COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 7, No. 2, 2017

ADAPTATION OF DISCRETE MACROMODELS OF ELECTRIC TRANSMISSION LINES TO MODERN COMPUTER TOOLS

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Abstract: The paper discusses the methods of creation of mathematical models of electric transmission lines. It is proposed to develop them on the basis of computer simulation data or data obtained by natural experiment using the "black box" approach in the form of state variables. The derived system of discrete equations in the form of state variables equivalent to the system of ordinary differential equations has been adapted to the MATLAB / Simulink software environment by creating the mathematical model using S-Function Block. The simulation results obtained with the use of proposed discrete macromodel are presented and prospects of its further application are discussed.

Key words: electric system, macromodel, transmission line, optimization, MATLAB/Simulink.

1. Introduction

Overhead and cable transmission lines are one of the important components of electric systems. In order to create their adequate mathematical models it is necessary to take into account different physical processes (electromagnetic, mechanical and thermal) inherent to them. Ensuring the growing needs in high quality elecric energy and, as a consequence, the development of new models corresponding to the actual state of maintained electrical equipment taking into account both technical features of their elements and the need for quick analysis of processes using these models is a relevant scientific and technical problem [1–3].

With respect to modern requirements, the models of transmission lines should take into account the influence of following processes:

1) mechanical processes, namely, mechnical strengths and vibration of wires caused by different types of loading (icing and influence of wind on time and space values of parameters);

2) electromagnetic processes (state of electromagnetic field around wires, line currents and voltages);

3) thermal processes (variations of conductor temperature in time and space).

In order to analyze dynamic processes of transmission lines, two types of their models in time domain are used [1]:

1) models with lumped parameters which are created on the basis of a ***T***- or ***Pi***- equivalent circuit with lumped parameters whose values are calculated at some frequency. In this case the longitudinal and transverse symmetry of resistances of the equivalent circuit and a constant number of circuit components are used as a basis for further calculations. Values of impedances of these circuits are calculated as a dependence on cross-section dimensions and the material of the wire. Values of inductances of overhead transmission lines depend on distances between wires and their connection, their diameter and magnetic properties of materials used at their manufacturing. Capcitances of lines are calculated as a dependence on the diameter of wires, distance between wires and distance to the ground.

2) models with distributed parameters which can be divided into two subgropus: models with constant parameters and models with frequency-dependent parameters.

Most mathematical models of transmission lines can be created with the use of the following methods:

– travelling wave method where the process under research in some point of transmission line is a sum of waves reaching this place in different moments;

– standing wave method where the unknown paramenter in the given point of the process can be described as a sum of parameters of processes of all harmonic components present in the given moment.

In the all above mentioned models of transmission lines, the assumption that parameters are linear has been made. It can be applied, if a corona effect, which is the source of damping and distortion of the oscillation waveforms in transmission lines, is not taken into account. This phenomenon and possibilities of its being taking into account are analyzed in [5]. The influence of the corona effect on switching overvoltages and their mathematical simulation is very complicated phenomenon, which has not been studied thoroughly. In addition, the calculation of the dynamic processes in transmission lines is complicated by the presence of a skin-effect in conductors whose consideration leads to complications of the mathematical form of the model.

Physical modeling or the alternative application of macromodeling on its basis, which make it possible to consider the results of natural experiments and the parameters of transmission lines, difficult for practical taking into account, can ensure required preconditions for the complete analysis of transmission lines and the development of their adequate mathematical models.

2. Mathematical macromodel of transmission line

Due to the necessity of taking into account complex phenomena typical for transmission lines and difficulties of estimation of their parameters, it is proposed to develop their mathematical macromodels as a "black boxes" in the form of discrete state equations based on experimental data obtained with the use of mathematical or natural experiment and the following equation:

$$\begin{cases} \vec{x}^{(k+1)} = \mathbf{F}\cdot\vec{x}^{(k)} + \mathbf{G}\cdot\vec{v}^{(k)} + \mathbf{\Phi}(\vec{x}^{(k)},\vec{v}^{(k)}) \\ \vec{y}^{(k+1)} = \mathbf{C}\cdot\vec{x}^{(k+1)} + \mathbf{D}\cdot\vec{v}^{(k+1)}, \end{cases} \qquad (1)$$

where **F, G, C, D** are matrices of corresponding sizes; **Φ** is some vector-function of several variables, $\vec{x}^{(k)}, \vec{v}^{(k)}, \vec{y}^{(k)}$ are vectors of internal, input and output variables respectively, *k* is a discrete number.

The main problem of the simulation of transmission lines is principal impossibilty to desribe them using a simple electric circuit with lumped parameters. As a result, in order to create macromodel of a transmission line based on the expert analysis, the following mathematical form of its description was selected:

$$\begin{cases} \vec{x}^{(k+1)} = \mathbf{F}\cdot\vec{x}^{(k-m)} + \mathbf{G}\cdot\vec{v}^{(k)} + \mathbf{\Phi}(\vec{x}^{(k-m)},\vec{v}^{(k)}) \\ \vec{y}^{(k+1)} = \mathbf{C}\cdot\vec{x}^{(k-m)} + \mathbf{D}\cdot\vec{v}^{(k+1)} \end{cases} \qquad (2)$$

that makes it possible to develop discrete mathematical macromodels of the transmission line with fixed length adequate to the set of ordinary differential equations.

Based on the results of the mathematical experiment, a discrete mathematical macromodel of a single-phase transmission line was developed, and the procedure of its creation is decribed in details in [4].

The simulation of the transmission line was conducted considering it as a part of the electric circuit with a "power source-transmission line-load" structure, as it is shown in Fig. 1. The submodel of the line was introduced in the form of currents and voltages of sending and receiving end of line obtained based on differential equations in coordinates of phase voltages and currents using travelling wave method and considering non-zero initial conditions in the following form:

$$\begin{aligned} &u_1(t) - Z_c i_1(t) = (u_2(t-\tau) - Z_c i_2(t-\tau))e^{-\alpha\tau} + \\ &+U(0)(1-1(t-\tau))e^{-\alpha\tau}, \end{aligned} \qquad (2)$$

$$\begin{aligned} &u_2(t) + Z_c i_2(t) = (u_1(t-\tau) + Z_c i_1(t-\tau))e^{-\alpha\tau} + \\ &+U(0)(1-1(t-\tau))e^{-\alpha\tau}, \end{aligned} \qquad (3)$$

where u_1, u_2, i_1, i_2 are voltages and currents of sending and receiving end of the line, L, C, Z_C, l are inductance, capacitance, wave impedance and line length respectively, and τ is a propagation time of electromagnetic wave along the line; $1(t-\tau)$ is the Heaviside function.

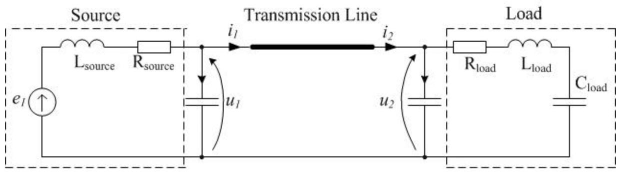

Fig. 1. Equivalent electric circuit for the analysis of transient processes of a transmission line.

To obtain the experimental data, the transmission line with following parameters was tested: Unom = 220 kV, l = 100 km, L0 = 951,7 mkH/km, C0 = 11.9 nF/km, Zc = 282.8 Ohm. The no-load, short circuit and loaded modes under DC and AC supply voltages were simulated. The model was implemented in MATLAB/Simulink using built-in operators of mathematical operations.

As a result of the application of optimization approach, the macromodel of the transmission line different from a classic state-variable form of the model due to the presence of variables $\vec{x}^{(-35)}$ was obtained in the following form:

$$\begin{cases} \mathbf{x}^{(k+1)} = \begin{pmatrix} 0 & 0.99 \\ -0.99 & 0 \end{pmatrix}\mathbf{x}^{(k-35)} + \begin{pmatrix} 0.001 & 0 \\ 0 & 0.2832 \end{pmatrix}\mathbf{v}^{(k)} \\ \mathbf{y}^{(k+1)} = \begin{pmatrix} 0 & 6.999 \\ 1977 & 0 \end{pmatrix}\mathbf{x}^{(k-35)} + \begin{pmatrix} 0.003535 & 0 \\ 0 & -282.5 \end{pmatrix}\mathbf{v}^{(k+1)} \end{cases} \qquad (4)$$

Voltage u1 and current i_2 were input variables, and current i1 and voltage u_2 were used as output variables during the macromodel creation. So, the vectors of input and output variables have the following form: $\vec{v} = (u_1, i_2)$, $\vec{y} = (i_1, u_2)$. The obtained model has a simple structure, which in the case of its use as a component of complex electrical circuits can enable fast modeling of their dynamic modes by modern software tools.

3. Adaptation of mathematical macromodels of transmission lines to modern software

In SimPowerSystem BlockSet of MATLAB/Simulink, such models of transmission line as a single- and three-phase line in the form of the Pi-equivalent circuit with lumped parameters and a multiphase line with distributed parameters and lumped losses are represented. The development of User-Defined Library with macromodels of elements, including transmission lines in the form of equation (1) or (2) will make it possible to conduct effective modeling of systems with models of real elements used in electric systems or networks.

The Discrete State-Space Unit intended for creating the model of the dynamic object is embedded into Simulink toolboxes and has the following mathematical form based on state space variables equations:

$$\begin{aligned} \vec{x}(\boldsymbol{n}+1) &= \mathbf{A}\vec{x}(\boldsymbol{n}) + \mathbf{B}\vec{u}(\boldsymbol{n}); \\ \vec{y}(\boldsymbol{n}) &= \mathbf{C}\vec{x}(\boldsymbol{n}) + \mathbf{D}\vec{u}(\boldsymbol{n}), \end{aligned} \tag{5}$$

where $\vec{x}$ is state variables vector; $\vec{u}$ is vector of input variables, $\vec{y}$ is vector of output signals, **A, B, C, D** are system, input, output and detour matrices respectively; n is the number of discrete.

Nevertheless, usage of Discrete State-Space block in the form of equation (4) is not suitable for the creation of nonlinear object models or models with a different structure. Therefore, for the adaptation of macromodels of electric engineering objects to MATLAB/Simulink softare it is necessary to use the technique of S-functions [6, 7].

Simulink-functions (S-functions) is a description of the block with one of the following programming languages: MATLAB, C, C^{++}, Ada or Fortran. Using these programming languages, we can create a description of the macromodel block of a required object and connect it with Simulink-model. In this case, the block created using S-function will not be different from a typical Simulink library block. Created blocks are typical S-functions compilled into executed (***.dll**) files which ensure high effectiveness of the models created on the basis of these blocks.

S-function block connected to the Simulink model is a typical library **S-function block** (from the **Functions&Tables** library).

Simulink-block is described unambiguously by sets of input variables $\vec{u}$, state variables $\vec{x}$ and output variables $\vec{y}$ as it is shown in Fig. 2.

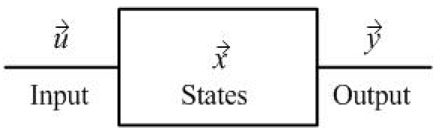

*Fig. 2. General view of the **Simulink**-block.*

The S-function block in mathematical form can be in general described using the following set of equations:

$$\vec{y} = f(t, \vec{x}, \vec{u});$$

$$\dot{\vec{x}}_C = f_d(t, \vec{x}, \vec{u});$$

$$x_{d_{k+1}} = f_u(t, \vec{x}, \vec{u}),$$

where $\vec{x} = \vec{x}_c + \vec{x}_d$ is a vector of input variables, $\vec{y}$ is a vector of output variables, $\dot{\vec{x}}_C$ are derivatives of continuos state variables, $x_{d_{k+1}}$ are discrete state variables.

The fragment of the program from the S-functions programming block with matrices **A, B, C, D** of the discrete macromodel is shown in Fig. 3.

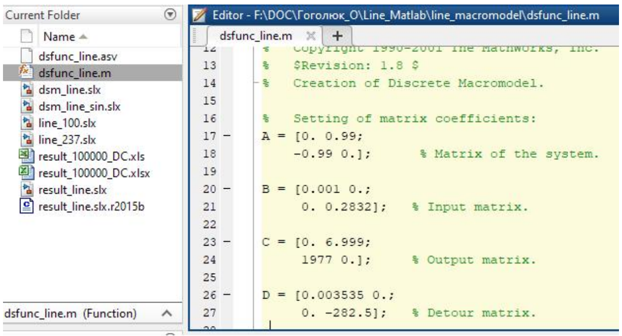

Fig. 3. Fragment of the block of the programmed S-function.

Thus, the electric circuit containing the transmission line which is shown in Fig. 1, will be implemented in Simulink environment in the form of the model where source and load will be traditional continuous elements and the transmission line is a discrete macromodel created using the programmable S-Function block and additional Controlled Voltage Sources as it is shown in Fig. 4.

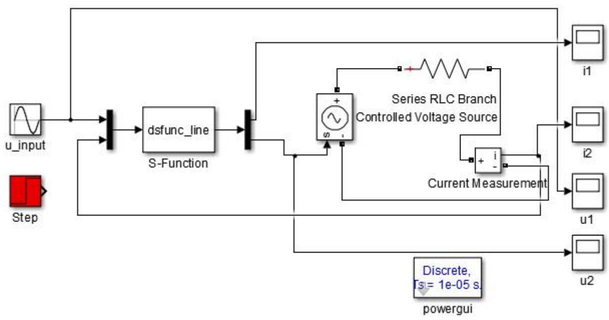

Fig. 4. Simulink model of electric system with transmission line macromodel.

The results of the simulation of this circuit when DC voltage is applyied to the line in no-load mode are shown in Fig. 5.

The simulation results of the transmission line described by the developed discrete macromodel and implemented into Simulink using the S-function block were compared with transient waveforms obtained with the model described in [4], and the comparison showed the full compliance of the obtained results.

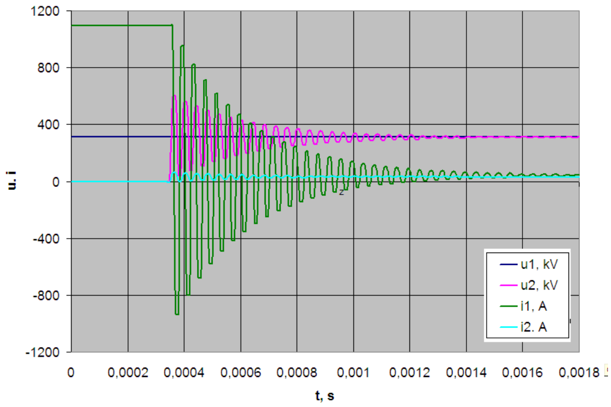

Fig. 5. Transient waveforms of the electric system under applied DC voltage.

4. Conclusions

Using the proposed method of adaptating the discrete mathematical macromodels in the form of programmable S-functions described using the example of the transmission line, the User can create its own library containing sepearate elements of electric circuits, as well as complex electrical systems in the form of the "black box" using state-space variables.

The use of discrete macromodels based on proposed method and their computer implementation will significantly reduce the computing time during its further implementation as a component for the analysis of electrical systems.

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АДАПТАЦІЯ ДИСКРЕТНИХ МАКРОМОДЕЛЕЙ ЛІНІЙ ЕЛЕКТРОПЕРЕСИЛАННЯ ДО СУЧАСНИХ КОМП'ЮТЕРНИХ СЕРЕДОВИЩ

Оксана Гоголюк

Розглянуто методи побудови математичних моделей ліній електропересилання. Запропоновано здійснювати їх побудову на основі експериментальних даних, отриманих шляхом комп'ютерного чи натурного експерименту у вигляді "чорної скриньки" у формі змінних стану. Отриману систему дискретних рівнянь у формі змінних стану, еквівалентну системі звичайних диференційних рівнянь, адаптовано до середовища MATLAB/Simulink шляхом програмування математичної форми моделі з використанням блоку S-функцій. Наведено результати комп'ютерного моделювання, отримані за допомогою дискретної макромоделі та описано можливості її подальшого використання.

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