of the data - 440 measurements - were taken on USA territory The rest of the points (261) are from : Europe - 87, Antarctica - 56, the Far East - 92, Australia - 8 and etc. - 43 (Lejay - 1936, 1938, 1939; Duerksen - 1949; Uotila - 1960).

On the bases of this data we found a unique and stable model with 4 point masses with the parameters shown in the Table 2.

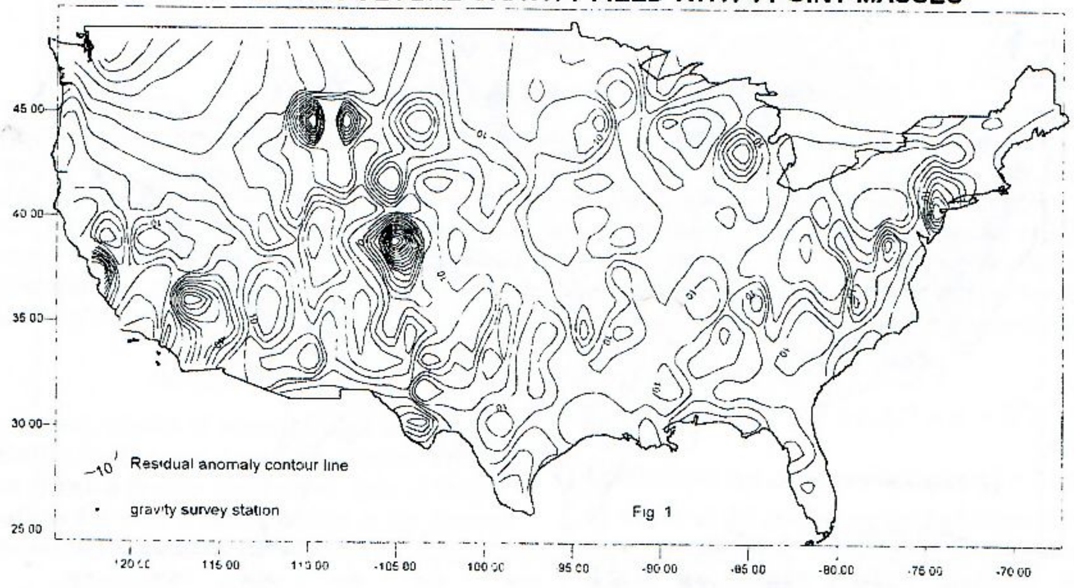
Table 2.

N	X_i [km]	Y_i [km]	Z_i [km]	f.M_i
1	129.069	-452.488	-23.279	42286.21
2	217.952	76.414	3.969	164689.40
3	-226.515	-52.757	7.717	161477.30
4	-158.079	499.096	-30.659	30157.03
1-4				398609.94

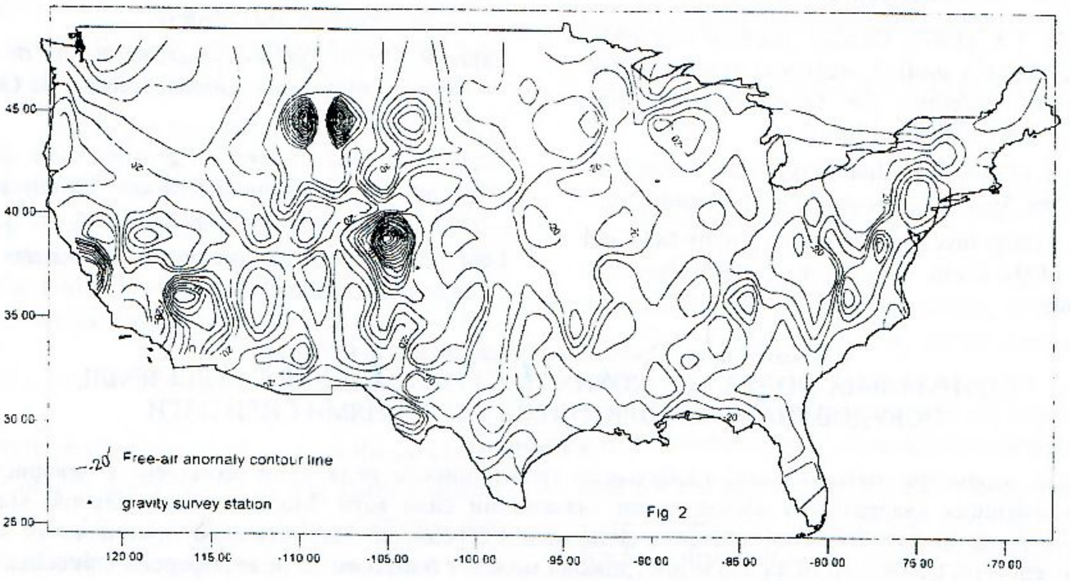
The field of this model approximates the real data of gravity at the 701 physical points with mean square error 40.01 mGal. The error of approximation of the American data is 30 mGal. The residual field can be used for qualitative interpretation in geology We have constructed a map of the residual anomalies for the territory of the USA (Fig.1).

The attempts for constructing a more complex and stable model did not give results, which means that most of the information have been extracted from the given set of data and that the information about the residual field is insufficient to solve the inverse problem. The residual field, though, could be used for qualitative interpretation in geology. For that purpose we have compiled a map of the residual anomalies on the territory of the USA. The comparison with the maps of the free-air anomaly (Fig.2) and the Bouguer anomaly (Fig.3) shows that the residual anomalies are close in pattern to the free-air anomalies. That is

USA RESIDUAL ANOMALY MAP AFTER APPROXIMATION OF THE GLOBAL GRAVITY FIELD WITH 4 POINT MASSES



USA FREE-AIR ANOMALY MAP



natural because real vertical gradient for the territory of the USA is close in value to the normal vertical gradient. The difference is that the global gravity field, which is subtracted from the measured field, is a field of an optimal, not an ideal model of the Earth. That means that the residual field will be reduced better from anomalies of non-geological origin. For other areas of the Earth, though, the differences between both anomalies can be considerable

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USA BOUGUER ANOMALY MAP

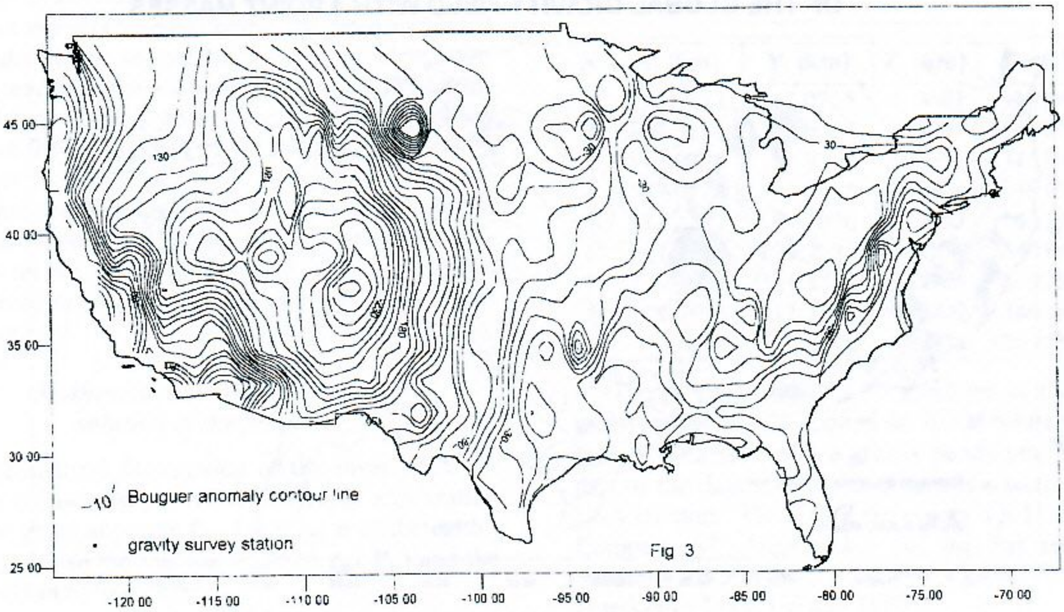

Fig 3

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ОПТИМАЛЬНА МОДЕЛЬ ТОЧКОВИХ МАС ГРАВІТАЦІЙНОГО ПОЛЯ ЗЕМЛІ, ПОБУДОВАНА ЗА АБСОЛЮТНИМИ ЗНАЧЕННЯМИ СИЛИ ВАГИ

Резюме

Оптимальні параметри точкових мас глобального гравітаційного поля були визначені з використанням методу найменших квадратів за абсолютними значеннями сили ваги. Модель з мінімальною кількістю параметрів представляє абсолютні значення сили ваги з середньою квадратичною помилкою 40 мГал, а тільки на території США – 30 мГал Поле побудованої моделі є близьким до поля референц-еліпсоїда.

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ОПТИМАЛЬНАЯ МОДЕЛЬ ТОЧЕЧНЫХ МАСС ГРАВИТАЦИОННОГО ПОЛЯ ЗЕМЛИ, ПОСТРОЕННАЯ ПО АБСОЛЮТНЫМ ЗНАЧЕНИЯМ СИЛЫ ТЯЖЕСТИ

Резюме

Оптимальные параметры точечных масс глобального гравитационного поля были определены с использованием метода наименьших квадратов по абсолютным значениям силы тяжести. Модель с минимальным количеством параметров представляет фбсолютные значения силы тяжести со средней квадратической ошибкой 40 мГал, а только на территории США – 30 мГал. Поле построенной модели близко к полю референц-еллипсоида